

# Computer algebra independent integration tests

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/6.2.2-e-x<sup>m</sup>+b-x<sup>n</sup><sup>p</sup>-cosh

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3.55	$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4} dx$	293
3.56	$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} dx$	298
3.57	$\int \frac{x^4 \cosh(c+dx)}{a+bx^2} dx$	303
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3.75	$\int \frac{\cosh(c+dx)}{(a+bx^2)^3} dx$	403
3.76	$\int \frac{\cosh(c+dx)}{x(a+bx^2)^3} dx$	410
3.77	$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3} dx$	418
3.78	$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)^3} dx$	426
3.79	$\int x^3 (a + bx^3) \cosh(c + dx) dx$	434
3.80	$\int x^2 (a + bx^3) \cosh(c + dx) dx$	439
3.81	$\int x (a + bx^3) \cosh(c + dx) dx$	443
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3.84	$\int \frac{(a+bx^3) \cosh(c+dx)}{x^2} dx$	455
3.85	$\int \frac{(a+bx^3) \cosh(c+dx)}{x^3} dx$	459
3.86	$\int \frac{(a+bx^3) \cosh(c+dx)}{x^4} dx$	463
3.87	$\int x (a + bx^3)^2 \cosh(c + dx) dx$	467
3.88	$\int (a + bx^3)^2 \cosh(c + dx) dx$	472
3.89	$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x} dx$	477
3.90	$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx$	482
3.91	$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3} dx$	487

3.92	$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4} dx$	492
3.93	$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} dx$	497
3.94	$\int \frac{x^4 \cosh(c+dx)}{a+bx^3} dx$	502
3.95	$\int \frac{x^3 \cosh(c+dx)}{a+bx^3} dx$	508
3.96	$\int \frac{x^2 \cosh(c+dx)}{a+bx^3} dx$	514
3.97	$\int \frac{x \cosh(c+dx)}{a+bx^3} dx$	519
3.98	$\int \frac{\cosh(c+dx)}{a+bx^3} dx$	524
3.99	$\int \frac{\cosh(c+dx)}{x(a+bx^3)} dx$	529
3.100	$\int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx$	534
3.101	$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)} dx$	539
3.102	$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2} dx$	545
3.103	$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} dx$	552
3.104	$\int \frac{x \cosh(c+dx)}{(a+bx^3)^2} dx$	558
3.105	$\int \frac{\cosh(c+dx)}{(a+bx^3)^2} dx$	565
3.106	$\int \frac{\cosh(c+dx)}{x(a+bx^3)^2} dx$	572
3.107	$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} dx$	579
3.108	$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} dx$	588
3.109	$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} dx$	598
3.110	$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3} dx$	606
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#### 4 Listing of Grading functions

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 111 ]. This is test number [ 166 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 111 )	% 0. ( 0 )
Mathematica	% 100. ( 111 )	% 0. ( 0 )
Maple	% 100. ( 111 )	% 0. ( 0 )
Maxima	% 57.66 ( 64 )	% 42.34 ( 47 )
Fricas	% 100. ( 111 )	% 0. ( 0 )
Sympy	% 23.42 ( 26 )	% 76.58 ( 85 )
Giac	% 63.96 ( 71 )	% 36.04 ( 40 )

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

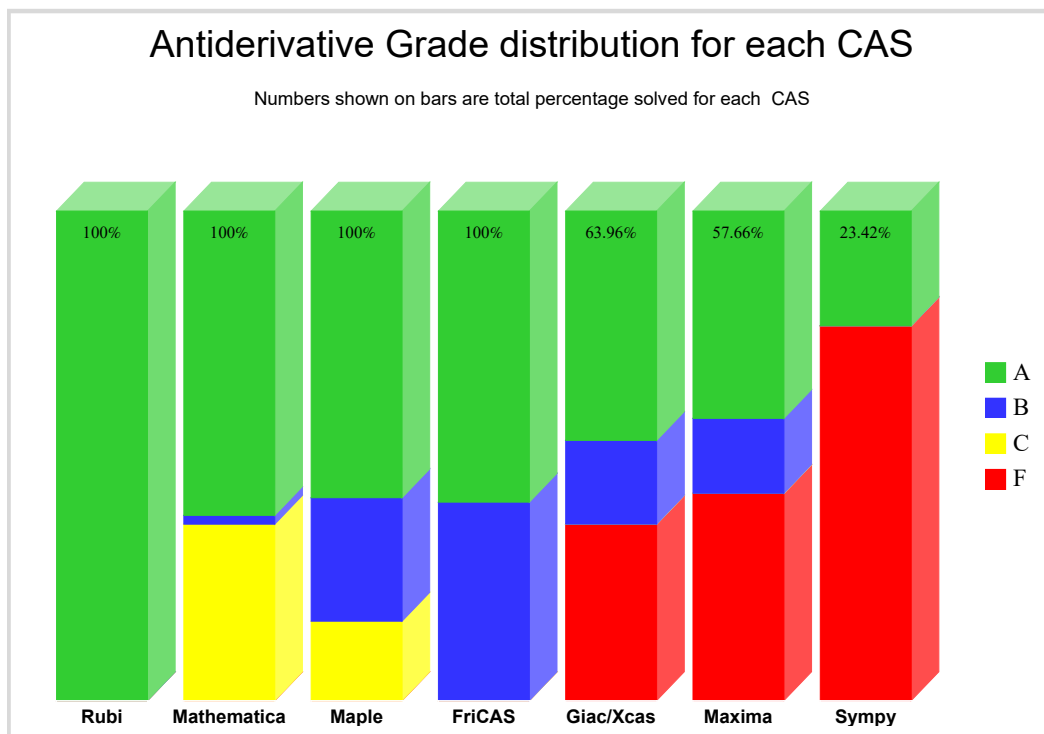


grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

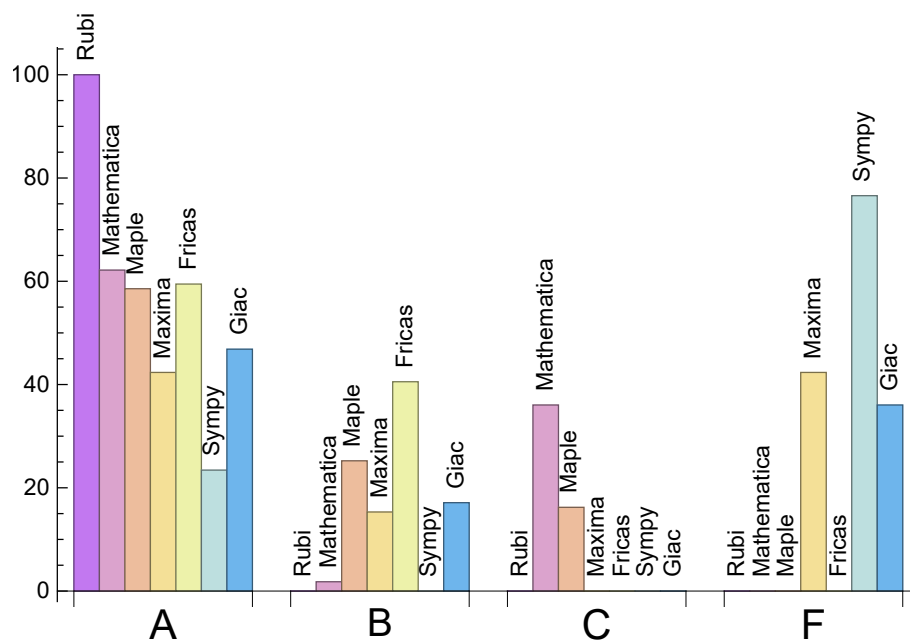
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	62.16	1.8	36.04	0.
Maple	58.56	25.23	16.22	0.
Maxima	42.34	15.32	0.	42.34
Fricas	59.46	40.54	0.	0.
Sympy	23.42	0.	0.	76.58
Giac	46.85	17.12	0.	36.04

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.55	268.55	1.	178.	1.
Mathematica	0.69	289.74	0.97	143.	0.86
Maple	0.06	412.83	1.76	288.	1.69
Maxima	1.24	247.95	2.25	229.5	2.3
Fricas	2.12	1453.68	4.07	439.	3.78
Sympy	4.78	138.96	1.27	127.5	1.23
Giac	1.21	322.94	2.28	244.	2.16

## 1.4 list of integrals that has no closed form antiderivative

{

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 78, 104, 106}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

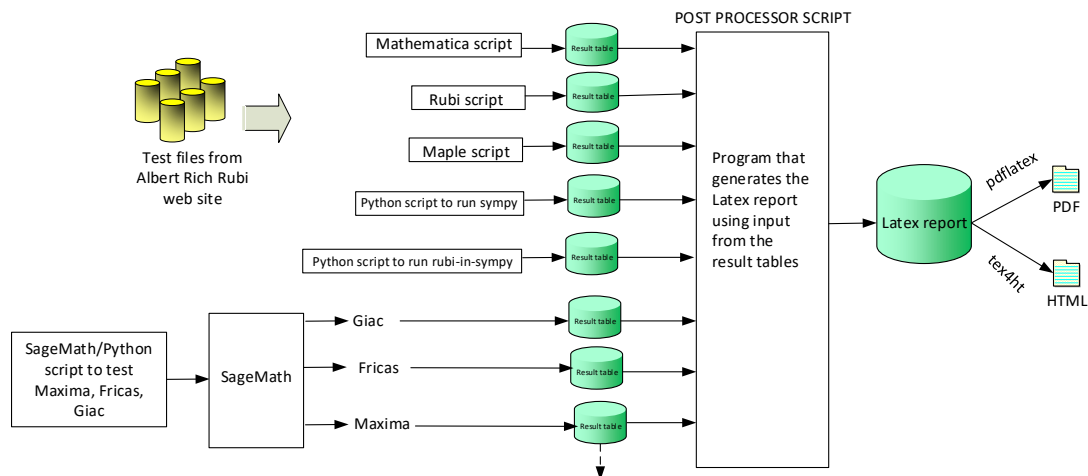
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

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# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 37, 38 }

C grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F grade: { }

## 2.1.3 Maple

A grade: { 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 37, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 83, 84, 85, 86, 91, 92, 93 }

B grade: { 1, 2, 10, 11, 12, 33, 34, 35, 36, 38, 39, 40, 41, 42, 49, 50, 51, 52, 72, 78, 79, 80, 81, 82, 87, 88, 89, 90 }

C grade: { 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F grade: { }

## 2.1.4 Maxima

A grade: { 1, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 22, 24, 25, 26, 27, 28, 29, 30, 31, 36, 40, 41, 43, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 79, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 2, 3, 4, 5, 11, 12, 13, 19, 20, 21, 23, 42, 44, 52, 80, 81, 83 }

C grade: { }

F grade: { 32, 33, 34, 35, 37, 38, 39, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 62, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade: { 30, 33, 35, 36, 37, 38, 39, 57, 58, 59, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

C grade: { }

F grade: { }

## 2.1.6 SymPy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 40, 41, 42, 43, 44, 49, 50, 51, 52, 79, 80, 81, 82, 83, 87, 88, 89 }

B grade: { }

C grade: { }

F grade: { 6, 7, 8, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, }

72, 73, 74, 75, 76, 77, 78, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

## 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 79, 80, 81, 82, 83, 85, 86, 87, 88, 92 }

B grade: { 12, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 52, 53, 84, 89, 90, 91, 93 }

C grade: { }

F grade: { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	82	356	313	193	151	205
normalized size	1	1.	0.66	2.87	2.52	1.56	1.22	1.65
time (sec)	N/A	0.321	0.14	0.011	1.149	1.942	3.639	1.213

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	65	224	265	151	117	157
normalized size	1	1.	0.69	2.38	2.82	1.61	1.24	1.67
time (sec)	N/A	0.217	0.117	0.007	1.124	1.923	2.242	1.164

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	45	122	216	111	82	107
normalized size	1	1.	0.7	1.91	3.38	1.73	1.28	1.67
time (sec)	N/A	0.114	0.088	0.008	1.13	1.919	1.95	1.224

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	53	92	73	46	62
normalized size	1	1.	0.96	1.89	3.29	2.61	1.64	2.21
time (sec)	N/A	0.021	0.051	0.009	1.139	1.983	0.629	1.203

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	39	52	131	142	34	63
normalized size	1	1.	1.39	1.86	4.68	5.07	1.21	2.25
time (sec)	N/A	0.149	0.028	0.038	1.201	1.967	4.547	1.182

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	59	77	111	186	0	97
normalized size	1	1.	1.26	1.64	2.36	3.96	0.	2.06
time (sec)	N/A	0.228	0.127	0.039	1.397	1.968	0.	1.151

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	78	139	89	278	0	181
normalized size	1	1.	0.89	1.58	1.01	3.16	0.	2.06
time (sec)	N/A	0.281	0.155	0.047	1.417	1.994	0.	1.152

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	205	105	328	0	269
normalized size	1	1.	0.83	1.55	0.8	2.48	0.	2.04
time (sec)	N/A	0.34	0.272	0.053	1.395	1.98	0.	1.148

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	140	271	111	366	0	359
normalized size	1	1.	0.84	1.63	0.67	2.2	0.	2.16
time (sec)	N/A	0.406	0.318	0.062	1.399	2.021	0.	1.265

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	100	463	444	273	228	319
normalized size	1	1.	0.54	2.52	2.41	1.48	1.24	1.73
time (sec)	N/A	0.346	0.242	0.007	1.184	2.019	3.531	1.177

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	87	283	371	203	172	231
normalized size	1	1.	0.65	2.11	2.77	1.51	1.28	1.72
time (sec)	N/A	0.214	0.188	0.01	1.176	2.008	2.114	1.143

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	56	147	182	140	112	151
normalized size	1	1.	1.14	3.	3.71	2.86	2.29	3.08
time (sec)	N/A	0.048	0.142	0.007	1.206	1.949	1.064	1.215

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	51	121	236	221	73	153
normalized size	1	1.	0.82	1.95	3.81	3.56	1.18	2.47
time (sec)	N/A	0.185	0.235	0.036	1.34	1.971	5.077	1.263

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	62	118	184	284	0	161
normalized size	1	1.	0.89	1.69	2.63	4.06	0.	2.3
time (sec)	N/A	0.247	0.23	0.049	1.393	1.994	0.	1.239

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	93	181	170	351	0	244
normalized size	1	1.	0.77	1.5	1.4	2.9	0.	2.02
time (sec)	N/A	0.339	0.373	0.052	1.372	1.95	0.	1.178

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	154	287	158	431	0	385
normalized size	1	1.	0.9	1.67	0.92	2.51	0.	2.24
time (sec)	N/A	0.426	0.448	0.06	1.431	2.016	0.	1.152

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	206	396	173	510	0	533
normalized size	1	1.	0.83	1.6	0.7	2.06	0.	2.15
time (sec)	N/A	0.521	0.524	0.072	1.398	1.973	0.	1.115

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	159	442	590	481	0	84
normalized size	1	1.	0.73	2.02	2.69	2.2	0.	0.38
time (sec)	N/A	0.484	0.631	0.133	1.448	2.093	0.	1.149

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	118	292	443	392	0	84
normalized size	1	1.	0.79	1.95	2.95	2.61	0.	0.56
time (sec)	N/A	0.329	0.43	0.034	1.276	2.094	0.	1.176

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	89	184	315	327	0	84
normalized size	1	1.	0.89	1.84	3.15	3.27	0.	0.84
time (sec)	N/A	0.262	0.283	0.03	1.378	2.156	0.	1.207

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	64	114	211	251	0	78
normalized size	1	1.	0.94	1.68	3.1	3.69	0.	1.15
time (sec)	N/A	0.176	0.136	0.023	1.333	1.997	0.	1.186

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	81	77	193	0	76
normalized size	1	1.	0.96	1.59	1.51	3.78	0.	1.49
time (sec)	N/A	0.08	0.061	0.019	1.316	2.086	0.	1.171

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	108	209	277	0	101
normalized size	1	1.	0.86	1.48	2.86	3.79	0.	1.38
time (sec)	N/A	0.259	0.141	0.031	1.437	2.066	0.	1.159

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	101	172	259	400	0	174
normalized size	1	1.	0.89	1.52	2.29	3.54	0.	1.54
time (sec)	N/A	0.369	0.381	0.048	1.456	2.047	0.	1.172

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	178	281	327	591	0	335
normalized size	1	1.	0.94	1.48	1.72	3.11	0.	1.76
time (sec)	N/A	0.485	0.487	0.05	1.626	2.085	0.	1.205

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	173	431	548	740	0	393
normalized size	1	1.	0.75	1.87	2.37	3.2	0.	1.7
time (sec)	N/A	0.536	1.201	0.122	1.516	2.08	0.	1.187

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	156	325	421	663	0	393
normalized size	1	1.	0.86	1.79	2.31	3.64	0.	2.16
time (sec)	N/A	0.423	0.914	0.046	1.343	2.033	0.	1.186



Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	115	254	319	570	0	387
normalized size	1	1.	0.78	1.73	2.17	3.88	0.	2.63
time (sec)	N/A	0.375	0.692	0.039	1.348	1.992	0.	1.229

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	97	215	240	414	0	373
normalized size	1	1.	0.78	1.72	1.92	3.31	0.	2.98
time (sec)	N/A	0.297	0.414	0.027	1.363	2.034	0.	1.24

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	132	109	316	0	201
normalized size	1	1.	0.92	1.86	1.54	4.45	0.	2.83
time (sec)	N/A	0.11	0.202	0.025	1.209	2.037	0.	1.211

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	241	254	306	579	0	439
normalized size	1	1.	1.61	1.69	2.04	3.86	0.	2.93
time (sec)	N/A	0.4	1.05	0.05	1.432	2.151	0.	1.211

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	183	312	0	802	0	576
normalized size	1	1.	0.98	1.68	0.	4.31	0.	3.1
time (sec)	N/A	0.501	1.414	0.058	0.	2.11	0.	1.286

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	236	571	0	1142	0	1019
normalized size	1	1.	0.89	2.16	0.	4.33	0.	3.86
time (sec)	N/A	0.618	1.	0.117	0.	2.509	0.	1.161

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	153	527	0	973	0	1000
normalized size	1	1.	0.63	2.19	0.	4.04	0.	4.15
time (sec)	N/A	0.53	0.928	0.047	0.	2.473	0.	1.181

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	158	435	0	770	0	714
normalized size	1	1.	0.89	2.44	0.	4.33	0.	4.01
time (sec)	N/A	0.37	0.578	0.036	0.	2.236	0.	1.234

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	276	128	518	0	402
normalized size	1	1.	0.85	2.65	1.23	4.98	0.	3.87
time (sec)	N/A	0.146	0.467	0.028	1.19	1.956	0.	1.183

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	614	488	0	1243	0	1130
normalized size	1	1.	2.34	1.86	0.	4.74	0.	4.31
time (sec)	N/A	0.561	4.592	0.055	0.	2.175	0.	1.211

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	710	643	0	1563	0	1358
normalized size	1	1.	2.38	2.16	0.	5.24	0.	4.56
time (sec)	N/A	0.69	1.44	0.073	0.	2.2	0.	1.253

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	627	760	0	1845	0	1578
normalized size	1	1.	1.66	2.02	0.	4.89	0.	4.19
time (sec)	N/A	0.821	1.807	0.082	0.	2.182	0.	1.319

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	92	447	338	212	168	235
normalized size	1	1.	0.66	3.22	2.43	1.53	1.21	1.69
time (sec)	N/A	0.255	0.149	0.009	1.071	2.016	5.485	1.204

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	74	298	289	174	134	186
normalized size	1	1.	0.68	2.73	2.65	1.6	1.23	1.71
time (sec)	N/A	0.187	0.129	0.009	1.03	2.026	2.615	1.198

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	57	183	288	132	99	136
normalized size	1	1.	0.72	2.32	3.65	1.67	1.25	1.72
time (sec)	N/A	0.121	0.1	0.007	1.086	1.984	1.32	1.178

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	97	116	97	65	95
normalized size	1	1.	0.78	1.9	2.27	1.9	1.27	1.86
time (sec)	N/A	0.067	0.067	0.008	1.019	1.995	0.674	1.153

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	81	165	188	49	103
normalized size	1	1.	1.34	1.98	4.02	4.59	1.2	2.51
time (sec)	N/A	0.098	0.11	0.035	1.169	2.063	3.565	1.176

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	81	108	203	0	108
normalized size	1	1.	1.	1.93	2.57	4.83	0.	2.57
time (sec)	N/A	0.107	0.088	0.046	1.188	2.084	0.	1.222

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	80	110	122	254	0	147
normalized size	1	1.	1.08	1.49	1.65	3.43	0.	1.99
time (sec)	N/A	0.178	0.147	0.044	1.191	2.023	0.	1.16

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	95	172	99	298	0	230
normalized size	1	1.	0.9	1.64	0.94	2.84	0.	2.19
time (sec)	N/A	0.234	0.225	0.059	1.213	1.98	0.	1.183

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	127	238	103	352	0	320
normalized size	1	1.	0.85	1.6	0.69	2.36	0.	2.15
time (sec)	N/A	0.29	0.274	0.066	1.213	2.007	0.	1.199

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	138	738	517	336	286	410
normalized size	1	1.	0.59	3.15	2.21	1.44	1.22	1.75
time (sec)	N/A	0.388	0.321	0.009	1.055	2.059	10.16	1.218

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	113	513	477	273	226	323
normalized size	1	1.	0.61	2.79	2.59	1.48	1.23	1.76
time (sec)	N/A	0.277	0.23	0.008	1.078	2.084	5.259	1.172

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	85	332	255	207	172	243
normalized size	1	1.	0.62	2.44	1.88	1.52	1.26	1.79
time (sec)	N/A	0.181	0.179	0.008	1.039	2.181	2.954	1.175

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	82	226	317	292	121	300
normalized size	1	1.	0.75	2.05	2.88	2.65	1.1	2.73
time (sec)	N/A	0.189	0.361	0.05	1.187	2.477	5.733	1.201

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	190	242	286	0	266
normalized size	1	1.	1.	2.	2.55	3.01	0.	2.8
time (sec)	N/A	0.177	0.263	0.069	1.204	2.014	0.	1.199

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	97	188	223	359	0	278
normalized size	1	1.	0.85	1.65	1.96	3.15	0.	2.44
time (sec)	N/A	0.225	0.348	0.084	1.347	1.993	0.	1.22

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	114	222	182	381	0	319
normalized size	1	1.	0.86	1.67	1.37	2.86	0.	2.4
time (sec)	N/A	0.259	0.402	0.092	1.237	2.044	0.	1.158

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	124	291	188	439	0	397
normalized size	1	1.	0.71	1.66	1.07	2.51	0.	2.27
time (sec)	N/A	0.36	0.439	0.116	1.277	1.974	0.	1.175

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	273	273	274	369	0	1296	0	0
normalized size	1	1.	1.	1.35	0.	4.75	0.	0.
time (sec)	N/A	0.729	0.419	0.189	0.	2.195	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	210	268	0	1100	0	0
normalized size	1	1.	1.	1.28	0.	5.26	0.	0.
time (sec)	N/A	0.358	0.349	0.06	0.	2.103	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	213	259	0	1087	0	0
normalized size	1	1.	0.94	1.15	0.	4.81	0.	0.
time (sec)	N/A	0.376	0.252	0.045	0.	2.169	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	171	200	0	455	0	0
normalized size	1	1.	0.97	1.13	0.	2.57	0.	0.
time (sec)	N/A	0.254	0.175	0.032	0.	2.092	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	213	213	180	212	0	624	0	0
normalized size	1	1.	0.85	1.	0.	2.93	0.	0.
time (sec)	N/A	0.259	0.201	0.021	0.	2.157	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	187	227	0	545	0	0
normalized size	1	1.	0.95	1.15	0.	2.77	0.	0.
time (sec)	N/A	0.372	0.31	0.043	0.	2.203	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	249	249	243	288	0	1323	0	0
normalized size	1	1.	0.98	1.16	0.	5.31	0.	0.
time (sec)	N/A	0.496	0.357	0.055	0.	2.499	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	257	330	0	1283	0	0
normalized size	1	1.	0.95	1.22	0.	4.75	0.	0.
time (sec)	N/A	0.504	0.53	0.063	0.	2.51	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	449	449	621	532	0	2488	0	0
normalized size	1	1.	1.38	1.18	0.	5.54	0.	0.
time (sec)	N/A	0.859	1.507	0.319	0.	2.313	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	431	431	582	495	0	2099	0	0
normalized size	1	1.	1.35	1.15	0.	4.87	0.	0.
time (sec)	N/A	0.701	0.74	0.095	0.	2.153	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	416	416	364	491	0	2433	0	0
normalized size	1	1.	0.88	1.18	0.	5.85	0.	0.
time (sec)	N/A	0.611	0.992	0.068	0.	2.196	0.	0.



Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	239	291	0	1393	0	0
normalized size	1	1.	1.	1.22	0.	5.83	0.	0.
time (sec)	N/A	0.325	0.514	0.044	0.	2.167	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	476	476	590	503	0	2431	0	0
normalized size	1	1.	1.24	1.06	0.	5.11	0.	0.
time (sec)	N/A	0.818	0.688	0.029	0.	2.274	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	363	546	0	2253	0	0
normalized size	1	1.	0.83	1.26	0.	5.18	0.	0.
time (sec)	N/A	0.839	4.804	0.074	0.	2.297	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	500	500	675	595	0	2786	0	0
normalized size	1	1.	1.35	1.19	0.	5.57	0.	0.
time (sec)	N/A	1.253	1.036	0.086	0.	2.337	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	476	476	648	820	0	3344	0	0
normalized size	1	1.	1.36	1.72	0.	7.03	0.	0.
time (sec)	N/A	1.062	1.824	0.376	0.	2.408	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	746	746	932	1064	0	4169	0	0
normalized size	1	1.	1.25	1.43	0.	5.59	0.	0.
time (sec)	N/A	1.1	2.509	0.108	0.	2.397	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	512	512	637	743	0	3306	0	0
normalized size	1	1.	1.24	1.45	0.	6.46	0.	0.
time (sec)	N/A	0.831	1.712	0.069	0.	2.358	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	856	856	933	1064	0	4342	0	0
normalized size	1	1.	1.09	1.24	0.	5.07	0.	0.
time (sec)	N/A	1.255	2.384	0.037	0.	2.333	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	730	730	1558	1090	0	4301	0	0
normalized size	1	1.	2.13	1.49	0.	5.89	0.	0.
time (sec)	N/A	1.682	3.262	0.105	0.	2.152	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	874	874	1359	1178	0	4894	0	0
normalized size	1	1.	1.55	1.35	0.	5.6	0.	0.
time (sec)	N/A	2.689	3.378	0.128	0.	2.172	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	791	791	998	1294	0	4891	0	0
normalized size	1	1.	1.26	1.64	0.	6.18	0.	0.
time (sec)	N/A	1.868	3.717	0.152	0.	2.207	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	100	551	362	240	185	259
normalized size	1	1.	0.65	3.58	2.35	1.56	1.2	1.68
time (sec)	N/A	0.297	0.161	0.008	1.046	1.764	9.675	1.14

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	84	389	360	200	151	211
normalized size	1	1.	0.68	3.14	2.9	1.61	1.22	1.7
time (sec)	N/A	0.228	0.133	0.007	1.042	1.733	6.047	1.189

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	66	257	265	155	116	161
normalized size	1	1.	0.7	2.73	2.82	1.65	1.23	1.71
time (sec)	N/A	0.159	0.11	0.007	1.042	1.719	3.262	1.368

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	49	158	140	119	82	119
normalized size	1	1.	0.74	2.39	2.12	1.8	1.24	1.8
time (sec)	N/A	0.103	0.081	0.007	1.029	1.813	1.408	1.224

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	49	113	189	211	66	147
normalized size	1	1.	0.88	2.02	3.38	3.77	1.18	2.62
time (sec)	N/A	0.128	0.164	0.032	1.231	1.7	4.863	1.273

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	110	138	223	0	150
normalized size	1	1.	1.	2.	2.51	4.05	0.	2.73
time (sec)	N/A	0.129	0.131	0.054	1.159	1.822	0.	1.32

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	86	114	117	238	0	159
normalized size	1	1.	1.25	1.65	1.7	3.45	0.	2.3
time (sec)	N/A	0.139	0.141	0.056	1.177	1.833	0.	1.256

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	73	143	128	277	0	190
normalized size	1	1.	0.8	1.57	1.41	3.04	0.	2.09
time (sec)	N/A	0.215	0.238	0.066	1.23	1.728	0.	1.272

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	139	818	517	358	284	409
normalized size	1	1.	0.59	3.5	2.21	1.53	1.21	1.75
time (sec)	N/A	0.395	0.291	0.008	1.071	1.759	15.388	1.233

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	111	592	328	285	226	329
normalized size	1	1.	0.6	3.18	1.76	1.53	1.22	1.77
time (sec)	N/A	0.287	0.22	0.009	1.046	1.741	9.363	1.381

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	108	335	390	365	168	447
normalized size	1	1.	0.68	2.09	2.44	2.28	1.05	2.79
time (sec)	N/A	0.282	0.468	0.086	1.233	1.786	11.71	1.213

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	143	296	317	358	0	416
normalized size	1	1.	1.	2.07	2.22	2.5	0.	2.91
time (sec)	N/A	0.252	0.348	0.102	1.197	1.775	0.	1.261

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	136	265	274	351	0	378
normalized size	1	1.	0.96	1.88	1.94	2.49	0.	2.68
time (sec)	N/A	0.233	0.348	0.115	1.2	1.752	0.	1.336

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	135	261	254	412	0	377
normalized size	1	1.	0.9	1.74	1.69	2.75	0.	2.51
time (sec)	N/A	0.279	0.57	0.128	1.251	1.76	0.	1.335

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	150	292	208	435	0	425
normalized size	1	1.	0.9	1.75	1.25	2.6	0.	2.54
time (sec)	N/A	0.308	0.539	0.174	1.247	1.804	0.	1.333

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	213	925	0	2627	0	0
normalized size	1	1.	0.57	2.48	0.	7.04	0.	0.
time (sec)	N/A	0.937	0.462	0.087	0.	2.136	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	198	671	0	2591	0	0
normalized size	1	1.	0.55	1.87	0.	7.24	0.	0.
time (sec)	N/A	0.648	0.26	0.036	0.	2.203	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	170	423	0	1322	0	0
normalized size	1	1.	0.6	1.49	0.	4.67	0.	0.
time (sec)	N/A	0.455	0.225	0.032	0.	1.952	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	180	280	0	1773	0	0
normalized size	1	1.	0.52	0.81	0.	5.14	0.	0.
time (sec)	N/A	0.412	0.211	0.027	0.	2.138	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	180	143	0	1770	0	0
normalized size	1	1.	0.52	0.41	0.	5.13	0.	0.
time (sec)	N/A	0.401	0.136	0.022	0.	2.042	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	186	138	0	1412	0	0
normalized size	1	1.	0.61	0.46	0.	4.66	0.	0.
time (sec)	N/A	0.523	0.268	0.036	0.	2.044	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	215	187	0	2921	0	0
normalized size	1	1.	0.56	0.49	0.	7.67	0.	0.
time (sec)	N/A	0.6	0.407	0.047	0.	2.189	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	237	240	0	3071	0	0
normalized size	1	1.	0.58	0.59	0.	7.49	0.	0.
time (sec)	N/A	0.639	0.377	0.054	0.	2.13	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	718	363	877	0	5017	0	0
normalized size	1	1.	0.51	1.22	0.	6.99	0.	0.
time (sec)	N/A	1.081	0.336	0.15	0.	2.408	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	203	594	0	3200	0	0
normalized size	1	1.	0.54	1.59	0.	8.58	0.	0.
time (sec)	N/A	0.601	0.16	0.044	0.	2.22	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	695	695	387	395	0	4891	0	0
normalized size	1	1.	0.56	0.57	0.	7.04	0.	0.
time (sec)	N/A	1.324	0.188	0.037	0.	2.395	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	739	739	387	226	0	5042	0	0
normalized size	1	1.	0.52	0.31	0.	6.82	0.	0.
time (sec)	N/A	1.323	0.194	0.026	0.	2.35	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	697	697	5530	338	0	4420	0	0
normalized size	1	1.	7.93	0.48	0.	6.34	0.	0.
time (sec)	N/A	1.472	8.611	0.072	0.	2.327	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	784	784	397	2448	0	6886	0	0
normalized size	1	1.	0.51	3.12	0.	8.78	0.	0.
time (sec)	N/A	1.626	0.595	0.52	0.	3.099	0.	0.



Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1105	1105	675	1927	0	10106	0	0
normalized size	1	1.	0.61	1.74	0.	9.15	0.	0.
time (sec)	N/A	1.853	0.524	0.143	0.	3.476	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	776	776	429	1456	0	6494	0	0
normalized size	1	1.	0.55	1.88	0.	8.37	0.	0.
time (sec)	N/A	2.659	0.636	0.112	0.	2.639	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	781	781	423	994	0	6869	0	0
normalized size	1	1.	0.54	1.27	0.	8.8	0.	0.
time (sec)	N/A	1.449	0.422	0.086	0.	2.537	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1147	1147	669	810	0	10103	0	0
normalized size	1	1.	0.58	0.71	0.	8.81	0.	0.
time (sec)	N/A	3.198	0.458	0.063	0.	3.085	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [111] had the largest ratio of [ 0.5294 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	4	1.	15	0.267
2	A	9	4	1.	15	0.267
3	A	7	4	1.	13	0.308
4	A	2	2	1.	12	0.167
5	A	6	5	1.	15	0.333
6	A	9	5	1.	15	0.333
7	A	11	5	1.	15	0.333
8	A	13	5	1.	15	0.333
9	A	15	5	1.	15	0.333
10	A	14	4	1.	17	0.235
11	A	11	4	1.	15	0.267
12	A	3	2	1.	14	0.143
13	A	8	7	1.	17	0.412
14	A	10	6	1.	17	0.353
15	A	14	5	1.	17	0.294
16	A	17	5	1.	17	0.294
17	A	20	5	1.	17	0.294
18	A	15	7	1.	17	0.412
19	A	11	7	1.	17	0.412
20	A	8	7	1.	17	0.412
21	A	6	5	1.	15	0.333
22	A	3	3	1.	14	0.214
23	A	8	4	1.	17	0.235
24	A	12	5	1.	17	0.294
25	A	17	5	1.	17	0.294
26	A	15	8	1.	17	0.471
27	A	12	8	1.	17	0.471
28	A	10	6	1.	17	0.353
29	A	9	5	1.	15	0.333
30	A	4	4	1.	14	0.286
31	A	12	5	1.	17	0.294

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	16	5	1.	17	0.294
33	A	15	6	1.	17	0.353
34	A	14	5	1.	17	0.294
35	A	11	5	1.	15	0.333
36	A	5	4	1.	14	0.286
37	A	17	5	1.	17	0.294
38	A	21	5	1.	17	0.294
39	A	26	5	1.	17	0.294
40	A	12	3	1.	17	0.176
41	A	10	3	1.	17	0.176
42	A	8	3	1.	15	0.2
43	A	6	3	1.	14	0.214
44	A	7	6	1.	17	0.353
45	A	7	6	1.	17	0.353
46	A	10	5	1.	17	0.294
47	A	12	5	1.	17	0.294
48	A	14	5	1.	17	0.294
49	A	17	3	1.	19	0.158
50	A	14	3	1.	17	0.176
51	A	11	3	1.	16	0.188
52	A	11	6	1.	19	0.316
53	A	10	7	1.	19	0.368
54	A	12	7	1.	19	0.368
55	A	13	6	1.	19	0.316
56	A	17	5	1.	19	0.263
57	A	14	7	1.	19	0.368
58	A	12	6	1.	19	0.316
59	A	11	6	1.	19	0.316
60	A	8	4	1.	17	0.235
61	A	8	4	1.	16	0.25
62	A	13	4	1.	19	0.21
63	A	14	6	1.	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	18	5	1.	19	0.263
65	A	24	9	1.	19	0.474
66	A	20	8	1.	19	0.421
67	A	17	6	1.	19	0.316
68	A	9	5	1.	17	0.294
69	A	18	5	1.	16	0.312
70	A	22	6	1.	19	0.316
71	A	32	6	1.	19	0.316
72	A	27	8	1.	19	0.421
73	A	28	7	1.	19	0.368
74	A	19	6	1.	17	0.353
75	A	28	5	1.	16	0.312
76	A	41	7	1.	19	0.368
77	A	60	6	1.	19	0.316
78	A	46	7	1.	19	0.368
79	A	13	4	1.	17	0.235
80	A	11	4	1.	17	0.235
81	A	9	4	1.	15	0.267
82	A	7	4	1.	14	0.286
83	A	8	6	1.	17	0.353
84	A	8	7	1.	17	0.412
85	A	8	6	1.	17	0.353
86	A	11	5	1.	17	0.294
87	A	17	4	1.	17	0.235
88	A	14	4	1.	16	0.25
89	A	14	7	1.	19	0.368
90	A	13	8	1.	19	0.421
91	A	12	8	1.	19	0.421
92	A	14	7	1.	19	0.368
93	A	15	7	1.	19	0.368
94	A	15	6	1.	19	0.316
95	A	14	6	1.	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	11	4	1.	19	0.21
97	A	11	4	1.	17	0.235
98	A	11	4	1.	16	0.25
99	A	16	4	1.	19	0.21
100	A	17	5	1.	19	0.263
101	A	18	6	1.	19	0.316
102	A	23	6	1.	19	0.316
103	A	12	5	1.	19	0.263
104	A	34	7	1.	17	0.412
105	A	36	8	1.	16	0.5
106	A	41	8	1.	19	0.421
107	A	36	8	1.	19	0.421
108	A	47	9	1.	19	0.474
109	A	71	10	1.	19	0.526
110	A	37	9	1.	19	0.474
111	A	89	9	1.	17	0.529



# Chapter 3

## Listing of integrals

### 3.1 $\int x^3(a + bx) \cosh(c + dx) dx$

**Optimal.** Leaf size=124

$$-\frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{6a \cosh(c + dx)}{d^4} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} - \frac{4bx^3 \cosh(c + dx)}{d^2}$$

[Out]  $(-6*a*Cosh[c + d*x])/d^4 - (24*b*x*Cosh[c + d*x])/d^4 - (3*a*x^2*Cosh[c + d*x])/d^2 - (4*b*x^3*Cosh[c + d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (6*a*x*Sinh[c + d*x])/d^3 + (12*b*x^2*Sinh[c + d*x])/d^3 + (a*x^3*Sinh[c + d*x])/d + (b*x^4*Sinh[c + d*x])/d$

---

**Rubi [A]** time = 0.320678, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6742, 3296, 2638, 2637}

$$-\frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{6a \cosh(c + dx)}{d^4} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} - \frac{4bx^3 \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(a + b*x)*Cosh[c + d*x], x]$

[Out]  $(-6*a*Cosh[c + d*x])/d^4 - (24*b*x*Cosh[c + d*x])/d^4 - (3*a*x^2*Cosh[c + d*x])/d^2 - (4*b*x^3*Cosh[c + d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (6*a*x*Sinh[c + d*x])/d^3 + (12*b*x^2*Sinh[c + d*x])/d^3 + (a*x^3*Sinh[c + d*x])/d + (b*x^4*Sinh[c + d*x])/d$

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3(a + bx) \cosh(c + dx) dx &= \int (ax^3 \cosh(c + dx) + bx^4 \cosh(c + dx)) dx \\
&= a \int x^3 \cosh(c + dx) dx + b \int x^4 \cosh(c + dx) dx \\
&= \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} - \frac{(3a) \int x^2 \sinh(c + dx) dx}{d} - \frac{(4b) \int x^3 \sinh(c + dx) dx}{d} \\
&= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} + \frac{(6a)}{d^2} \\
&= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} + \frac{12bx^2 \sinh(c + dx)}{d^3} + \frac{ax}{d^3} \\
&= -\frac{6a \cosh(c + dx)}{d^4} - \frac{24bx \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{6ax}{d^3} \\
&= -\frac{6a \cosh(c + dx)}{d^4} - \frac{24bx \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{24bx \sinh(c + dx)}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.139607, size = 82, normalized size = 0.66

$$\frac{(ad^2x(d^2x^2 + 6) + b(d^4x^4 + 12d^2x^2 + 24)) \sinh(c + dx) - d(3a(d^2x^2 + 2) + 4bx(d^2x^2 + 6)) \cosh(c + dx)}{d^5}$$



Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*x)\*Cosh[c + d\*x],x]

[Out]  $(-(d*(3*a*(2 + d^2*x^2) + 4*b*x*(6 + d^2*x^2))*Cosh[c + d*x]) + (a*d^2*x*(6 + d^2*x^2) + b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5$

**Maple [B]** time = 0.011, size = 356, normalized size = 2.9

$$\frac{1}{d^4} \left( \frac{b((dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c))}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x+a)\*cosh(d\*x+c),x)

[Out]  $1/d^4*(b/d*((d*x+c)^4*\sinh(d*x+c)-4*(d*x+c)^3*\cosh(d*x+c)+12*(d*x+c)^2*\sinh(d*x+c)-24*(d*x+c)*\cosh(d*x+c)+24*\sinh(d*x+c))-4*b*c/d*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))+6*b/d*c^2*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-4*b/d*c^3*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+b*c^4/d*\sinh(d*x+c)+a*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))-3*a*c*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))+3*a*c^2*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))-a*c^3*\sinh(d*x+c))$

**Maxima [A]** time = 1.14939, size = 313, normalized size = 2.52

$$-\frac{1}{40} d \left( \frac{5(d^4 x^4 e^c - 4d^3 x^3 e^c + 12d^2 x^2 e^c - 24dx e^c + 24e^c) a e^{(dx)}}{d^5} + \frac{5(d^4 x^4 + 4d^3 x^3 + 12d^2 x^2 + 24dx + 24) a e^{(-dx-c)}}{d^5} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x+a)\*cosh(d\*x+c),x, algorithm="maxima")

[Out]  $-1/40*d*(5*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*e^{(d*x)}/d^5 + 5*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*e^{(-d*x - c)}/d^5 + 4*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^{(d*x)}/d^6 + 4*(d^5*x^5 + 5*d^4*x^4 + 2$

$$0*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^{(-d*x - c)/d^6} + 1/20*(4*b*x^5 + 5*a*x^4)*\cosh(d*x + c)$$

**Fricas [A]** time = 1.94238, size = 193, normalized size = 1.56

$$\frac{(4bd^3x^3 + 3ad^3x^2 + 24bdx + 6ad)\cosh(dx + c) - (bd^4x^4 + ad^4x^3 + 12bd^2x^2 + 6ad^2x + 24b)\sinh(dx + c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x+a)\*cosh(d\*x+c),x, algorithm="fricas")

[Out] -((4\*b\*d^3\*x^3 + 3\*a\*d^3\*x^2 + 24\*b\*d\*x + 6\*a\*d)\*cosh(d\*x + c) - (b\*d^4\*x^4 + a\*d^4\*x^3 + 12\*b\*d^2\*x^2 + 6\*a\*d^2\*x + 24\*b)\*sinh(d\*x + c))/d^5

**Sympy [A]** time = 3.63855, size = 151, normalized size = 1.22

$$\left\{ \begin{array}{l} \frac{ax^3 \sinh(c+dx)}{d} - \frac{3ax^2 \cosh(c+dx)}{d^2} + \frac{6ax \sinh(c+dx)}{d^3} - \frac{6a \cosh(c+dx)}{d^4} + \frac{bx^4 \sinh(c+dx)}{d} - \frac{4bx^3 \cosh(c+dx)}{d^2} + \frac{12bx^2 \sinh(c+dx)}{d^3} - \frac{24bx \cosh(c+dx)}{d^4} \\ \left( \frac{ax^4}{4} + \frac{bx^5}{5} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(b\*x+a)\*cosh(d\*x+c),x)

[Out] Piecewise((a\*x\*\*3\*sinh(c + d\*x)/d - 3\*a\*x\*\*2\*cosh(c + d\*x)/d\*\*2 + 6\*a\*x\*sinh(c + d\*x)/d\*\*3 - 6\*a\*cosh(c + d\*x)/d\*\*4 + b\*x\*\*4\*sinh(c + d\*x)/d - 4\*b\*x\*\*3\*cosh(c + d\*x)/d\*\*2 + 12\*b\*x\*\*2\*sinh(c + d\*x)/d\*\*3 - 24\*b\*x\*cosh(c + d\*x)/d\*\*4 + 24\*b\*sinh(c + d\*x)/d\*\*5, Ne(d, 0)), ((a\*x\*\*4/4 + b\*x\*\*5/5)\*cosh(c), True))

**Giac [A]** time = 1.21328, size = 205, normalized size = 1.65

$$\frac{(bd^4x^4 + ad^4x^3 - 4bd^3x^3 - 3ad^3x^2 + 12bd^2x^2 + 6ad^2x - 24bdx - 6ad + 24b)e^{(dx+c)}}{2d^5} - \frac{(bd^4x^4 + ad^4x^3 + 4bd^3x^3 + 3ad^3x^2 + 12bd^2x^2 + 6ad^2x + 24bdx + 6ad + 24b)\cosh(dx + c) - (bd^4x^4 + ad^4x^3 + 4bd^3x^3 + 3ad^3x^2 + 12bd^2x^2 + 6ad^2x + 24bdx + 6ad + 24b)\sinh(dx + c)}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^4*x^4 + a*d^4*x^3 - 4*b*d^3*x^3 - 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*a*d^2*x - 24*b*d*x - 6*a*d + 24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + a*d^4*x^3 + 4*b*d^3*x^3 + 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*a*d^2*x + 24*b*d*x + 6*a*d + 24*b)*e^(-d*x - c)/d^5
```

## 3.2 $\int x^2(a + bx) \cosh(c + dx) dx$

**Optimal.** Leaf size=94

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{6b \cosh(c + dx)}{d^4} + \frac{bx^3 \sinh(c + dx)}{d}$$

[Out]  $(-6*b*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x^2*Sinh[c + d*x])/d + (b*x^3*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.216928, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6742, 3296, 2637, 2638}

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{6b \cosh(c + dx)}{d^4} + \frac{bx^3 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*x)*Cosh[c + d*x], x]$

[Out]  $(-6*b*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x^2*Sinh[c + d*x])/d + (b*x^3*Sinh[c + d*x])/d$

### Rule 6742

$\text{Int}[u_, x\_Symbol] \text{ :> With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] \text{ /; SumQ}[v]]$

### Rule 3296

$\text{Int}[\{(c_.) + (d_.)*(x_.)\}^{(m_.)} \sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \text{ :> -Simp}[\{(c + d*x)^m \cos[e + f*x]\}/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)} \cos[e + f*x], x], x] \text{ /; FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x\_Symbol] \text{ :> Simp}[\text{Sin}[c + d*x]/d, x] \text{ /; FreeQ}\{c, d\}, x]$

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \int x^2(a + bx) \cosh(c + dx) dx &= \int (ax^2 \cosh(c + dx) + bx^3 \cosh(c + dx)) dx \\
 &= a \int x^2 \cosh(c + dx) dx + b \int x^3 \cosh(c + dx) dx \\
 &= \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} - \frac{(2a) \int x \sinh(c + dx) dx}{d} - \frac{(3b) \int x^2 \sinh(c + dx) dx}{d} \\
 &= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} + \frac{(2a)}{d^3} \\
 &= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{ax^2 \sinh(c + dx)}{d^3} \\
 &= -\frac{6b \cosh(c + dx)}{d^4} - \frac{2ax \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{ax^2 \sinh(c + dx)}{d^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.117328, size = 65, normalized size = 0.69

$$\frac{d \left( a \left( d^2 x^2 + 2 \right) + bx \left( d^2 x^2 + 6 \right) \right) \sinh(c + dx) - \left( 2ad^2 x + 3b \left( d^2 x^2 + 2 \right) \right) \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x)\*Cosh[c + d\*x],x]

[Out]  $(-((2*a*d^2*x + 3*b*(2 + d^2*x^2))*Cosh[c + d*x]) + d*(a*(2 + d^2*x^2) + b*x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4$

**Maple [B]** time = 0.007, size = 224, normalized size = 2.4

$$\frac{1}{d^3} \left( \frac{b \left( (dx + c)^3 \sinh(dx + c) - 3(dx + c)^2 \cosh(dx + c) + 6(dx + c) \sinh(dx + c) - 6 \cosh(dx + c) \right)}{d} - 3 \frac{cb \left( (dx + c)^2 \right)}{d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x+a)*cosh(d*x+c),x)`

[Out]  $\frac{1}{d^3} \left( \frac{b}{d} \left( (d*x+c)^3 \sinh(d*x+c) - 3(d*x+c)^2 \cosh(d*x+c) + 6(d*x+c) \sinh(d*x+c) - 6 \cosh(d*x+c) \right) - 3*b*c/d \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) + 3*b/d*c^2 \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) - b*c^3/d \sinh(d*x+c) + a \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) - 2*a*c \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) + a*c^2 \sinh(d*x+c) \right)$

**Maxima [B]** time = 1.1237, size = 265, normalized size = 2.82

$$-\frac{1}{24}d \left( \frac{4(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)ae^{(dx)}}{d^4} + \frac{4(d^3x^3 + 3d^2x^2 + 6dx + 6)ae^{(-dx-c)}}{d^4} + \frac{3(d^4x^4e^c - 4d^3x^3e^c + 12d^2x^2e^c - 12dx + 6)e^{(-dx-c)}}{d^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")`

[Out]  $-1/24*d*(4*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^{(d*x)}/d^4 + 4*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^{(-d*x - c)}/d^4 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b*e^{(d*x)}/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b*e^{(-d*x - c)}/d^5) + 1/12*(3*b*x^4 + 4*a*x^3)*cosh(d*x + c)$

**Fricas [A]** time = 1.92279, size = 151, normalized size = 1.61

$$\frac{(3bd^2x^2 + 2ad^2x + 6b) \cosh(dx + c) - (bd^3x^3 + ad^3x^2 + 6bdx + 2ad) \sinh(dx + c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")`

[Out]  $-((3*b*d^2*x^2 + 2*a*d^2*x + 6*b)*cosh(d*x + c) - (b*d^3*x^3 + a*d^3*x^2 + 6*b*d*x + 2*a*d)*sinh(d*x + c))/d^4$

**Sympy [A]** time = 2.24197, size = 117, normalized size = 1.24

$$\begin{cases} \frac{ax^2 \sinh(c+dx)}{d} - \frac{2ax \cosh(c+dx)}{d^2} + \frac{2a \sinh(c+dx)}{d^3} + \frac{bx^3 \sinh(c+dx)}{d} - \frac{3bx^2 \cosh(c+dx)}{d^2} + \frac{6bx \sinh(c+dx)}{d^3} - \frac{6b \cosh(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left(\frac{ax^3}{3} + \frac{bx^4}{4}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x+a)\*cosh(d\*x+c),x)

[Out] Piecewise((a\*x\*\*2\*sinh(c + d\*x)/d - 2\*a\*x\*cosh(c + d\*x)/d\*\*2 + 2\*a\*sinh(c + d\*x)/d\*\*3 + b\*x\*\*3\*sinh(c + d\*x)/d - 3\*b\*x\*\*2\*cosh(c + d\*x)/d\*\*2 + 6\*b\*x\*sinh(c + d\*x)/d\*\*3 - 6\*b\*cosh(c + d\*x)/d\*\*4, Ne(d, 0)), ((a\*x\*\*3/3 + b\*x\*\*4/4)\*cosh(c), True))

**Giac [A]** time = 1.16419, size = 157, normalized size = 1.67

$$\frac{(bd^3x^3 + ad^3x^2 - 3bd^2x^2 - 2ad^2x + 6bdx + 2ad - 6b)e^{(dx+c)}}{2d^4} - \frac{(bd^3x^3 + ad^3x^2 + 3bd^2x^2 + 2ad^2x + 6bdx + 2ad + 6b)e^{-(dx+c)}}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x+a)\*cosh(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(b\*d^3\*x^3 + a\*d^3\*x^2 - 3\*b\*d^2\*x^2 - 2\*a\*d^2\*x + 6\*b\*d\*x + 2\*a\*d - 6\*b)\*e^(d\*x + c)/d^4 - 1/2\*(b\*d^3\*x^3 + a\*d^3\*x^2 + 3\*b\*d^2\*x^2 + 2\*a\*d^2\*x + 6\*b\*d\*x + 2\*a\*d + 6\*b)\*e^(-d\*x - c)/d^4

### 3.3 $\int x(a + bx) \cosh(c + dx) dx$

**Optimal.** Leaf size=64

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{2b \sinh(c + dx)}{d^3} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{bx^2 \sinh(c + dx)}{d}$$

[Out]  $-\frac{(a \cosh[c + d*x])}{d^2} - \frac{(2*b*x*\cosh[c + d*x])}{d^2} + \frac{(2*b*\sinh[c + d*x])}{d^3} + \frac{(a*x*\sinh[c + d*x])}{d} + \frac{(b*x^2*\sinh[c + d*x])}{d}$

**Rubi [A]** time = 0.113946, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 13,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {6742, 3296, 2638, 2637}

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{2b \sinh(c + dx)}{d^3} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{bx^2 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x)\*Cosh[c + d\*x], x]

[Out]  $-\frac{(a \cosh[c + d*x])}{d^2} - \frac{(2*b*x*\cosh[c + d*x])}{d^2} + \frac{(2*b*\sinh[c + d*x])}{d^3} + \frac{(a*x*\sinh[c + d*x])}{d} + \frac{(b*x^2*\sinh[c + d*x])}{d}$

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m \* Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1) \* Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 2637



```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int x(a + bx) \cosh(c + dx) dx &= \int (ax \cosh(c + dx) + bx^2 \cosh(c + dx)) dx \\
 &= a \int x \cosh(c + dx) dx + b \int x^2 \cosh(c + dx) dx \\
 &= \frac{ax \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d} - \frac{a \int \sinh(c + dx) dx}{d} - \frac{(2b) \int x \sinh(c + dx) dx}{d} \\
 &= -\frac{a \cosh(c + dx)}{d^2} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d} + \frac{(2b) \int \cos}{d} \\
 &= -\frac{a \cosh(c + dx)}{d^2} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{2b \sinh(c + dx)}{d^3} + \frac{ax \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d}
 \end{aligned}$$

**Mathematica [A]** time = 0.0879951, size = 45, normalized size = 0.7

$$\frac{(ad^2x + b(d^2x^2 + 2)) \sinh(c + dx) - d(a + 2bx) \cosh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)*Cosh[c + d*x], x]
```

```
[Out] (-(d*(a + 2*b*x)*Cosh[c + d*x]) + (a*d^2*x + b*(2 + d^2*x^2))*Sinh[c + d*x])/d^3
```

**Maple [A]** time = 0.008, size = 122, normalized size = 1.9

$$\frac{1}{d^2} \left( \frac{b((dx + c)^2 \sinh(dx + c) - 2(dx + c) \cosh(dx + c) + 2 \sinh(dx + c))}{d} - 2 \frac{cb((dx + c) \sinh(dx + c) - \cosh(dx + c))}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*cosh(d*x+c), x)
```

```
[Out] 1/d^2*(b/d*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-2*b*c/d*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+b
```

$$*c^2/d*\sinh(d*x+c)-c*a*\sinh(d*x+c))$$

**Maxima [B]** time = 1.13037, size = 216, normalized size = 3.38

$$-\frac{1}{12}d\left(\frac{3(d^2x^2e^c - 2dxe^c + 2e^c)ae^{(dx)}}{d^3} + \frac{3(d^2x^2 + 2dx + 2)ae^{(-dx-c)}}{d^3} + \frac{2(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)be^{(dx)}}{d^4} + \frac{2(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)be^{(-dx-c)}}{d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x+a)\*cosh(d\*x+c),x, algorithm="maxima")

[Out]  $-1/12*d*(3*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*e^{(d*x)}/d^3 + 3*(d^2*x^2 + 2*d*x + 2)*a*e^{(-d*x - c)}/d^3 + 2*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*b*e^{(d*x)}/d^4 + 2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b*e^{(-d*x - c)}/d^4 + 1/6*(2*b*x^3 + 3*a*x^2)*\cosh(d*x + c)$

**Fricas [A]** time = 1.9195, size = 111, normalized size = 1.73

$$\frac{(2bdx + ad)\cosh(dx + c) - (bd^2x^2 + ad^2x + 2b)\sinh(dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x+a)\*cosh(d\*x+c),x, algorithm="fricas")

[Out]  $-((2*b*d*x + a*d)*\cosh(d*x + c) - (b*d^2*x^2 + a*d^2*x + 2*b)*\sinh(d*x + c))/d^3$

**Sympy [A]** time = 1.95005, size = 82, normalized size = 1.28

$$\begin{cases} \frac{ax \sinh(c+dx)}{d} - \frac{a \cosh(c+dx)}{d^2} + \frac{bx^2 \sinh(c+dx)}{d} - \frac{2bx \cosh(c+dx)}{d^2} + \frac{2b \sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^3}{3}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x+a)\*cosh(d\*x+c),x)

[Out] Piecewise((a\*x\*sinh(c + d\*x)/d - a\*cosh(c + d\*x)/d\*\*2 + b\*x\*\*2\*sinh(c + d\*x)/d - 2\*b\*x\*cosh(c + d\*x)/d\*\*2 + 2\*b\*sinh(c + d\*x)/d\*\*3, Ne(d, 0)), ((a\*x\*\*2/2 + b\*x\*\*3/3)\*cosh(c), True))

**Giac [A]** time = 1.22437, size = 107, normalized size = 1.67

$$\frac{(bd^2x^2 + ad^2x - 2bdx - ad + 2b)e^{(dx+c)}}{2d^3} - \frac{(bd^2x^2 + ad^2x + 2bdx + ad + 2b)e^{(-dx-c)}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x+a)\*cosh(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(b\*d^2\*x^2 + a\*d^2\*x - 2\*b\*d\*x - a\*d + 2\*b)\*e^(d\*x + c)/d^3 - 1/2\*(b\*d^2\*x^2 + a\*d^2\*x + 2\*b\*d\*x + a\*d + 2\*b)\*e^(-d\*x - c)/d^3

### 3.4 $\int (a + bx) \cosh(c + dx) dx$

**Optimal.** Leaf size=28

$$\frac{(a + bx) \sinh(c + dx)}{d} - \frac{b \cosh(c + dx)}{d^2}$$

[Out]  $-\frac{b \cosh[c + d*x]}{d^2} + \frac{(a + b*x) \sinh[c + d*x]}{d}$

**Rubi [A]** time = 0.0205861, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3296, 2638}

$$\frac{(a + bx) \sinh(c + dx)}{d} - \frac{b \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x)*\text{Cosh}[c + d*x], x]$

[Out]  $-\frac{b \cosh[c + d*x]}{d^2} + \frac{(a + b*x) \sinh[c + d*x]}{d}$

#### Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)} \sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m \cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)} \cos[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{GtQ}[m, 0]$

#### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\cos[c + d*x]/d, x] /; \text{FreeQ}\{c, d\}, x]$

#### Rubi steps

$$\begin{aligned} \int (a + bx) \cosh(c + dx) dx &= \frac{(a + bx) \sinh(c + dx)}{d} - \frac{b \int \sinh(c + dx) dx}{d} \\ &= -\frac{b \cosh(c + dx)}{d^2} + \frac{(a + bx) \sinh(c + dx)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0513288, size = 27, normalized size = 0.96

$$\frac{d(a + bx) \sinh(c + dx) - b \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)\*Cosh[c + d\*x], x]

[Out]  $(-(b*\text{Cosh}[c + d*x]) + d*(a + b*x)*\text{Sinh}[c + d*x])/d^2$

**Maple [A]** time = 0.009, size = 53, normalized size = 1.9

$$\frac{1}{d} \left( \frac{b((dx + c) \sinh(dx + c) - \cosh(dx + c))}{d} - \frac{cb \sinh(dx + c)}{d} + a \sinh(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*cosh(d\*x+c), x)

[Out]  $1/d*(b/d*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))-b*c/d*\sinh(d*x+c)+a*\sinh(d*x+c))$

**Maxima [B]** time = 1.13935, size = 92, normalized size = 3.29

$$\frac{ae^{(dx+c)}}{2d} + \frac{(dxe^c - e^c)be^{(dx)}}{2d^2} - \frac{(dx+1)be^{(-dx-c)}}{2d^2} - \frac{ae^{(-dx-c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c), x, algorithm="maxima")

[Out]  $1/2*a*e^{(d*x + c)}/d + 1/2*(d*x*e^c - e^c)*b*e^{(d*x)}/d^2 - 1/2*(d*x + 1)*b*e^{(-d*x - c)}/d^2 - 1/2*a*e^{(-d*x - c)}/d$

**Fricas [A]** time = 1.98283, size = 73, normalized size = 2.61

$$\frac{b \cosh(dx + c) - (bdx + ad) \sinh(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c),x, algorithm="fricas")

[Out]  $-(b*\cosh(d*x + c) - (b*d*x + a*d)*\sinh(d*x + c))/d^2$

**Sympy [A]** time = 0.629005, size = 46, normalized size = 1.64

$$\begin{cases} \frac{a \sinh(c+dx)}{d} + \frac{bx \sinh(c+dx)}{d} - \frac{b \cosh(c+dx)}{d^2} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^2}{2}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c),x)

[Out] Piecewise((a\*sinh(c + d\*x)/d + b\*x\*sinh(c + d\*x)/d - b\*cosh(c + d\*x)/d\*\*2, Ne(d, 0)), ((a\*x + b\*x\*\*2/2)\*cosh(c), True))

**Giac [A]** time = 1.20289, size = 62, normalized size = 2.21

$$\frac{(bdx + ad - b)e^{(dx+c)}}{2d^2} - \frac{(bdx + ad + b)e^{(-dx-c)}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c),x, algorithm="giac")

[Out]  $1/2*(b*d*x + a*d - b)*e^{(d*x + c)}/d^2 - 1/2*(b*d*x + a*d + b)*e^{(-d*x - c)}/d^2$

$$3.5 \quad \int \frac{(a+bx) \cosh(c+dx)}{x} dx$$

**Optimal.** Leaf size=28

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) + \frac{b \sinh(c+dx)}{d}$$

[Out] a\*Cosh[c]\*CoshIntegral[d\*x] + (b\*Sinh[c + d\*x])/d + a\*Sinh[c]\*SinhIntegral[d\*x]

**Rubi [A]** time = 0.148764, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 2637, 3303, 3298, 3301}

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) + \frac{b \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x)\*Cosh[c + d\*x])/x,x]

[Out] a\*Cosh[c]\*CoshIntegral[d\*x] + (b\*Sinh[c + d\*x])/d + a\*Sinh[c]\*SinhIntegral[d\*x]

#### Rule 6742

Int[u\_, x\_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] :=> Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :=> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + bx) \cosh(c + dx)}{x} dx &= \int \left( b \cosh(c + dx) + \frac{a \cosh(c + dx)}{x} \right) dx \\ &= a \int \frac{\cosh(c + dx)}{x} dx + b \int \cosh(c + dx) dx \\ &= \frac{b \sinh(c + dx)}{d} + (a \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (a \sinh(c)) \int \frac{\sinh(dx)}{x} dx \\ &= a \cosh(c) \text{Chi}(dx) + \frac{b \sinh(c + dx)}{d} + a \sinh(c) \text{Shi}(dx) \end{aligned}$$

**Mathematica [A]** time = 0.0279314, size = 39, normalized size = 1.39

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) + \frac{b \sinh(c) \cosh(dx)}{d} + \frac{b \cosh(c) \sinh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x,x]
```

```
[Out] a*Cosh[c]*CoshIntegral[d*x] + (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d + a*Sinh[c]*SinhIntegral[d*x]
```

**Maple [A]** time = 0.038, size = 52, normalized size = 1.9

$$-\frac{ae^{-c} \text{Ei}(1, dx)}{2} - \frac{be^{-dx-c}}{2d} - \frac{ae^c \text{Ei}(1, -dx)}{2} + \frac{be^{dx+c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((b*x+a)*cosh(d*x+c)/x,x)`

[Out]  $-1/2*a*\exp(-c)*\text{Ei}(1,d*x)-1/2*b/d*\exp(-d*x-c)-1/2*a*\exp(c)*\text{Ei}(1,-d*x)+1/2*b/d*\exp(d*x+c)$

**Maxima [B]** time = 1.20109, size = 131, normalized size = 4.68

$$-\frac{1}{2} \left( b \left( \frac{(dx e^c - e^c) e^{(dx)}}{d^2} + \frac{(dx + 1) e^{(-dx-c)}}{d^2} \right) + \frac{2 a \cosh(dx + c) \log(x)}{d} - \frac{(\text{Ei}(-dx) e^{(-c)} + \text{Ei}(dx) e^c) a}{d} \right) d + (bx + a \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="maxima")`

[Out]  $-1/2*(b*((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x + 1)*e^{(-d*x - c)}/d^2) + 2*a*\cosh(d*x + c)*\log(x)/d - (\text{Ei}(-d*x)*e^{(-c)} + \text{Ei}(d*x)*e^c)*a/d)*d + (b*x + a*\log(x))*\cosh(d*x + c)$

**Fricas [A]** time = 1.96747, size = 142, normalized size = 5.07

$$\frac{(ad\text{Ei}(dx) + ad\text{Ei}(-dx)) \cosh(c) + 2b \sinh(dx + c) + (ad\text{Ei}(dx) - ad\text{Ei}(-dx)) \sinh(c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="fricas")`

[Out]  $1/2*((a*d*\text{Ei}(d*x) + a*d*\text{Ei}(-d*x))*\cosh(c) + 2*b*\sinh(d*x + c) + (a*d*\text{Ei}(d*x) - a*d*\text{Ei}(-d*x))*\sinh(c))/d$

**Sympy [A]** time = 4.54689, size = 34, normalized size = 1.21

$$a \sinh(c) \text{Shi}(dx) + a \cosh(c) \text{Chi}(dx) + b \left( \begin{cases} \frac{\sinh(c+dx)}{d} & \text{for } d \neq 0 \\ x \cosh(c) & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x)
```

```
[Out] a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True))
```

**Giac [A]** time = 1.1822, size = 63, normalized size = 2.25

$$\frac{ad\text{Ei}(-dx)e^{(-c)} + ad\text{Ei}(dx)e^c + be^{(dx+c)} - be^{(-dx-c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

```
[Out] 1/2*(a*d*Ei(-d*x)*e^(-c) + a*d*Ei(d*x)*e^c + b*e^(d*x + c) - b*e^(-d*x - c))/d
```

$$3.6 \quad \int \frac{(a+bx) \cosh(c+dx)}{x^2} dx$$

**Optimal.** Leaf size=47

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c + dx)}{x} + b \cosh(c)\text{Chi}(dx) + b \sinh(c)\text{Shi}(dx)$$

[Out]  $-\left(\frac{a \cosh[c + d x]}{x}\right) + b \cosh[c] \text{CoshIntegral}[d x] + a d \text{CoshIntegral}[d x] \text{Sinh}[c] + a d \cosh[c] \text{SinhIntegral}[d x] + b \sinh[c] \text{SinhIntegral}[d x]$

**Rubi [A]** time = 0.227998, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c + dx)}{x} + b \cosh(c)\text{Chi}(dx) + b \sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\left(\frac{(a + b x) \cosh[c + d x]}{x^2}\right), x]$

[Out]  $-\left(\frac{a \cosh[c + d x]}{x}\right) + b \cosh[c] \text{CoshIntegral}[d x] + a d \text{CoshIntegral}[d x] \text{Sinh}[c] + a d \cosh[c] \text{SinhIntegral}[d x] + b \sinh[c] \text{SinhIntegral}[d x]$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \text{ :> With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] \text{ /; SumQ}[v]]$

#### Rule 3297

$\text{Int}[\left(\frac{(c_. + (d_.)(x_))^{(m_)} \sin[(e_.) + (f_.)(x_)]}{(c + d x)^{(m + 1)} \sin[e + f x]} \right) / (d(m + 1)), x] - \text{Dist}[f / (d(m + 1)), \text{Int}[\left(\frac{(c + d x)^{(m + 1)} \cos[e + f x]}{(c + d x)^{(m + 1)} \cos[e + f x]} \right), x], x] \text{ /; FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{LtQ}[m, -1]$

#### Rule 3303

$\text{Int}[\sin[(e_.) + (f_.)(x_)] / ((c_.) + (d_.)(x_)), x\_Symbol] \text{ :> Dist}[\text{Cos}[(d * e - c * f) / d], \text{Int}[\text{Sin}[(c * f) / d + f * x] / (c + d * x), x], x] + \text{Dist}[\text{Sin}[(d * e - c * f) / d], \text{Int}[\text{Cos}[(c * f) / d + f * x] / (c + d * x), x], x] \text{ /; FreeQ}\{c, d, e, f\}, x] \ \&\&$

NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx) \cosh(c + dx)}{x^2} dx &= \int \left( \frac{a \cosh(c + dx)}{x^2} + \frac{b \cosh(c + dx)}{x} \right) dx \\
 &= a \int \frac{\cosh(c + dx)}{x^2} dx + b \int \frac{\cosh(c + dx)}{x} dx \\
 &= -\frac{a \cosh(c + dx)}{x} + (ad) \int \frac{\sinh(c + dx)}{x} dx + (b \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (b \sinh(c)) \int \frac{\sinh(dx)}{x} dx \\
 &= -\frac{a \cosh(c + dx)}{x} + b \cosh(c) \text{Chi}(dx) + b \sinh(c) \text{Shi}(dx) + (ad \cosh(c)) \int \frac{\sinh(dx)}{x} dx + (ad \sinh(c)) \int \frac{\cosh(dx)}{x} dx \\
 &= -\frac{a \cosh(c + dx)}{x} + b \cosh(c) \text{Chi}(dx) + ad \text{Chi}(dx) \sinh(c) + ad \cosh(c) \text{Shi}(dx) + b \sinh(c) \text{Chi}(dx)
 \end{aligned}$$

**Mathematica [A]** time = 0.127054, size = 59, normalized size = 1.26

$$ad(\sinh(c)\text{Chi}(dx) + \cosh(c)\text{Shi}(dx)) - \frac{a \sinh(c) \sinh(dx)}{x} - \frac{a \cosh(c) \cosh(dx)}{x} + b \cosh(c)\text{Chi}(dx) + b \sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)\*Cosh[c + d\*x])/x^2,x]

[Out] -((a\*Cosh[c]\*Cosh[d\*x])/x) + b\*Cosh[c]\*CoshIntegral[d\*x] - (a\*Sinh[c]\*Sinh[d\*x])/x + b\*Sinh[c]\*SinhIntegral[d\*x] + a\*d\*(CoshIntegral[d\*x]\*Sinh[c] + Cosh[c]\*SinhIntegral[d\*x])

**Maple [A]** time = 0.039, size = 77, normalized size = 1.6

$$-\frac{ae^{-dx-c}}{2x} + \frac{dae^{-c}\text{Ei}(1, dx)}{2} - \frac{be^{-c}\text{Ei}(1, dx)}{2} - \frac{ae^{dx+c}}{2x} - \frac{dae^c\text{Ei}(1, -dx)}{2} - \frac{be^c\text{Ei}(1, -dx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*cosh(d\*x+c)/x^2,x)

[Out]  $-1/2*a*\exp(-d*x-c)/x+1/2*d*a*\exp(-c)*\text{Ei}(1,d*x)-1/2*b*\exp(-c)*\text{Ei}(1,d*x)-1/2*a/\exp(d*x+c)-1/2*d*a*\exp(c)*\text{Ei}(1,-d*x)-1/2*b*\exp(c)*\text{Ei}(1,-d*x)$

**Maxima [A]** time = 1.39651, size = 111, normalized size = 2.36

$$-\frac{1}{2} \left( (\text{Ei}(-dx)e^{(-c)} - \text{Ei}(dx)e^c)a + \frac{2b \cosh(dx+c) \log(x)}{d} - \frac{(\text{Ei}(-dx)e^{(-c)} + \text{Ei}(dx)e^c)b}{d} \right) d + \left( b \log(x) - \frac{a}{x} \right) \cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x^2,x, algorithm="maxima")

[Out]  $-1/2*((\text{Ei}(-d*x)*e^{(-c)} - \text{Ei}(d*x)*e^c)*a + 2*b*\cosh(d*x + c)*\log(x)/d - (\text{Ei}(-d*x)*e^{(-c)} + \text{Ei}(d*x)*e^c)*b/d)*d + (b*\log(x) - a/x)*\cosh(d*x + c)$

**Fricas [A]** time = 1.96812, size = 186, normalized size = 3.96

$$\frac{2a \cosh(dx+c) - ((ad+b)x\text{Ei}(dx) - (ad-b)x\text{Ei}(-dx)) \cosh(c) - ((ad+b)x\text{Ei}(dx) + (ad-b)x\text{Ei}(-dx)) \sinh(c)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x^2,x, algorithm="fricas")

[Out]  $-1/2*(2*a*\cosh(d*x + c) - ((a*d + b)*x*\text{Ei}(d*x) - (a*d - b)*x*\text{Ei}(-d*x))*\cosh(c) - ((a*d + b)*x*\text{Ei}(d*x) + (a*d - b)*x*\text{Ei}(-d*x))*\sinh(c))/x$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx) \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x\*\*2,x)

[Out] Integral((a + b\*x)\*cosh(c + d\*x)/x\*\*2, x)

**Giac [A]** time = 1.15134, size = 97, normalized size = 2.06

$$\frac{adx\text{Ei}(-dx)e^{(-c)} - adx\text{Ei}(dx)e^c - bx\text{Ei}(-dx)e^{(-c)} - bx\text{Ei}(dx)e^c + ae^{(dx+c)} + ae^{(-dx-c)}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x^2,x, algorithm="giac")

[Out] -1/2\*(a\*d\*x\*Ei(-d\*x)\*e^(-c) - a\*d\*x\*Ei(d\*x)\*e^c - b\*x\*Ei(-d\*x)\*e^(-c) - b\*x\*Ei(d\*x)\*e^c + a\*e^(d\*x + c) + a\*e^(-d\*x - c))/x

$$3.7 \quad \int \frac{(a+bx) \cosh(c+dx)}{x^3} dx$$

**Optimal.** Leaf size=88

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + bd \sinh(c)\text{Chi}(dx) + bd \cosh(c)\text{Shi}(dx)$$

[Out]  $-(a*\text{Cosh}[c + d*x])/(2*x^2) - (b*\text{Cosh}[c + d*x])/x + (a*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] - (a*d*\text{Sinh}[c + d*x])/(2*x) + b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + (a*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

**Rubi [A]** time = 0.28109, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + bd \sinh(c)\text{Chi}(dx) + bd \cosh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + b*x)*\text{Cosh}[c + d*x]}{x^3}, x]$

[Out]  $-(a*\text{Cosh}[c + d*x])/(2*x^2) - (b*\text{Cosh}[c + d*x])/x + (a*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] - (a*d*\text{Sinh}[c + d*x])/(2*x) + b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + (a*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

### Rule 3297

$\text{Int}[\frac{((c_.) + (d_.)*(x_))^{(m_)}*\sin[(e_.) + (f_.)*(x_)]}{(c + d*x)^{(m+1)}*\sin[e + f*x]}], x\_Symbol] \rightarrow \text{Simp}[\frac{(c + d*x)^{(m+1)}*\sin[e + f*x]}{(d*(m+1))}, x] - \text{Dist}[f/(d*(m+1)), \text{Int}[\frac{(c + d*x)^{(m+1)}*\cos[e + f*x]}{(d*(m+1))}, x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{LtQ}[m, -1]$

### Rule 3303

$\text{Int}[\frac{\sin[(e_.) + (f_.)*(x_)]}{(c_.) + (d_.)*(x_)}], x\_Symbol] \rightarrow \text{Dist}[\frac{\cos[(d*e - c*f)}{d}], \text{Int}[\frac{\sin[(c*f)}{d + f*x]}{(c + d*x)}, x], x] + \text{Dist}[\frac{\sin[(d*e - c*f)}{d}], \text{Int}[\frac{\cos[(c*f)}{d + f*x]}{(c + d*x)}, x], x]$

/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned} \int \frac{(a + bx) \cosh(c + dx)}{x^3} dx &= \int \left( \frac{a \cosh(c + dx)}{x^3} + \frac{b \cosh(c + dx)}{x^2} \right) dx \\ &= a \int \frac{\cosh(c + dx)}{x^3} dx + b \int \frac{\cosh(c + dx)}{x^2} dx \\ &= -\frac{a \cosh(c + dx)}{2x^2} - \frac{b \cosh(c + dx)}{x} + \frac{1}{2}(ad) \int \frac{\sinh(c + dx)}{x^2} dx + (bd) \int \frac{\sinh(c + dx)}{x} dx \\ &= -\frac{a \cosh(c + dx)}{2x^2} - \frac{b \cosh(c + dx)}{x} - \frac{ad \sinh(c + dx)}{2x} + \frac{1}{2}(ad^2) \int \frac{\cosh(c + dx)}{x} dx + (bd) \int \frac{\sinh(c + dx)}{x} dx \\ &= -\frac{a \cosh(c + dx)}{2x^2} - \frac{b \cosh(c + dx)}{x} + bd \operatorname{Chi}(dx) \sinh(c) - \frac{ad \sinh(c + dx)}{2x} + bd \cosh(c) \operatorname{Shi}(dx) \\ &= -\frac{a \cosh(c + dx)}{2x^2} - \frac{b \cosh(c + dx)}{x} + \frac{1}{2} ad^2 \cosh(c) \operatorname{Chi}(dx) + bd \operatorname{Chi}(dx) \sinh(c) - \frac{ad \sinh(c + dx)}{2x} + bd \cosh(c) \operatorname{Shi}(dx) \end{aligned}$$

**Mathematica [A]** time = 0.155352, size = 78, normalized size = 0.89

$$\frac{dx^2 \operatorname{Chi}(dx)(ad \cosh(c) + 2b \sinh(c)) + dx^2 \operatorname{Shi}(dx)(ad \sinh(c) + 2b \cosh(c)) - adx \sinh(c + dx) - a \cosh(c + dx) - 2bx \cosh(c)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)\*Cosh[c + d\*x])/x^3, x]

[Out] (-(a\*Cosh[c + d\*x]) - 2\*b\*x\*Cosh[c + d\*x] + d\*x^2\*CoshIntegral[d\*x]\*(a\*d\*Cos  
sh[c] + 2\*b\*Sinh[c]) - a\*d\*x\*Sinh[c + d\*x] + d\*x^2\*(2\*b\*Cosh[c] + a\*d\*Sinh[c]



c])\*SinhIntegral[d\*x])/(2\*x^2)

**Maple [A]** time = 0.047, size = 139, normalized size = 1.6

$$\frac{dae^{-dx-c}}{4x} - \frac{ae^{-dx-c}}{4x^2} - \frac{d^2ae^{-c}\text{Ei}(1,dx)}{4} - \frac{be^{-dx-c}}{2x} + \frac{dbe^{-c}\text{Ei}(1,dx)}{2} - \frac{ae^{dx+c}}{4x^2} - \frac{ade^{dx+c}}{4x} - \frac{d^2ae^c\text{Ei}(1,-dx)}{4} - \frac{be^{dx+c}}{2x} - \frac{d}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*cosh(d\*x+c)/x^3,x)

[Out]  $\frac{1}{4}d*a*\exp(-d*x-c)/x - \frac{1}{4}a*\exp(-d*x-c)/x^2 - \frac{1}{4}d^2*a*\exp(-c)*\text{Ei}(1,d*x) - \frac{1}{2}b*\exp(-d*x-c)/x + \frac{1}{2}d*b*\exp(-c)*\text{Ei}(1,d*x) - \frac{1}{4}a/x^2*\exp(d*x+c) - \frac{1}{4}d*a/x*\exp(d*x+c) - \frac{1}{4}d^2*a*\exp(c)*\text{Ei}(1,-d*x) - \frac{1}{2}b/x*\exp(d*x+c) - \frac{1}{2}d*b*\exp(c)*\text{Ei}(1,-d*x)$

**Maxima [A]** time = 1.417, size = 89, normalized size = 1.01

$$\frac{1}{4} \left( ade^{(-c)}\Gamma(-1,dx) + ade^c\Gamma(-1,-dx) - 2b\text{Ei}(-dx)e^{(-c)} + 2b\text{Ei}(dx)e^c \right) d - \frac{(2bx+a)\cosh(dx+c)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x^3,x, algorithm="maxima")

[Out]  $\frac{1}{4}*(a*d*e^{(-c)}*\text{gamma}(-1,d*x) + a*d*e^c*\text{gamma}(-1,-d*x) - 2*b*\text{Ei}(-d*x)*e^{(-c)} + 2*b*\text{Ei}(d*x)*e^c)*d - \frac{1}{2}*(2*b*x + a)*\cosh(d*x + c)/x^2$

**Fricas [A]** time = 1.99403, size = 278, normalized size = 3.16

$$\frac{2adx \sinh(dx+c) + 2(2bx+a)\cosh(dx+c) - \left( (ad^2 + 2bd)x^2\text{Ei}(dx) + (ad^2 - 2bd)x^2\text{Ei}(-dx) \right) \cosh(c) - \left( (ad^2 + 2bd)x^2\text{Ei}(dx) + (ad^2 - 2bd)x^2\text{Ei}(-dx) \right) \cosh(c)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x^3,x, algorithm="fricas")

[Out] 
$$-1/4*(2*a*d*x*\sinh(d*x + c) + 2*(2*b*x + a)*\cosh(d*x + c) - ((a*d^2 + 2*b*d)*x^2*Ei(d*x) + (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*\cosh(c) - ((a*d^2 + 2*b*d)*x^2*Ei(d*x) - (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*\sinh(c))/x^2$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*cosh(d*x+c)/x**3,x)`

[Out] Exception raised: ValueError

**Giac [A]** time = 1.15187, size = 181, normalized size = 2.06

$$\frac{ad^2x^2Ei(-dx)e^{(-c)} + ad^2x^2Ei(dx)e^c - 2bdx^2Ei(-dx)e^{(-c)} + 2bdx^2Ei(dx)e^c - adxe^{(dx+c)} + adxe^{(-dx-c)} - 2bx e^{(dx+c)} - 2bx e^{(-dx-c)}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="giac")`

[Out] 
$$1/4*(a*d^2*x^2*Ei(-d*x)*e^{(-c)} + a*d^2*x^2*Ei(d*x)*e^c - 2*b*d*x^2*Ei(-d*x)*e^{(-c)} + 2*b*d*x^2*Ei(d*x)*e^c - a*d*x*e^{(d*x + c)} + a*d*x*e^{(-d*x - c)} - 2*b*x*e^{(d*x + c)} - 2*b*x*e^{(-d*x - c)} - a*e^{(d*x + c)} - a*e^{(-d*x - c)})/x^2$$

$$3.8 \quad \int \frac{(a+bx) \cosh(c+dx)}{x^4} dx$$

**Optimal.** Leaf size=132

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{ad \sinh(c+dx)}{6x^2} - \frac{a \cosh(c+dx)}{3x^3} + \frac{1}{2}bd^2 \cosh(c)C$$

[Out]  $-(a*\text{Cosh}[c + d*x])/(3*x^3) - (b*\text{Cosh}[c + d*x])/(2*x^2) - (a*d^2*\text{Cosh}[c + d*x])/(6*x) + (b*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (a*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 - (a*d*\text{Sinh}[c + d*x])/(6*x^2) - (b*d*\text{Sinh}[c + d*x])/(2*x) + (a*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6 + (b*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

**Rubi [A]** time = 0.339799, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{ad \sinh(c+dx)}{6x^2} - \frac{a \cosh(c+dx)}{3x^3} + \frac{1}{2}bd^2 \cosh(c)C$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x)\*Cosh[c + d\*x])/x^4, x]

[Out]  $-(a*\text{Cosh}[c + d*x])/(3*x^3) - (b*\text{Cosh}[c + d*x])/(2*x^2) - (a*d^2*\text{Cosh}[c + d*x])/(6*x) + (b*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (a*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 - (a*d*\text{Sinh}[c + d*x])/(6*x^2) - (b*d*\text{Sinh}[c + d*x])/(2*x) + (a*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6 + (b*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx) \cosh(c + dx)}{x^4} dx &= \int \left( \frac{a \cosh(c + dx)}{x^4} + \frac{b \cosh(c + dx)}{x^3} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^4} dx + b \int \frac{\cosh(c + dx)}{x^3} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{2x^2} + \frac{1}{3}(ad) \int \frac{\sinh(c + dx)}{x^3} dx + \frac{1}{2}(bd) \int \frac{\sinh(c + dx)}{x^2} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad \sinh(c + dx)}{6x^2} - \frac{bd \sinh(c + dx)}{2x} + \frac{1}{6}(ad^2) \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{6x} - \frac{ad \sinh(c + dx)}{6x^2} - \frac{bd \sinh(c + dx)}{2x} \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{6x} + \frac{1}{2}bd^2 \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c)}{6x^2} \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{6x} + \frac{1}{2}bd^2 \cosh(c) \text{Chi}(dx) + \frac{1}{6}ad^3 \text{Chi}(dx)
\end{aligned}$$

**Mathematica [A]** time = 0.2723, size = 110, normalized size = 0.83

$$\frac{-d^2 x^3 \text{Chi}(dx)(ad \sinh(c) + 3b \cosh(c)) - d^2 x^3 \text{Shi}(dx)(ad \cosh(c) + 3b \sinh(c)) + ad^2 x^2 \cosh(c + dx) + adx \sinh(c + dx)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Cosh[c + d*x])/x^4,x]
```

[Out]  $-(2*a*\text{Cosh}[c + d*x] + 3*b*x*\text{Cosh}[c + d*x] + a*d^2*x^2*\text{Cosh}[c + d*x] - d^2*x^3*\text{CoshIntegral}[d*x]*(3*b*\text{Cosh}[c] + a*d*\text{Sinh}[c]) + a*d*x*\text{Sinh}[c + d*x] + 3*b*d*x^2*\text{Sinh}[c + d*x] - d^2*x^3*(a*d*\text{Cosh}[c] + 3*b*\text{Sinh}[c])*SinhIntegral[d*x])/(6*x^3)$

**Maple [A]** time = 0.053, size = 205, normalized size = 1.6

$$-\frac{ad^2e^{-dx-c}}{12x} + \frac{dae^{-dx-c}}{12x^2} - \frac{ae^{-dx-c}}{6x^3} + \frac{d^3ae^{-c}\text{Ei}(1, dx)}{12} + \frac{bde^{-dx-c}}{4x} - \frac{be^{-dx-c}}{4x^2} - \frac{d^2be^{-c}\text{Ei}(1, dx)}{4} - \frac{ae^{dx+c}}{6x^3} - \frac{ade^{dx+c}}{12x^2} - \frac{ad^2e^{dx+c}}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)*cosh(d*x+c)/x^4, x)`

[Out]  $-1/12*d^2*a*\exp(-d*x-c)/x+1/12*d*a*\exp(-d*x-c)/x^2-1/6*a*\exp(-d*x-c)/x^3+1/12*d^3*a*\exp(-c)*\text{Ei}(1, d*x)+1/4*d*b*\exp(-d*x-c)/x-1/4*b*\exp(-d*x-c)/x^2-1/4*d^2*b*\exp(-c)*\text{Ei}(1, d*x)-1/6*a/x^3*\exp(d*x+c)-1/12*d*a/x^2*\exp(d*x+c)-1/12*d^2*a/x*\exp(d*x+c)-1/12*d^3*a*\exp(c)*\text{Ei}(1, -d*x)-1/4*b/x^2*\exp(d*x+c)-1/4*d*b/x*\exp(d*x+c)-1/4*d^2*b*\exp(c)*\text{Ei}(1, -d*x)$

**Maxima [A]** time = 1.39474, size = 105, normalized size = 0.8

$$\frac{1}{12} \left( 2ad^2e^{(-c)}\Gamma(-2, dx) - 2ad^2e^c\Gamma(-2, -dx) + 3bde^{(-c)}\Gamma(-1, dx) + 3bde^c\Gamma(-1, -dx) \right) d - \frac{(3bx + 2a) \cosh(dx + c)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)*cosh(d*x+c)/x^4, x, algorithm="maxima")`

[Out]  $1/12*(2*a*d^2*e^{(-c)}*\text{gamma}(-2, d*x) - 2*a*d^2*e^c*\text{gamma}(-2, -d*x) + 3*b*d*e^{(-c)}*\text{gamma}(-1, d*x) + 3*b*d*e^c*\text{gamma}(-1, -d*x))*d - 1/6*(3*b*x + 2*a)*\cosh(d*x + c)/x^3$

**Fricas [A]** time = 1.98033, size = 328, normalized size = 2.48

$$\frac{2(ad^2x^2 + 3bx + 2a) \cosh(dx + c) - \left( (ad^3 + 3bd^2)x^3\text{Ei}(dx) - (ad^3 - 3bd^2)x^3\text{Ei}(-dx) \right) \cosh(c) + 2(3bdx^2 + adx)}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x^4,x, algorithm="fricas")

[Out] 
$$\frac{-1/12*(2*(a*d^2*x^2 + 3*b*x + 2*a)*\cosh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*Ei(d*x) - (a*d^3 - 3*b*d^2)*x^3*Ei(-d*x))*\cosh(c) + 2*(3*b*d*x^2 + a*d*x)*\sinh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*Ei(d*x) + (a*d^3 - 3*b*d^2)*x^3*Ei(-d*x))*\sinh(c)}{x^3}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x\*\*4,x)

[Out] Timed out

**Giac [A]** time = 1.14764, size = 269, normalized size = 2.04

$$\frac{ad^3x^3Ei(-dx)e^{(-c)} - ad^3x^3Ei(dx)e^c - 3bd^2x^3Ei(-dx)e^{(-c)} - 3bd^2x^3Ei(dx)e^c + ad^2x^2e^{(dx+c)} + ad^2x^2e^{(-dx-c)} + 3bdx^2e^{(dx+c)} + 3bdx^2e^{(-dx-c)}}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x^4,x, algorithm="giac")

[Out] 
$$\frac{-1/12*(a*d^3*x^3*Ei(-d*x)*e^{(-c)} - a*d^3*x^3*Ei(d*x)*e^c - 3*b*d^2*x^3*Ei(-d*x)*e^{(-c)} - 3*b*d^2*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^{(d*x + c)} + a*d^2*x^2*e^{(-d*x - c)} + 3*b*d*x^2*e^{(d*x + c)} - 3*b*d*x^2*e^{(-d*x - c)} + a*d*x*e^{(d*x + c)} - a*d*x*e^{(-d*x - c)} + 3*b*x*e^{(d*x + c)} + 3*b*x*e^{(-d*x - c)} + 2*a*e^{(d*x + c)} + 2*a*e^{(-d*x - c)})}{x^3}$$

$$3.9 \quad \int \frac{(a+bx) \cosh(c+dx)}{x^5} dx$$

**Optimal.** Leaf size=166

$$\frac{1}{24}ad^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}ad^4 \sinh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{24x^2} - \frac{ad^3 \sinh(c+dx)}{24x} - \frac{ad \sinh(c+dx)}{12x^3} - \frac{a \cosh(c+dx)}{4x^4}$$

[Out]  $-(a*\text{Cosh}[c + d*x])/(4*x^4) - (b*\text{Cosh}[c + d*x])/(3*x^3) - (a*d^2*\text{Cosh}[c + d*x])/(24*x^2) - (b*d^2*\text{Cosh}[c + d*x])/(6*x) + (a*d^4*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/24 + (b*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 - (a*d*\text{Sinh}[c + d*x])/(12*x^3) - (b*d*\text{Sinh}[c + d*x])/(6*x^2) - (a*d^3*\text{Sinh}[c + d*x])/(24*x) + (b*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6 + (a*d^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/24$

**Rubi [A]** time = 0.40618, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{24}ad^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}ad^4 \sinh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{24x^2} - \frac{ad^3 \sinh(c+dx)}{24x} - \frac{ad \sinh(c+dx)}{12x^3} - \frac{a \cosh(c+dx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x)\*Cosh[c + d\*x])/x^5, x]

[Out]  $-(a*\text{Cosh}[c + d*x])/(4*x^4) - (b*\text{Cosh}[c + d*x])/(3*x^3) - (a*d^2*\text{Cosh}[c + d*x])/(24*x^2) - (b*d^2*\text{Cosh}[c + d*x])/(6*x) + (a*d^4*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/24 + (b*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 - (a*d*\text{Sinh}[c + d*x])/(12*x^3) - (b*d*\text{Sinh}[c + d*x])/(6*x^2) - (a*d^3*\text{Sinh}[c + d*x])/(24*x) + (b*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6 + (a*d^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/24$

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx) \cosh(c + dx)}{x^5} dx &= \int \left( \frac{a \cosh(c + dx)}{x^5} + \frac{b \cosh(c + dx)}{x^4} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^5} dx + b \int \frac{\cosh(c + dx)}{x^4} dx \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{3x^3} + \frac{1}{4}(ad) \int \frac{\sinh(c + dx)}{x^4} dx + \frac{1}{3}(bd) \int \frac{\sinh(c + dx)}{x^3} dx \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{3x^3} - \frac{ad \sinh(c + dx)}{12x^3} - \frac{bd \sinh(c + dx)}{6x^2} + \frac{1}{12}(ad^2) \int \frac{\cosh(c + dx)}{x^3} dx \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{24x^2} - \frac{bd^2 \cosh(c + dx)}{6x} - \frac{ad \sinh(c + dx)}{12x^3} \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{24x^2} - \frac{bd^2 \cosh(c + dx)}{6x} - \frac{ad \sinh(c + dx)}{12x^3} \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{24x^2} - \frac{bd^2 \cosh(c + dx)}{6x} + \frac{1}{6}bd^3 \text{Chi}(dx) \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{24x^2} - \frac{bd^2 \cosh(c + dx)}{6x} + \frac{1}{24}ad^4 \cosh(c + dx)
\end{aligned}$$

**Mathematica [A]** time = 0.318458, size = 140, normalized size = 0.84

$$\frac{-d^3 x^4 \text{Chi}(dx)(ad \cosh(c) + 4b \sinh(c)) - d^3 x^4 \text{Shi}(dx)(ad \sinh(c) + 4b \cosh(c)) + ad^3 x^3 \sinh(c + dx) + ad^2 x^2 \cosh(c + dx)}{24x^4}$$



Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)\*Cosh[c + d\*x])/x^5,x]

[Out]  $-(6*a*\text{Cosh}[c + d*x] + 8*b*x*\text{Cosh}[c + d*x] + a*d^2*x^2*\text{Cosh}[c + d*x] + 4*b*d^2*x^3*\text{Cosh}[c + d*x] - d^3*x^4*\text{CoshIntegral}[d*x]*(a*d*\text{Cosh}[c] + 4*b*\text{Sinh}[c]) + 2*a*d*x*\text{Sinh}[c + d*x] + 4*b*d*x^2*\text{Sinh}[c + d*x] + a*d^3*x^3*\text{Sinh}[c + d*x] - d^3*x^4*(4*b*\text{Cosh}[c] + a*d*\text{Sinh}[c])*\text{SinhIntegral}[d*x])/(24*x^4)$

**Maple [A]** time = 0.062, size = 271, normalized size = 1.6

$$\frac{ad^3e^{-dx-c}}{48x} - \frac{ad^2e^{-dx-c}}{48x^2} + \frac{dae^{-dx-c}}{24x^3} - \frac{ae^{-dx-c}}{8x^4} - \frac{d^4ae^{-c}\text{Ei}(1,dx)}{48} - \frac{bd^2e^{-dx-c}}{12x} + \frac{bde^{-dx-c}}{12x^2} - \frac{be^{-dx-c}}{6x^3} + \frac{d^3be^{-c}\text{Ei}(1,dx)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*cosh(d\*x+c)/x^5,x)

[Out]  $1/48*d^3*a*\exp(-d*x-c)/x - 1/48*d^2*a*\exp(-d*x-c)/x^2 + 1/24*d*a*\exp(-d*x-c)/x^3 - 1/8*a*\exp(-d*x-c)/x^4 - 1/48*d^4*a*\exp(-c)*\text{Ei}(1,d*x) - 1/12*d^2*b*\exp(-d*x-c)/x + 1/12*d*b*\exp(-d*x-c)/x^2 - 1/6*b*\exp(-d*x-c)/x^3 + 1/12*d^3*b*\exp(-c)*\text{Ei}(1,d*x) - 1/8*a/x^4*\exp(d*x+c) - 1/24*d*a/x^3*\exp(d*x+c) - 1/48*d^2*a/x^2*\exp(d*x+c) - 1/48*d^3*a/x*\exp(d*x+c) - 1/48*d^4*a*\exp(c)*\text{Ei}(1,-d*x) - 1/6*b/x^3*\exp(d*x+c) - 1/12*d*b/x^2*\exp(d*x+c) - 1/12*d^2*b/x*\exp(d*x+c) - 1/12*d^3*b*\exp(c)*\text{Ei}(1,-d*x)$

**Maxima [A]** time = 1.39888, size = 111, normalized size = 0.67

$$\frac{1}{24} \left( 3ad^3e^{(-c)}\Gamma(-3,dx) + 3ad^3e^c\Gamma(-3,-dx) + 4bd^2e^{(-c)}\Gamma(-2,dx) - 4bd^2e^c\Gamma(-2,-dx) \right) d - \frac{(4bx + 3a)\cosh(dx + c)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x^5,x, algorithm="maxima")

[Out]  $1/24*(3*a*d^3*e^{(-c)}*\text{gamma}(-3,d*x) + 3*a*d^3*e^c*\text{gamma}(-3,-d*x) + 4*b*d^2*e^{(-c)}*\text{gamma}(-2,d*x) - 4*b*d^2*e^c*\text{gamma}(-2,-d*x))*d - 1/12*(4*b*x + 3*a)*\cosh(d*x + c)/x^4$

**Fricas [A]** time = 2.02097, size = 366, normalized size = 2.2

$$\frac{2(4bd^2x^3 + ad^2x^2 + 8bx + 6a) \cosh(dx + c) - ((ad^4 + 4bd^3)x^4 \operatorname{Ei}(dx) + (ad^4 - 4bd^3)x^4 \operatorname{Ei}(-dx)) \cosh(c) + 2(ad^3x^3 + 4bd^2x^2 + 2ad^2x) \sinh(dx + c) - ((ad^4 + 4bd^3)x^4 \operatorname{Ei}(dx) - (ad^4 - 4bd^3)x^4 \operatorname{Ei}(-dx)) \sinh(c)}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x^5,x, algorithm="fricas")

[Out] -1/48\*(2\*(4\*b\*d^2\*x^3 + a\*d^2\*x^2 + 8\*b\*x + 6\*a)\*cosh(d\*x + c) - ((a\*d^4 + 4\*b\*d^3)\*x^4\*Ei(d\*x) + (a\*d^4 - 4\*b\*d^3)\*x^4\*Ei(-d\*x))\*cosh(c) + 2\*(a\*d^3\*x^3 + 4\*b\*d^2\*x^2 + 2\*a\*d\*x)\*sinh(d\*x + c) - ((a\*d^4 + 4\*b\*d^3)\*x^4\*Ei(d\*x) - (a\*d^4 - 4\*b\*d^3)\*x^4\*Ei(-d\*x))\*sinh(c))/x^4

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x\*\*5,x)

[Out] Timed out

**Giac [A]** time = 1.26538, size = 359, normalized size = 2.16

$$\frac{ad^4x^4\operatorname{Ei}(-dx)e^{(-c)} + ad^4x^4\operatorname{Ei}(dx)e^c - 4bd^3x^4\operatorname{Ei}(-dx)e^{(-c)} + 4bd^3x^4\operatorname{Ei}(dx)e^c - ad^3x^3e^{(dx+c)} + ad^3x^3e^{(-dx-c)} - 4bd^2x^3e^{(dx+c)} - 4bd^2x^3e^{(-dx-c)} + 2ad^2x^2e^{(dx+c)} + 2ad^2x^2e^{(-dx-c)} - 4bd^2x^2e^{(dx+c)} - 4bd^2x^2e^{(-dx-c)} + 4bd^2x^2e^{(dx+c)} + 4bd^2x^2e^{(-dx-c)} - 2ad^2x^2e^{(dx+c)} - 2ad^2x^2e^{(-dx-c)} - 8bd^2x^2e^{(dx+c)} - 8bd^2x^2e^{(-dx-c)} - 6ad^2x^2e^{(dx+c)} - 6ad^2x^2e^{(-dx-c)})}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*cosh(d\*x+c)/x^5,x, algorithm="giac")

[Out] 1/48\*(a\*d^4\*x^4\*Ei(-d\*x)\*e^(-c) + a\*d^4\*x^4\*Ei(d\*x)\*e^c - 4\*b\*d^3\*x^4\*Ei(-d\*x)\*e^(-c) + 4\*b\*d^3\*x^4\*Ei(d\*x)\*e^c - a\*d^3\*x^3\*e^(d\*x + c) + a\*d^3\*x^3\*e^(-d\*x - c) - 4\*b\*d^2\*x^3\*e^(d\*x + c) - 4\*b\*d^2\*x^3\*e^(-d\*x - c) - a\*d^2\*x^2\*e^(d\*x + c) - a\*d^2\*x^2\*e^(-d\*x - c) - 4\*b\*d\*x^2\*e^(d\*x + c) + 4\*b\*d\*x^2\*e^(-d\*x - c) - 2\*a\*d\*x^2\*e^(d\*x + c) + 2\*a\*d\*x^2\*e^(-d\*x - c) - 8\*b\*x^2\*e^(d\*x + c) - 8\*b\*x^2\*e^(-d\*x - c) - 6\*a\*e^(d\*x + c) - 6\*a\*e^(-d\*x - c))/x^4

### 3.10 $\int x^2(a + bx)^2 \cosh(c + dx) dx$

**Optimal.** Leaf size=184

$$\frac{2a^2 \sinh(c + dx)}{d^3} - \frac{2a^2 x \cosh(c + dx)}{d^2} + \frac{a^2 x^2 \sinh(c + dx)}{d} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{12ab \cosh(c + dx)}{d^4}$$

[Out]  $(-12*a*b*Cosh[c + d*x])/d^4 - (24*b^2*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Cosh[c + d*x])/d^2 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + (24*b^2*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (12*a*b*x*Sinh[c + d*x])/d^3 + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/d + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.346497, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {6742, 3296, 2637, 2638}

$$\frac{2a^2 \sinh(c + dx)}{d^3} - \frac{2a^2 x \cosh(c + dx)}{d^2} + \frac{a^2 x^2 \sinh(c + dx)}{d} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{12ab \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*x)^2*Cosh[c + d*x], x]$

[Out]  $(-12*a*b*Cosh[c + d*x])/d^4 - (24*b^2*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Cosh[c + d*x])/d^2 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + (24*b^2*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (12*a*b*x*Sinh[c + d*x])/d^3 + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/d + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

#### Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^(m_.)*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^(m - 1)*\cos[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{GtQ}[m, 0]$

#### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int x^2(a + bx)^2 \cosh(c + dx) dx &= \int (a^2x^2 \cosh(c + dx) + 2abx^3 \cosh(c + dx) + b^2x^4 \cosh(c + dx)) dx \\
 &= a^2 \int x^2 \cosh(c + dx) dx + (2ab) \int x^3 \cosh(c + dx) dx + b^2 \int x^4 \cosh(c + dx) dx \\
 &= \frac{a^2x^2 \sinh(c + dx)}{d} + \frac{2abx^3 \sinh(c + dx)}{d} + \frac{b^2x^4 \sinh(c + dx)}{d} - \frac{(2a^2) \int x \sinh(c + dx) dx}{d} \\
 &= -\frac{2a^2x \cosh(c + dx)}{d^2} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{4b^2x^3 \cosh(c + dx)}{d^2} + \frac{a^2x^2 \sinh(c + dx)}{d} + \dots \\
 &= -\frac{2a^2x \cosh(c + dx)}{d^2} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{4b^2x^3 \cosh(c + dx)}{d^2} + \frac{2a^2 \sinh(c + dx)}{d^3} + \dots \\
 &= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{24b^2x \cosh(c + dx)}{d^4} - \frac{2a^2x \cosh(c + dx)}{d^2} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \dots \\
 &= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{24b^2x \cosh(c + dx)}{d^4} - \frac{2a^2x \cosh(c + dx)}{d^2} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \dots
 \end{aligned}$$

**Mathematica [A]** time = 0.241922, size = 100, normalized size = 0.54

$$\frac{(a^2d^2(d^2x^2 + 2) + 2abd^2x(d^2x^2 + 6) + b^2(d^4x^4 + 12d^2x^2 + 24)) \sinh(c + dx) - 2d(a + 2bx)(ad^2x + b(d^2x^2 + 6)) \cosh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^2*Cosh[c + d*x], x]
```

```
[Out] (-2*d*(a + 2*b*x)*(a*d^2*x + b*(6 + d^2*x^2))*Cosh[c + d*x] + (a^2*d^2*(2 +
d^2*x^2) + 2*a*b*d^2*x*(6 + d^2*x^2) + b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Si
nh[c + d*x])/d^5
```

**Maple [B]** time = 0.007, size = 463, normalized size = 2.5

$$\frac{1}{d^3} \left( \frac{b^2 \left( (dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \right)}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x+a)^2\*cosh(d\*x+c),x)

[Out] 1/d^3\*(b^2/d^2\*((d\*x+c)^4\*sinh(d\*x+c)-4\*(d\*x+c)^3\*cosh(d\*x+c)+12\*(d\*x+c)^2\*sinh(d\*x+c)-24\*(d\*x+c)\*cosh(d\*x+c)+24\*sinh(d\*x+c))-4\*b^2/d^2\*c\*((d\*x+c)^3\*sinh(d\*x+c)-3\*(d\*x+c)^2\*cosh(d\*x+c)+6\*(d\*x+c)\*sinh(d\*x+c)-6\*cosh(d\*x+c))+6\*b^2\*c^2/d^2\*((d\*x+c)^2\*sinh(d\*x+c)-2\*(d\*x+c)\*cosh(d\*x+c)+2\*sinh(d\*x+c))-4\*b^2/d^2\*c^3\*((d\*x+c)\*sinh(d\*x+c)-cosh(d\*x+c))+2\*b/d\*a\*((d\*x+c)^3\*sinh(d\*x+c)-3\*(d\*x+c)^2\*cosh(d\*x+c)+6\*(d\*x+c)\*sinh(d\*x+c)-6\*cosh(d\*x+c))-6\*b\*c/d\*a\*((d\*x+c)^2\*sinh(d\*x+c)-2\*(d\*x+c)\*cosh(d\*x+c)+2\*sinh(d\*x+c))+6\*b/d\*a\*c^2\*((d\*x+c)\*sinh(d\*x+c)-cosh(d\*x+c))+b^2\*c^4/d^2\*sinh(d\*x+c)-2\*b\*c^3/d\*a\*sinh(d\*x+c)+a^2\*((d\*x+c)^2\*sinh(d\*x+c)-2\*(d\*x+c)\*cosh(d\*x+c)+2\*sinh(d\*x+c))-2\*a^2\*c\*((d\*x+c)\*sinh(d\*x+c)-cosh(d\*x+c))+a^2\*c^2\*sinh(d\*x+c))

**Maxima [A]** time = 1.18353, size = 444, normalized size = 2.41

$$-\frac{1}{60} d \left( \frac{10(d^3 x^3 e^c - 3d^2 x^2 e^c + 6dx e^c - 6e^c) a^2 e^{(dx)}}{d^4} + \frac{10(d^3 x^3 + 3d^2 x^2 + 6dx + 6) a^2 e^{(-dx-c)}}{d^4} + \frac{15(d^4 x^4 e^c - 4d^3 x^3 e^c + 12d^2 x^2 e^c - 24dx e^c + 24e^c) a b e^{(dx)}}{d^5} + \frac{15(d^4 x^4 + 4d^3 x^3 + 12d^2 x^2 + 24dx + 24) a b e^{(-dx-c)}}{d^5} + \frac{6(d^5 x^5 e^c - 5d^4 x^4 e^c + 20d^3 x^3 e^c - 60d^2 x^2 e^c + 120dx e^c - 120e^c) b^2 e^{(dx)}}{d^6} + \frac{6(d^5 x^5 + 5d^4 x^4 + 20d^3 x^3 + 60d^2 x^2 + 120dx + 120) b^2 e^{(-dx-c)}}{d^6} + \frac{1}{30} (6b^2 x^5 + 15a b x^4 + 10a^2 x^3) \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x+a)^2\*cosh(d\*x+c),x, algorithm="maxima")

[Out] -1/60\*d\*(10\*(d^3\*x^3\*e^c - 3\*d^2\*x^2\*e^c + 6\*d\*x\*e^c - 6\*e^c)\*a^2\*e^(d\*x)/d^4 + 10\*(d^3\*x^3 + 3\*d^2\*x^2 + 6\*d\*x + 6)\*a^2\*e^(-d\*x - c)/d^4 + 15\*(d^4\*x^4\*e^c - 4\*d^3\*x^3\*e^c + 12\*d^2\*x^2\*e^c - 24\*d\*x\*e^c + 24\*e^c)\*a\*b\*e^(d\*x)/d^5 + 15\*(d^4\*x^4 + 4\*d^3\*x^3 + 12\*d^2\*x^2 + 24\*d\*x + 24)\*a\*b\*e^(-d\*x - c)/d^5 + 6\*(d^5\*x^5\*e^c - 5\*d^4\*x^4\*e^c + 20\*d^3\*x^3\*e^c - 60\*d^2\*x^2\*e^c + 120\*d\*x\*e^c - 120\*e^c)\*b^2\*e^(d\*x)/d^6 + 6\*(d^5\*x^5 + 5\*d^4\*x^4 + 20\*d^3\*x^3 + 60\*d^2\*x^2 + 120\*d\*x + 120)\*b^2\*e^(-d\*x - c)/d^6) + 1/30\*(6\*b^2\*x^5 + 15\*a\*b\*x^4 + 10\*a^2\*x^3)\*cosh(d\*x + c)

**Fricas [A]** time = 2.01887, size = 273, normalized size = 1.48

$$\frac{2 \left( 2 b^2 d^3 x^3 + 3 a b d^3 x^2 + 6 a b d + (a^2 d^3 + 12 b^2 d) x \right) \cosh(dx + c) - \left( b^2 d^4 x^4 + 2 a b d^4 x^3 + 12 a b d^2 x + 2 a^2 d^2 + (a^2 d^4 + 12 b^2 d^2) x^2 + 24 b^2 \right) \sinh(dx + c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x+a)^2\*cosh(d\*x+c),x, algorithm="fricas")

[Out]  $-(2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2 + 6*a*b*d + (a^2*d^3 + 12*b^2*d)*x)*\cosh(d*x + c) - (b^2*d^4*x^4 + 2*a*b*d^4*x^3 + 12*a*b*d^2*x + 2*a^2*d^2 + (a^2*d^4 + 12*b^2*d^2)*x^2 + 24*b^2)*\sinh(d*x + c))/d^5$

**Sympy [A]** time = 3.53062, size = 228, normalized size = 1.24

$$\left\{ \begin{array}{l} \frac{a^2 x^2 \sinh(c+dx)}{d^2} - \frac{2a^2 x \cosh(c+dx)}{d^2} + \frac{2a^2 \sinh(c+dx)}{d^3} + \frac{2abx^3 \sinh(c+dx)}{d} - \frac{6abx^2 \cosh(c+dx)}{d^2} + \frac{12abx \sinh(c+dx)}{d^3} - \frac{12ab \cosh(c+dx)}{d^4} + \frac{b^2 x^4 \sinh(c+dx)}{d^5} \\ \left( \frac{a^2 x^3}{3} + \frac{abx^4}{2} + \frac{b^2 x^5}{5} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x+a)\*\*2\*cosh(d\*x+c),x)

[Out] Piecewise((a\*\*2\*x\*\*2\*sinh(c + d\*x)/d - 2\*a\*\*2\*x\*cosh(c + d\*x)/d\*\*2 + 2\*a\*\*2\*sinh(c + d\*x)/d\*\*3 + 2\*a\*b\*x\*\*3\*sinh(c + d\*x)/d - 6\*a\*b\*x\*\*2\*cosh(c + d\*x)/d\*\*2 + 12\*a\*b\*x\*sinh(c + d\*x)/d\*\*3 - 12\*a\*b\*cosh(c + d\*x)/d\*\*4 + b\*\*2\*x\*\*4\*sinh(c + d\*x)/d - 4\*b\*\*2\*x\*\*3\*cosh(c + d\*x)/d\*\*2 + 12\*b\*\*2\*x\*\*2\*sinh(c + d\*x)/d\*\*3 - 24\*b\*\*2\*x\*cosh(c + d\*x)/d\*\*4 + 24\*b\*\*2\*sinh(c + d\*x)/d\*\*5, Ne(d, 0)), ((a\*\*2\*x\*\*3/3 + a\*b\*x\*\*4/2 + b\*\*2\*x\*\*5/5)\*cosh(c), True))

**Giac [A]** time = 1.17721, size = 319, normalized size = 1.73

$$\frac{(b^2 d^4 x^4 + 2 a b d^4 x^3 + a^2 d^4 x^2 - 4 b^2 d^3 x^3 - 6 a b d^3 x^2 - 2 a^2 d^3 x + 12 b^2 d^2 x^2 + 12 a b d^2 x + 2 a^2 d^2 - 24 b^2 d x - 12 a b d + 24 b^2) \cosh(dx + c) - (b^2 d^4 x^4 + 2 a b d^4 x^3 + a^2 d^4 x^2 - 4 b^2 d^3 x^3 - 6 a b d^3 x^2 - 2 a^2 d^3 x + 12 b^2 d^2 x^2 + 12 a b d^2 x + 2 a^2 d^2 - 24 b^2 d x - 12 a b d + 24 b^2) \sinh(dx + c)}{2 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x+a)^2\*cosh(d\*x+c),x, algorithm="giac")

[Out]  $\frac{1}{2}(b^2d^4x^4 + 2ab^2d^4x^3 + a^2d^4x^2 - 4b^2d^3x^3 - 6ab^2d^3x^2 - 2a^2d^3x + 12b^2d^2x^2 + 12ab^2d^2x + 2a^2d^2 - 24b^2dx - 12abd + 24b^2)e^{(dx + c)}/d^5 - \frac{1}{2}(b^2d^4x^4 + 2ab^2d^4x^3 + a^2d^4x^2 + 4b^2d^3x^3 + 6ab^2d^3x^2 + 2a^2d^3x + 12b^2d^2x^2 + 12ab^2d^2x + 2a^2d^2 + 24b^2dx + 12abd + 24b^2)e^{(-dx - c)}/d^5$

5

### 3.11 $\int x(a + bx)^2 \cosh(c + dx) dx$

**Optimal.** Leaf size=134

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} + \frac{4ab \sinh(c + dx)}{d^3} - \frac{4abx \cosh(c + dx)}{d^2} + \frac{2abx^2 \sinh(c + dx)}{d} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2}$$

[Out]  $(-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/d^2 - (4*a*b*x*Cosh[c + d*x])/d^2 - (3*b^2*x^2*Cosh[c + d*x])/d^2 + (4*a*b*Sinh[c + d*x])/d^3 + (6*b^2*x*Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (2*a*b*x^2*Sinh[c + d*x])/d + (b^2*x^3*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.214288, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {6742, 3296, 2638, 2637}

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} + \frac{4ab \sinh(c + dx)}{d^3} - \frac{4abx \cosh(c + dx)}{d^2} + \frac{2abx^2 \sinh(c + dx)}{d} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x)^2\*Cosh[c + d\*x],x]

[Out]  $(-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/d^2 - (4*a*b*x*Cosh[c + d*x])/d^2 - (3*b^2*x^2*Cosh[c + d*x])/d^2 + (4*a*b*Sinh[c + d*x])/d^3 + (6*b^2*x*Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (2*a*b*x^2*Sinh[c + d*x])/d + (b^2*x^3*Sinh[c + d*x])/d$

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2638



Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int x(a + bx)^2 \cosh(c + dx) dx &= \int (a^2x \cosh(c + dx) + 2abx^2 \cosh(c + dx) + b^2x^3 \cosh(c + dx)) dx \\
 &= a^2 \int x \cosh(c + dx) dx + (2ab) \int x^2 \cosh(c + dx) dx + b^2 \int x^3 \cosh(c + dx) dx \\
 &= \frac{a^2x \sinh(c + dx)}{d} + \frac{2abx^2 \sinh(c + dx)}{d} + \frac{b^2x^3 \sinh(c + dx)}{d} - \frac{a^2 \int \sinh(c + dx) dx}{d} - \frac{(2ab) \int x \sinh(c + dx) dx}{d} - \frac{b^2 \int x^2 \sinh(c + dx) dx}{d} \\
 &= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{3b^2x^2 \cosh(c + dx)}{d^2} + \frac{a^2x \sinh(c + dx)}{d} + \frac{2abx^2 \sinh(c + dx)}{d} + \frac{b^2x^3 \sinh(c + dx)}{d} \\
 &= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{3b^2x^2 \cosh(c + dx)}{d^2} + \frac{4ab \sinh(c + dx)}{d^3} + \frac{6b^2x \sinh(c + dx)}{d^3} + \frac{b^2x^2 \sinh(c + dx)}{d^3} \\
 &= -\frac{6b^2 \cosh(c + dx)}{d^4} - \frac{a^2 \cosh(c + dx)}{d^2} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{3b^2x^2 \cosh(c + dx)}{d^2} - \frac{3b^2x^2 \cosh(c + dx)}{d^2} + \frac{4ab \sinh(c + dx)}{d^3} + \frac{6b^2x \sinh(c + dx)}{d^3} + \frac{b^2x^2 \sinh(c + dx)}{d^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.188351, size = 87, normalized size = 0.65

$$\frac{d(a^2d^2x + 2ab(d^2x^2 + 2) + b^2x(d^2x^2 + 6)) \sinh(c + dx) - (a^2d^2 + 4abd^2x + 3b^2(d^2x^2 + 2)) \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x)^2\*Cosh[c + d\*x], x]

[Out] (-((a^2\*d^2 + 4\*a\*b\*d^2\*x + 3\*b^2\*(2 + d^2\*x^2))\*Cosh[c + d\*x]) + d\*(a^2\*d^2\*x + 2\*a\*b\*(2 + d^2\*x^2) + b^2\*x\*(6 + d^2\*x^2))\*Sinh[c + d\*x])/d^4

**Maple [B]** time = 0.01, size = 283, normalized size = 2.1

$$\frac{1}{d^2} \left( \frac{b^2 \left( (dx + c)^3 \sinh(dx + c) - 3(dx + c)^2 \cosh(dx + c) + 6(dx + c) \sinh(dx + c) - 6 \cosh(dx + c) \right)}{d^2} - 3 \frac{cb^2 \left( (dx + c)^3 \sinh(dx + c) - 3(dx + c)^2 \cosh(dx + c) + 6(dx + c) \sinh(dx + c) - 6 \cosh(dx + c) \right)}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x+a)^2*cosh(d*x+c),x)`

[Out]  $1/d^2*(b^2/d^2*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))-3*b^2/d^2*c*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))+2*b/d*a*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))+3*b^2*c^2/d^2*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))-4*b*c/d*a*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+a^2*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))-b^2*c^3/d^2*\sinh(d*x+c)+2*b*c^2/d*a*\sinh(d*x+c)-c*a^2*\sinh(d*x+c))$

**Maxima [B]** time = 1.17618, size = 371, normalized size = 2.77

$$-\frac{1}{24}d\left(\frac{6(d^2x^2e^c - 2dxe^c + 2e^c)a^2e^{(dx)}}{d^3} + \frac{6(d^2x^2 + 2dx + 2)a^2e^{(-dx-c)}}{d^3} + \frac{8(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)abe^{(dx)}}{d^4} + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")`

[Out]  $-1/24*d*(6*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^{(d*x)}/d^3 + 6*(d^2*x^2 + 2*d*x + 2)*a^2*e^{(-d*x - c)}/d^3 + 8*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*b*e^{(d*x)}/d^4 + 8*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*b*e^{(-d*x - c)}/d^4 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^{(d*x)}/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^{(-d*x - c)}/d^5 + 1/12*(3*b^2*x^4 + 8*a*b*x^3 + 6*a^2*x^2)*\cosh(d*x + c)$

**Fricas [A]** time = 2.00752, size = 203, normalized size = 1.51

$$\frac{(3b^2d^2x^2 + 4abd^2x + a^2d^2 + 6b^2)\cosh(dx + c) - (b^2d^3x^3 + 2abd^3x^2 + 4abd + (a^2d^3 + 6b^2d)x)\sinh(dx + c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")`

[Out]  $-((3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 + 6*b^2)*\cosh(d*x + c) - (b^2*d^3*x^3 + 2*a*b*d^3*x^2 + 4*a*b*d + (a^2*d^3 + 6*b^2*d)*x)*\sinh(d*x + c))/d^4$

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**Sympy [A]** time = 2.11423, size = 172, normalized size = 1.28

$$\left\{ \frac{a^2 x \sinh(c+dx)}{d} - \frac{a^2 \cosh(c+dx)}{d^2} + \frac{2abx^2 \sinh(c+dx)}{d} - \frac{4abx \cosh(c+dx)}{d^2} + \frac{4ab \sinh(c+dx)}{d^3} + \frac{b^2 x^3 \sinh(c+dx)}{d} - \frac{3b^2 x^2 \cosh(c+dx)}{d^2} + \frac{6b^2 x \sinh(c+dx)}{d^3} \right\} \left( \frac{a^2 x^2}{2} + \frac{2abx^3}{3} + \frac{b^2 x^4}{4} \right) \cosh(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x+a)\*\*2\*cosh(d\*x+c), x)

[Out] Piecewise((a\*\*2\*x\*sinh(c + d\*x)/d - a\*\*2\*cosh(c + d\*x)/d\*\*2 + 2\*a\*b\*x\*\*2\*sinh(c + d\*x)/d - 4\*a\*b\*x\*cosh(c + d\*x)/d\*\*2 + 4\*a\*b\*sinh(c + d\*x)/d\*\*3 + b\*\*2\*x\*\*3\*sinh(c + d\*x)/d - 3\*b\*\*2\*x\*\*2\*cosh(c + d\*x)/d\*\*2 + 6\*b\*\*2\*x\*sinh(c + d\*x)/d\*\*3 - 6\*b\*\*2\*cosh(c + d\*x)/d\*\*4, Ne(d, 0)), ((a\*\*2\*x\*\*2/2 + 2\*a\*b\*x\*\*3/3 + b\*\*2\*x\*\*4/4)\*cosh(c), True))

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**Giac [A]** time = 1.14316, size = 231, normalized size = 1.72

$$\frac{(b^2 d^3 x^3 + 2 abd^3 x^2 + a^2 d^3 x - 3 b^2 d^2 x^2 - 4 abd^2 x - a^2 d^2 + 6 b^2 dx + 4 abd - 6 b^2) e^{(dx+c)}}{2 d^4} - \frac{(b^2 d^3 x^3 + 2 abd^3 x^2 + a^2 d^3 x + 3 b^2 d^2 x^2 + 4 abd^2 x + a^2 d^2 + 6 b^2 dx + 4 abd - 6 b^2) e^{-(d*x - c)}}{2 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x+a)^2\*cosh(d\*x+c), x, algorithm="giac")

[Out] 1/2\*(b^2\*d^3\*x^3 + 2\*a\*b\*d^3\*x^2 + a^2\*d^3\*x - 3\*b^2\*d^2\*x^2 - 4\*a\*b\*d^2\*x - a^2\*d^2 + 6\*b^2\*d\*x + 4\*a\*b\*d - 6\*b^2)\*e^(d\*x + c)/d^4 - 1/2\*(b^2\*d^3\*x^3 + 2\*a\*b\*d^3\*x^2 + a^2\*d^3\*x + 3\*b^2\*d^2\*x^2 + 4\*a\*b\*d^2\*x + a^2\*d^2 + 6\*b^2\*d\*x + 4\*a\*b\*d - 6\*b^2)\*e^(-d\*x - c)/d^4

### 3.12 $\int (a + bx)^2 \cosh(c + dx) dx$

**Optimal.** Leaf size=49

$$-\frac{2b(a + bx) \cosh(c + dx)}{d^2} + \frac{(a + bx)^2 \sinh(c + dx)}{d} + \frac{2b^2 \sinh(c + dx)}{d^3}$$

[Out]  $(-2*b*(a + b*x)*Cosh[c + d*x])/d^2 + (2*b^2*Sinh[c + d*x])/d^3 + ((a + b*x)^2*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.0479988, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3296, 2637}

$$-\frac{2b(a + bx) \cosh(c + dx)}{d^2} + \frac{(a + bx)^2 \sinh(c + dx)}{d} + \frac{2b^2 \sinh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x)^2*\text{Cosh}[c + d*x], x]$

[Out]  $(-2*b*(a + b*x)*Cosh[c + d*x])/d^2 + (2*b^2*Sinh[c + d*x])/d^3 + ((a + b*x)^2*Sinh[c + d*x])/d$

#### Rule 3296

$\text{Int}[(c + d*x)^m \sin(e + f*x), x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m \cos(e + f*x)/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{m-1} \cos(e + f*x), x], x] /; \text{FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{GtQ}[m, 0]$

#### Rule 2637

$\text{Int}[\sin(\pi/2 + (c + d*x)), x\_Symbol] \rightarrow \text{Simp}[\sin(c + d*x)/d, x] /; \text{FreeQ}\{c, d, x\}$

#### Rubi steps

$$\begin{aligned}
\int (a + bx)^2 \cosh(c + dx) dx &= \frac{(a + bx)^2 \sinh(c + dx)}{d} - \frac{(2b) \int (a + bx) \sinh(c + dx) dx}{d} \\
&= -\frac{2b(a + bx) \cosh(c + dx)}{d^2} + \frac{(a + bx)^2 \sinh(c + dx)}{d} + \frac{(2b^2) \int \cosh(c + dx) dx}{d^2} \\
&= -\frac{2b(a + bx) \cosh(c + dx)}{d^2} + \frac{2b^2 \sinh(c + dx)}{d^3} + \frac{(a + bx)^2 \sinh(c + dx)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.142182, size = 56, normalized size = 1.14

$$\frac{(a^2 d^2 + 2abd^2 x + b^2(d^2 x^2 + 2)) \sinh(c + dx) - 2bd(a + bx) \cosh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^2\*Cosh[c + d\*x], x]

[Out] (-2\*b\*d\*(a + b\*x)\*Cosh[c + d\*x] + (a^2\*d^2 + 2\*a\*b\*d^2\*x + b^2\*(2 + d^2\*x^2))\*Sinh[c + d\*x])/d^3

**Maple [B]** time = 0.007, size = 147, normalized size = 3.

$$\frac{1}{d} \left( \frac{b^2 \left( (dx + c)^2 \sinh(dx + c) - 2(dx + c) \cosh(dx + c) + 2 \sinh(dx + c) \right)}{d^2} - 2 \frac{cb^2 \left( (dx + c) \sinh(dx + c) - \cosh(dx + c) \right)}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^2\*cosh(d\*x+c), x)

[Out] 1/d\*(b^2/d^2\*((d\*x+c)^2\*sinh(d\*x+c)-2\*(d\*x+c)\*cosh(d\*x+c)+2\*sinh(d\*x+c))-2\*b^2/d^2\*c\*((d\*x+c)\*sinh(d\*x+c)-cosh(d\*x+c))+2\*b/d\*a\*((d\*x+c)\*sinh(d\*x+c)-cosh(d\*x+c))+b^2\*c^2/d^2\*sinh(d\*x+c)-2\*b\*c/d\*a\*sinh(d\*x+c)+a^2\*sinh(d\*x+c))

**Maxima [B]** time = 1.20621, size = 182, normalized size = 3.71

$$\frac{a^2 e^{(dx+c)}}{2d} + \frac{(dx e^c - e^c) a b e^{(dx)}}{d^2} - \frac{(dx + 1) a b e^{(-dx-c)}}{d^2} - \frac{a^2 e^{(-dx-c)}}{2d} + \frac{(d^2 x^2 e^c - 2 dx e^c + 2 e^c) b^2 e^{(dx)}}{2d^3} - \frac{(d^2 x^2 + 2 dx + 2) b^2 e^{(dx)}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c),x, algorithm="maxima")

[Out]  $\frac{1}{2}a^2e^{(d*x+c)}/d + (d*x*e^c - e^c)*a*b*e^{(d*x)}/d^2 - (d*x+1)*a*b*e^{(-d*x-c)}/d^2 - \frac{1}{2}a^2e^{(-d*x-c)}/d + \frac{1}{2}(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*b^2*e^{(d*x)}/d^3 - \frac{1}{2}(d^2*x^2 + 2*d*x + 2)*b^2*e^{(-d*x-c)}/d^3$

**Fricas [A]** time = 1.94877, size = 140, normalized size = 2.86

$$\frac{2(b^2dx + abd) \cosh(dx + c) - (b^2d^2x^2 + 2abd^2x + a^2d^2 + 2b^2) \sinh(dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c),x, algorithm="fricas")

[Out]  $-(2*(b^2*d*x + a*b*d)*\cosh(d*x + c) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 + 2*b^2)*\sinh(d*x + c))/d^3$

**Sympy [A]** time = 1.0641, size = 112, normalized size = 2.29

$$\begin{cases} \frac{a^2 \sinh(c+dx)}{d} + \frac{2abx \sinh(c+dx)}{d} - \frac{2ab \cosh(c+dx)}{d^2} + \frac{b^2x^2 \sinh(c+dx)}{d} - \frac{2b^2x \cosh(c+dx)}{d^2} + \frac{2b^2 \sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*cosh(d\*x+c),x)

[Out] Piecewise((a\*\*2\*sinh(c + d\*x)/d + 2\*a\*b\*x\*sinh(c + d\*x)/d - 2\*a\*b\*cosh(c + d\*x)/d\*\*2 + b\*\*2\*x\*\*2\*sinh(c + d\*x)/d - 2\*b\*\*2\*x\*cosh(c + d\*x)/d\*\*2 + 2\*b\*\*2\*sinh(c + d\*x)/d\*\*3, Ne(d, 0)), ((a\*\*2\*x + a\*b\*x\*\*2 + b\*\*2\*x\*\*3/3)\*cosh(c), True))

**Giac [B]** time = 1.21508, size = 151, normalized size = 3.08

$$\frac{(b^2d^2x^2 + 2abd^2x + a^2d^2 - 2b^2dx - 2abd + 2b^2)e^{(dx+c)}}{2d^3} - \frac{(b^2d^2x^2 + 2abd^2x + a^2d^2 + 2b^2dx + 2abd + 2b^2)e^{(-dx-c)}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2*d*x - 2*a*b*d + 2*b^2)*e^(d*x + c)/d^3 - 1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 + 2*b^2*d*x + 2*a*b*d + 2*b^2)*e^(-d*x - c)/d^3
```

### 3.13 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x} dx$

**Optimal.** Leaf size=62

$$a^2 \cosh(c) \text{Chi}(dx) + a^2 \sinh(c) \text{Shi}(dx) + \frac{2ab \sinh(c+dx)}{d} - \frac{b^2 \cosh(c+dx)}{d^2} + \frac{b^2 x \sinh(c+dx)}{d}$$

[Out]  $-\left(\frac{b^2 \cosh[c+dx]}{d^2}\right) + a^2 \cosh[c] \text{CoshIntegral}[dx] + \left(\frac{2ab \sinh[c+dx]}{d} + \frac{b^2 x \sinh[c+dx]}{d}\right) + \frac{b^2 \cosh[c+dx]}{d^2} + a^2 \sinh[c] \text{SinhIntegral}[dx]$

**Rubi [A]** time = 0.184752, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {6742, 2637, 3303, 3298, 3301, 3296, 2638}

$$a^2 \cosh(c) \text{Chi}(dx) + a^2 \sinh(c) \text{Shi}(dx) + \frac{2ab \sinh(c+dx)}{d} - \frac{b^2 \cosh(c+dx)}{d^2} + \frac{b^2 x \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\left(\frac{(a+bx)^2 \cosh[c+dx]}{x}\right), x]$

[Out]  $-\left(\frac{b^2 \cosh[c+dx]}{d^2}\right) + a^2 \cosh[c] \text{CoshIntegral}[dx] + \left(\frac{2ab \sinh[c+dx]}{d} + \frac{b^2 x \sinh[c+dx]}{d}\right) + \frac{b^2 \cosh[c+dx]}{d^2} + a^2 \sinh[c] \text{SinhIntegral}[dx]$

#### Rule 6742

$\text{Int}[u, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c\_.) + (d\_.)*(x\_)], x\_Symbol] \rightarrow \text{Simp}[\sin[c+dx]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

#### Rule 3303

$\text{Int}[\sin[(e\_.) + (f\_.)*(x\_)] / ((c\_.) + (d\_.)*(x\_)), x\_Symbol] \rightarrow \text{Dist}[\cos[(d*e - c*f)/d], \text{Int}[\sin[(c*f)/d + f*x] / (c + d*x), x], x] + \text{Dist}[\sin[(d*e - c*f)/d], \text{Int}[\cos[(c*f)/d + f*x] / (c + d*x), x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{NeQ}[d*e - c*f, 0]$



Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
  := Simp[I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
  && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
  := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
  && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
  ((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
  e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[
  {c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx)^2 \cosh(c + dx)}{x} dx &= \int \left( 2ab \cosh(c + dx) + \frac{a^2 \cosh(c + dx)}{x} + b^2 x \cosh(c + dx) \right) dx \\
 &= a^2 \int \frac{\cosh(c + dx)}{x} dx + (2ab) \int \cosh(c + dx) dx + b^2 \int x \cosh(c + dx) dx \\
 &= \frac{2ab \sinh(c + dx)}{d} + \frac{b^2 x \sinh(c + dx)}{d} - \frac{b^2 \int \sinh(c + dx) dx}{d} + (a^2 \cosh(c)) \int \frac{\cosh(dx)}{x} dx \\
 &= -\frac{b^2 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) + \frac{2ab \sinh(c + dx)}{d} + \frac{b^2 x \sinh(c + dx)}{d} + a^2 \sinh(c) \text{Shi}(dx)
 \end{aligned}$$

**Mathematica [A]** time = 0.234957, size = 51, normalized size = 0.82

$$a^2 \cosh(c) \text{Chi}(dx) + a^2 \sinh(c) \text{Shi}(dx) + \frac{b(d(2a + bx) \sinh(c + dx) - b \cosh(c + dx))}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x,x]
```

[Out]  $a^2 \text{Cosh}[c] \text{CoshIntegral}[d*x] + (b*(-(b*\text{Cosh}[c + d*x]) + d*(2*a + b*x)*\text{Sinh}[c + d*x]))/d^2 + a^2 \text{Sinh}[c] \text{SinhIntegral}[d*x]$

**Maple [A]** time = 0.036, size = 121, normalized size = 2.

$$\frac{a^2 e^{-c} \text{Ei}(1, dx)}{2} - \frac{b^2 e^{-dx-c} x}{2d} - \frac{b^2 e^{-dx-c}}{2d^2} - \frac{ab e^{-dx-c}}{d} - \frac{a^2 e^c \text{Ei}(1, -dx)}{2} + \frac{e^{dx+c} b^2 x}{2d} - \frac{e^{dx+c} b^2}{2d^2} + \frac{ab e^{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^2*cosh(d*x+c)/x,x)`

[Out]  $-1/2*a^2*\exp(-c)*\text{Ei}(1,d*x)-1/2*b^2/d*\exp(-d*x-c)*x-1/2*b^2/d^2*\exp(-d*x-c)-a*b/d*\exp(-d*x-c)-1/2*a^2*\exp(c)*\text{Ei}(1,-d*x)+1/2*b^2/d*\exp(d*x+c)*x-1/2*b^2/d^2*\exp(d*x+c)+a*b/d*\exp(d*x+c)$

**Maxima [B]** time = 1.34019, size = 236, normalized size = 3.81

$$-\frac{1}{4} \left( 4ab \left( \frac{(dx e^c - e^c) e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2} \right) + b^2 \left( \frac{(d^2 x^2 e^c - 2 dx e^c + 2 e^c) e^{(dx)}}{d^3} + \frac{(d^2 x^2 + 2 dx + 2) e^{(-dx-c)}}{d^3} \right) \right) + \frac{4a^2 \cosh(c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")`

[Out]  $-1/4*(4*a*b*((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x + 1)*e^{(-d*x - c)}/d^2) + b^2*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2)*e^{(-d*x - c)}/d^3) + 4*a^2*\cosh(d*x + c)*\log(x)/d - 2*(\text{Ei}(-d*x)*e^{-c} + \text{Ei}(d*x)*e^c)*a^2/d*d + 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*\log(x))*\cosh(d*x + c)$

**Fricas [A]** time = 1.97115, size = 221, normalized size = 3.56

$$\frac{2b^2 \cosh(dx + c) - (a^2 d^2 \text{Ei}(dx) + a^2 d^2 \text{Ei}(-dx)) \cosh(c) - 2(b^2 dx + 2abd) \sinh(dx + c) - (a^2 d^2 \text{Ei}(dx) - a^2 d^2 \text{Ei}(-dx))}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c)/x,x, algorithm="fricas")

[Out] 
$$-1/2*(2*b^2*cosh(d*x + c) - (a^2*d^2*Ei(d*x) + a^2*d^2*Ei(-d*x))*cosh(c) - 2*(b^2*d*x + 2*a*b*d)*sinh(d*x + c) - (a^2*d^2*Ei(d*x) - a^2*d^2*Ei(-d*x))*sinh(c))/d^2$$

**Sympy [A]** time = 5.07679, size = 73, normalized size = 1.18

$$a^2 \sinh(c) \operatorname{Shi}(dx) + a^2 \cosh(c) \operatorname{Chi}(dx) + 2ab \left( \begin{cases} \frac{\sinh(c+dx)}{d} & \text{for } d \neq 0 \\ x \cosh(c) & \text{otherwise} \end{cases} \right) + b^2 \left( \begin{cases} \frac{x \sinh(c+dx)}{d} - \frac{\cosh(c+dx)}{d^2} & \text{for } d \neq 0 \\ \frac{x^2 \cosh(c)}{2} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*cosh(d\*x+c)/x,x)

[Out] 
$$a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True)) + b**2*Piecewise((x*sinh(c + d*x)/d - cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True))$$

**Giac [A]** time = 1.26265, size = 153, normalized size = 2.47

$$\frac{a^2 d^2 Ei(-dx) e^{-c} + a^2 d^2 Ei(dx) e^c + b^2 dx e^{(dx+c)} - b^2 dx e^{(-dx-c)} + 2 ab d e^{(dx+c)} - 2 ab d e^{(-dx-c)} - b^2 e^{(dx+c)} - b^2 e^{(-dx-c)}}{2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c)/x,x, algorithm="giac")

[Out] 
$$1/2*(a^2*d^2*Ei(-d*x)*e^{-c} + a^2*d^2*Ei(d*x)*e^c + b^2*d*x*e^{(d*x + c)} - b^2*d*x*e^{(-d*x - c)} + 2*a*b*d*e^{(d*x + c)} - 2*a*b*d*e^{(-d*x - c)} - b^2*e^{(d*x + c)} - b^2*e^{(-d*x - c)})/d^2$$

### 3.14 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^2} dx$

**Optimal.** Leaf size=70

$$a^2 d \sinh(c) \text{Chi}(dx) + a^2 d \cosh(c) \text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + 2ab \cosh(c) \text{Chi}(dx) + 2ab \sinh(c) \text{Shi}(dx) + \frac{b^2 \sinh(c+dx)}{d}$$

[Out]  $-\left(\frac{a^2 \text{Cosh}[c + d*x]}{x}\right) + 2*a*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + a^2*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + (b^2*\text{Sinh}[c + d*x])/d + a^2*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + 2*a*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x]$

**Rubi [A]** time = 0.246734, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {6742, 2637, 3297, 3303, 3298, 3301}

$$a^2 d \sinh(c) \text{Chi}(dx) + a^2 d \cosh(c) \text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + 2ab \cosh(c) \text{Chi}(dx) + 2ab \sinh(c) \text{Shi}(dx) + \frac{b^2 \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\left((a + b*x)^2*\text{Cosh}[c + d*x]\right)/x^2, x]$

[Out]  $-\left(\frac{a^2 \text{Cosh}[c + d*x]}{x}\right) + 2*a*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + a^2*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + (b^2*\text{Sinh}[c + d*x])/d + a^2*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + 2*a*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x]$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

#### Rule 3297

$\text{Int}[\left((c_.) + (d_.)*(x_.)\right)^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[\left((c + d*x)^{(m+1)}*\sin[e + f*x]/(d*(m+1)), x\right) - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\cos[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{LtQ}[m, -1]$

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx)^2 \cosh(c + dx)}{x^2} dx &= \int \left( b^2 \cosh(c + dx) + \frac{a^2 \cosh(c + dx)}{x^2} + \frac{2ab \cosh(c + dx)}{x} \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^2} dx + (2ab) \int \frac{\cosh(c + dx)}{x} dx + b^2 \int \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{x} + \frac{b^2 \sinh(c + dx)}{d} + (a^2 d) \int \frac{\sinh(c + dx)}{x} dx + (2ab \cosh(c)) \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a^2 \cosh(c + dx)}{x} + 2ab \cosh(c) \text{Chi}(dx) + \frac{b^2 \sinh(c + dx)}{d} + 2ab \sinh(c) \text{Shi}(dx) + (a^2 d) \int \frac{\sinh(c + dx)}{x} dx \\
&= -\frac{a^2 \cosh(c + dx)}{x} + 2ab \cosh(c) \text{Chi}(dx) + a^2 d \text{Chi}(dx) \sinh(c) + \frac{b^2 \sinh(c + dx)}{d} + a^2 d \int \frac{\sinh(c + dx)}{x} dx
\end{aligned}$$

**Mathematica [A]** time = 0.229813, size = 62, normalized size = 0.89

$$-\frac{a^2 \cosh(c + dx)}{x} + a \text{Chi}(dx)(ad \sinh(c) + 2b \cosh(c)) + a \text{Shi}(dx)(ad \cosh(c) + 2b \sinh(c)) + \frac{b^2 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)^2\*Cosh[c + d\*x])/x^2,x]

[Out]  $-\left(\frac{a^2 \cosh[c + dx]}{x}\right) + a \operatorname{CoshIntegral}[dx] * (2b \cosh[c] + a d \sinh[c]) + \frac{b^2 \sinh[c + dx]}{d} + a(a d \cosh[c] + 2b \sinh[c]) \operatorname{SinhIntegral}[dx]$

**Maple [A]** time = 0.049, size = 118, normalized size = 1.7

$$-\frac{a^2 e^{-dx-c}}{2x} + \frac{da^2 e^{-c} \operatorname{Ei}(1, dx)}{2} - \frac{b^2 e^{-dx-c}}{2d} - abe^{-c} \operatorname{Ei}(1, dx) - \frac{e^{dx+c} a^2}{2x} - \frac{da^2 e^c \operatorname{Ei}(1, -dx)}{2} + \frac{e^{dx+c} b^2}{2d} - abe^c \operatorname{Ei}(1, -dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^2*cosh(d*x+c)/x^2,x)`

[Out]  $-1/2*a^2*\exp(-d*x-c)/x+1/2*d*a^2*\exp(-c)*\operatorname{Ei}(1,d*x)-1/2*b^2/d*\exp(-d*x-c)-a*b*\exp(-c)*\operatorname{Ei}(1,d*x)-1/2*a^2/x*\exp(d*x+c)-1/2*d*a^2*\exp(c)*\operatorname{Ei}(1,-d*x)+1/2*b^2/d*\exp(d*x+c)-a*b*\exp(c)*\operatorname{Ei}(1,-d*x)$

**Maxima [A]** time = 1.39303, size = 184, normalized size = 2.63

$$-\frac{1}{2} \left( \left( \operatorname{Ei}(-dx) e^{(-c)} - \operatorname{Ei}(dx) e^c \right) a^2 + b^2 \left( \frac{(dx e^c - e^c) e^{(dx)}}{d^2} + \frac{(dx + 1) e^{(-dx-c)}}{d^2} \right) + \frac{4ab \cosh(dx + c) \log(x)}{d} - \frac{2 \left( \operatorname{Ei}(-dx) e^{(-c)} \right)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")`

[Out]  $-1/2*((\operatorname{Ei}(-d*x)*e^{(-c)} - \operatorname{Ei}(d*x)*e^c)*a^2 + b^2*((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x + 1)*e^{(-d*x - c)}/d^2) + 4*a*b*\cosh(d*x + c)*\log(x)/d - 2*(\operatorname{Ei}(-d*x)*e^{(-c)} + \operatorname{Ei}(d*x)*e^c)*a*b/d)*d + (b^2*x + 2*a*b*\log(x) - a^2/x)*\cosh(d*x + c)$

**Fricas [A]** time = 1.99353, size = 284, normalized size = 4.06

$$\frac{2a^2 d \cosh(dx + c) - 2b^2 x \sinh(dx + c) - \left( (a^2 d^2 + 2abd) x \operatorname{Ei}(dx) - (a^2 d^2 - 2abd) x \operatorname{Ei}(-dx) \right) \cosh(c) - \left( (a^2 d^2 + 2abd) x \operatorname{Ei}(-dx) - (a^2 d^2 - 2abd) x \operatorname{Ei}(dx) \right) \sinh(c)}{2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c)/x^2,x, algorithm="fricas")

[Out] 
$$-1/2*(2*a^2*d*cosh(d*x + c) - 2*b^2*x*sinh(d*x + c) - ((a^2*d^2 + 2*a*b*d)*x*Ei(d*x) - (a^2*d^2 - 2*a*b*d)*x*Ei(-d*x))*cosh(c) - ((a^2*d^2 + 2*a*b*d)*x*Ei(d*x) + (a^2*d^2 - 2*a*b*d)*x*Ei(-d*x))*sinh(c))/(d*x)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx)^2 \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*cosh(d\*x+c)/x\*\*2,x)

[Out] Integral((a + b\*x)\*\*2\*cosh(c + d\*x)/x\*\*2, x)

**Giac [A]** time = 1.23853, size = 161, normalized size = 2.3

$$\frac{a^2 d^2 x Ei(-dx) e^{-c} - a^2 d^2 x Ei(dx) e^c - 2 ab dx Ei(-dx) e^{-c} - 2 ab dx Ei(dx) e^c + a^2 d e^{(dx+c)} - b^2 x e^{(dx+c)} + a^2 d e^{(-dx-c)} + 2 dx}{2 dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c)/x^2,x, algorithm="giac")

[Out] 
$$-1/2*(a^2*d^2*x*Ei(-d*x)*e^{-c} - a^2*d^2*x*Ei(d*x)*e^c - 2*a*b*d*x*Ei(-d*x)*e^{-c} - 2*a*b*d*x*Ei(d*x)*e^c + a^2*d*e^{(d*x + c)} - b^2*x*e^{(d*x + c)} + a^2*d*e^{(-d*x - c)} + b^2*x*e^{(-d*x - c)}))/(d*x)$$

### 3.15 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^3} dx$

**Optimal.** Leaf size=121

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + 2abd \sinh(c)\text{Chi}(dx) + 2abd \cosh(c)\text{Shi}(dx)$$

[Out]  $-(a^2*\text{Cosh}[c+d*x])/(2*x^2) - (2*a*b*\text{Cosh}[c+d*x])/x + b^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (a^2*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + 2*a*b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] - (a^2*d*\text{Sinh}[c+d*x])/(2*x) + 2*a*b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + b^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + (a^2*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

**Rubi [A]** time = 0.338688, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + 2abd \sinh(c)\text{Chi}(dx) + 2abd \cosh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*x)^2*\text{Cosh}[c+d*x])/x^3,x]$

[Out]  $-(a^2*\text{Cosh}[c+d*x])/(2*x^2) - (2*a*b*\text{Cosh}[c+d*x])/x + b^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (a^2*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + 2*a*b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] - (a^2*d*\text{Sinh}[c+d*x])/(2*x) + 2*a*b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + b^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + (a^2*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 3297

$\text{Int}[(c_. + (d_.)*(x_.))^(m_.)*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^(m + 1)*\text{Sin}[e + f*x]/(d*(m + 1)), x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^(m + 1)*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{LtQ}[m, -1]$

#### Rule 3303



```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2 \cosh(c+dx)}{x^3} dx &= \int \left( \frac{a^2 \cosh(c+dx)}{x^3} + \frac{2ab \cosh(c+dx)}{x^2} + \frac{b^2 \cosh(c+dx)}{x} \right) dx \\
&= a^2 \int \frac{\cosh(c+dx)}{x^3} dx + (2ab) \int \frac{\cosh(c+dx)}{x^2} dx + b^2 \int \frac{\cosh(c+dx)}{x} dx \\
&= -\frac{a^2 \cosh(c+dx)}{2x^2} - \frac{2ab \cosh(c+dx)}{x} + \frac{1}{2} (a^2 d) \int \frac{\sinh(c+dx)}{x^2} dx + (2abd) \int \frac{\sinh(c+dx)}{x} dx \\
&= -\frac{a^2 \cosh(c+dx)}{2x^2} - \frac{2ab \cosh(c+dx)}{x} + b^2 \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c+dx)}{2x} + b^2 \sinh(c) \text{Shi}(dx) \\
&= -\frac{a^2 \cosh(c+dx)}{2x^2} - \frac{2ab \cosh(c+dx)}{x} + b^2 \cosh(c) \text{Chi}(dx) + 2abd \text{Chi}(dx) \sinh(c) - \frac{a^2 d \sinh(c+dx)}{2x} \\
&= -\frac{a^2 \cosh(c+dx)}{2x^2} - \frac{2ab \cosh(c+dx)}{x} + b^2 \cosh(c) \text{Chi}(dx) + \frac{1}{2} a^2 d^2 \cosh(c) \text{Chi}(dx) + \frac{1}{2} a^2 d \sinh(c) \text{Shi}(dx)
\end{aligned}$$

**Mathematica [A]** time = 0.373391, size = 93, normalized size = 0.77

$$\frac{1}{2} \left( \text{Chi}(dx) \left( \cosh(c) (a^2 d^2 + 2b^2) + 4abd \sinh(c) \right) + \text{Shi}(dx) \left( \sinh(c) (a^2 d^2 + 2b^2) + 4abd \cosh(c) \right) - \frac{a((a+4bx) \cosh(c) + (a-4bx) \sinh(c))}{2x} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Cosh[c + d*x])/x^3, x]
```

[Out]  $(\text{CoshIntegral}[d*x]*((2*b^2 + a^2*d^2)*\text{Cosh}[c] + 4*a*b*d*\text{Sinh}[c]) - (a*((a + 4*b*x)*\text{Cosh}[c + d*x] + a*d*x*\text{Sinh}[c + d*x])))/x^2 + (4*a*b*d*\text{Cosh}[c] + (2*b^2 + a^2*d^2)*\text{Sinh}[c])*\text{SinhIntegral}[d*x])/2$

**Maple [A]** time = 0.052, size = 181, normalized size = 1.5

$$-\frac{d^2 a^2 e^{-c} \text{Ei}(1, dx)}{4} - \frac{a b e^{-dx-c}}{x} + d a b e^{-c} \text{Ei}(1, dx) - \frac{b^2 e^{-c} \text{Ei}(1, dx)}{2} + \frac{d a^2 e^{-dx-c}}{4x} - \frac{a^2 e^{-dx-c}}{4x^2} - \frac{a b e^{dx+c}}{x} - d a b e^c \text{Ei}(1, -dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x+a)^2*\cosh(d*x+c)/x^3,x)$

[Out]  $-1/4*d^2*a^2*\exp(-c)*\text{Ei}(1,d*x)-a*b*\exp(-d*x-c)/x+d*a*b*\exp(-c)*\text{Ei}(1,d*x)-1/2*b^2*\exp(-c)*\text{Ei}(1,d*x)+1/4*d*a^2*\exp(-d*x-c)/x-1/4*a^2*\exp(-d*x-c)/x^2-a*b/x*\exp(d*x+c)-d*a*b*\exp(c)*\text{Ei}(1,-d*x)-1/2*b^2*\exp(c)*\text{Ei}(1,-d*x)-1/4*d^2*a^2*\exp(c)*\text{Ei}(1,-d*x)-1/4*a^2/x^2*\exp(d*x+c)-1/4*d*a^2/x*\exp(d*x+c)$

**Maxima [A]** time = 1.37234, size = 170, normalized size = 1.4

$$\frac{1}{4} \left( (d e^{-c} \Gamma(-1, dx) + d e^c \Gamma(-1, -dx)) a^2 - 4 (\text{Ei}(-dx) e^{-c} - \text{Ei}(dx) e^c) a b - \frac{4 b^2 \cosh(dx + c) \log(x)}{d} + \frac{2 (\text{Ei}(-dx) e^{-c})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x+a)^2*\cosh(d*x+c)/x^3,x, \text{algorithm}="maxima")$

[Out]  $1/4*((d*e^{-c})*\text{gamma}(-1, d*x) + d*e^c*\text{gamma}(-1, -d*x))*a^2 - 4*(\text{Ei}(-d*x)*e^{-c} - \text{Ei}(d*x)*e^c)*a*b - 4*b^2*\cosh(d*x + c)*\log(x)/d + 2*(\text{Ei}(-d*x)*e^{-c} + \text{Ei}(d*x)*e^c)*b^2/d*d + 1/2*(2*b^2*\log(x) - (4*a*b*x + a^2)/x^2)*\cosh(d*x + c)$

**Fricas [A]** time = 1.94994, size = 351, normalized size = 2.9

$$\frac{2 a^2 dx \sinh(dx + c) + 2 (4 a b x + a^2) \cosh(dx + c) - ((a^2 d^2 + 4 a b d + 2 b^2) x^2 \text{Ei}(dx) + (a^2 d^2 - 4 a b d + 2 b^2) x^2 \text{Ei}(-dx))}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c)/x^3,x, algorithm="fricas")

[Out] 
$$-1/4*(2*a^2*d*x*\sinh(d*x + c) + 2*(4*a*b*x + a^2)*\cosh(d*x + c) - ((a^2*d^2 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) + (a^2*d^2 - 4*a*b*d + 2*b^2)*x^2*Ei(-d*x)) * \cosh(c) - ((a^2*d^2 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) - (a^2*d^2 - 4*a*b*d + 2*b^2)*x^2*Ei(-d*x))*\sinh(c))/x^2$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx)^2 \cosh(c + dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*cosh(d\*x+c)/x\*\*3,x)

[Out] Integral((a + b\*x)\*\*2\*cosh(c + d\*x)/x\*\*3, x)

**Giac [A]** time = 1.17773, size = 244, normalized size = 2.02

$$\frac{a^2 d^2 x^2 Ei(-dx) e^{-c} + a^2 d^2 x^2 Ei(dx) e^c - 4 ab dx^2 Ei(-dx) e^{-c} + 4 ab dx^2 Ei(dx) e^c + 2 b^2 x^2 Ei(-dx) e^{-c} + 2 b^2 x^2 Ei(dx) e^c}{4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c)/x^3,x, algorithm="giac")

[Out] 
$$1/4*(a^2*d^2*x^2*Ei(-d*x)*e^{-c} + a^2*d^2*x^2*Ei(d*x)*e^c - 4*a*b*d*x^2*Ei(-d*x)*e^{-c} + 4*a*b*d*x^2*Ei(d*x)*e^c + 2*b^2*x^2*Ei(-d*x)*e^{-c} + 2*b^2*x^2*Ei(d*x)*e^c - a^2*d*x*e^{(d*x + c)} + a^2*d*x*e^{(-d*x - c)} - 4*a*b*x*e^{(d*x + c)} - 4*a*b*x*e^{(-d*x - c)} - a^2*e^{(d*x + c)} - a^2*e^{(-d*x - c)})/x^2$$

### 3.16 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^4} dx$

**Optimal.** Leaf size=172

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2d \sinh(c+dx)}{6x^2} - \frac{a^2 \cosh(c+dx)}{3x^3} + abd^2 \cosh(c)$$

[Out]  $-(a^2 \cdot \text{Cosh}[c + d \cdot x]) / (3 \cdot x^3) - (a \cdot b \cdot \text{Cosh}[c + d \cdot x]) / x^2 - (b^2 \cdot \text{Cosh}[c + d \cdot x]) / x - (a^2 \cdot d^2 \cdot \text{Cosh}[c + d \cdot x]) / (6 \cdot x) + a \cdot b \cdot d^2 \cdot \text{Cosh}[c] \cdot \text{CoshIntegral}[d \cdot x] + b^2 \cdot d \cdot \text{CoshIntegral}[d \cdot x] \cdot \text{Sinh}[c] + (a^2 \cdot d^3 \cdot \text{CoshIntegral}[d \cdot x] \cdot \text{Sinh}[c]) / 6 - (a^2 \cdot d \cdot \text{Sinh}[c + d \cdot x]) / (6 \cdot x^2) - (a \cdot b \cdot d \cdot \text{Sinh}[c + d \cdot x]) / x + b^2 \cdot d \cdot \text{Cosh}[c] \cdot \text{SinhIntegral}[d \cdot x] + (a^2 \cdot d^3 \cdot \text{Cosh}[c] \cdot \text{SinhIntegral}[d \cdot x]) / 6 + a \cdot b \cdot d^2 \cdot \text{Sinh}[c] \cdot \text{SinhIntegral}[d \cdot x]$

**Rubi [A]** time = 0.42641, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2d \sinh(c+dx)}{6x^2} - \frac{a^2 \cosh(c+dx)}{3x^3} + abd^2 \cosh(c)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b \cdot x)^2 \cdot \text{Cosh}[c + d \cdot x] / x^4, x]$

[Out]  $-(a^2 \cdot \text{Cosh}[c + d \cdot x]) / (3 \cdot x^3) - (a \cdot b \cdot \text{Cosh}[c + d \cdot x]) / x^2 - (b^2 \cdot \text{Cosh}[c + d \cdot x]) / x - (a^2 \cdot d^2 \cdot \text{Cosh}[c + d \cdot x]) / (6 \cdot x) + a \cdot b \cdot d^2 \cdot \text{Cosh}[c] \cdot \text{CoshIntegral}[d \cdot x] + b^2 \cdot d \cdot \text{CoshIntegral}[d \cdot x] \cdot \text{Sinh}[c] + (a^2 \cdot d^3 \cdot \text{CoshIntegral}[d \cdot x] \cdot \text{Sinh}[c]) / 6 - (a^2 \cdot d \cdot \text{Sinh}[c + d \cdot x]) / (6 \cdot x^2) - (a \cdot b \cdot d \cdot \text{Sinh}[c + d \cdot x]) / x + b^2 \cdot d \cdot \text{Cosh}[c] \cdot \text{SinhIntegral}[d \cdot x] + (a^2 \cdot d^3 \cdot \text{Cosh}[c] \cdot \text{SinhIntegral}[d \cdot x]) / 6 + a \cdot b \cdot d^2 \cdot \text{Sinh}[c] \cdot \text{SinhIntegral}[d \cdot x]$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 3297

$\text{Int}[(c \cdot \_) + (d \cdot \cdot)(x \cdot \cdot)]^{(m \cdot \cdot)} \cdot \sin[(e \cdot \cdot) + (f \cdot \cdot)(x \cdot \cdot)], x\_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^{(m + 1)} \cdot \text{Sin}[e + f \cdot x] / (d \cdot (m + 1)), x] - \text{Dist}[f / (d \cdot (m + 1)), \text{Int}[(c + d \cdot x)^{(m + 1)} \cdot \text{Cos}[e + f \cdot x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{LtQ}[m, -1]$

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2 \cosh(c+dx)}{x^4} dx &= \int \left( \frac{a^2 \cosh(c+dx)}{x^4} + \frac{2ab \cosh(c+dx)}{x^3} + \frac{b^2 \cosh(c+dx)}{x^2} \right) dx \\
&= a^2 \int \frac{\cosh(c+dx)}{x^4} dx + (2ab) \int \frac{\cosh(c+dx)}{x^3} dx + b^2 \int \frac{\cosh(c+dx)}{x^2} dx \\
&= -\frac{a^2 \cosh(c+dx)}{3x^3} - \frac{ab \cosh(c+dx)}{x^2} - \frac{b^2 \cosh(c+dx)}{x} + \frac{1}{3} (a^2 d) \int \frac{\sinh(c+dx)}{x^3} dx + \\
&= -\frac{a^2 \cosh(c+dx)}{3x^3} - \frac{ab \cosh(c+dx)}{x^2} - \frac{b^2 \cosh(c+dx)}{x} - \frac{a^2 d \sinh(c+dx)}{6x^2} - \frac{abd \sinh(c+dx)}{x} \\
&= -\frac{a^2 \cosh(c+dx)}{3x^3} - \frac{ab \cosh(c+dx)}{x^2} - \frac{b^2 \cosh(c+dx)}{x} - \frac{a^2 d^2 \cosh(c+dx)}{6x} + b^2 d \text{Chi}(a \\
&= -\frac{a^2 \cosh(c+dx)}{3x^3} - \frac{ab \cosh(c+dx)}{x^2} - \frac{b^2 \cosh(c+dx)}{x} - \frac{a^2 d^2 \cosh(c+dx)}{6x} + abd^2 \cos \\
&= -\frac{a^2 \cosh(c+dx)}{3x^3} - \frac{ab \cosh(c+dx)}{x^2} - \frac{b^2 \cosh(c+dx)}{x} - \frac{a^2 d^2 \cosh(c+dx)}{6x} + abd^2 \cos
\end{aligned}$$

**Mathematica [A]** time = 0.44756, size = 154, normalized size = 0.9

---


$$-dx^3 \text{Chi}(dx) \left( \sinh(c) (a^2 d^2 + 6b^2) + 6abd \cosh(c) \right) - dx^3 \text{Shi}(dx) \left( a^2 d^2 \cosh(c) + 6abd \sinh(c) + 6b^2 \cosh(c) \right) + a^2 d^2$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)^2\*Cosh[c + d\*x])/x^4,x]

[Out] 
$$\frac{-(2a^2 \cosh[c + dx] + 6abx \cosh[c + dx] + 6b^2 x^2 \cosh[c + dx] + a^2 d^2 x^2 \cosh[c + dx] - dx^3 \operatorname{CoshIntegral}[dx] * (6abd \cosh[c] + (6b^2 + a^2 d^2) \sinh[c]) + a^2 dx \sinh[c + dx] + 6abd x^2 \sinh[c + dx] - dx^3 (6b^2 \cosh[c] + a^2 d^2 \cosh[c] + 6abd \sinh[c]) * \operatorname{SinhIntegral}[dx])}{(6x^3)}$$

**Maple [A]** time = 0.06, size = 287, normalized size = 1.7

$$\frac{d^3 a^2 e^{-c} \operatorname{Ei}(1, dx)}{12} - \frac{a^2 d^2 e^{-dx-c}}{12x} - \frac{a^2 e^{-dx-c}}{6x^3} + \frac{da^2 e^{-dx-c}}{12x^2} - \frac{b^2 e^{-dx-c}}{2x} + \frac{bdae^{-dx-c}}{2x} - \frac{abe^{-dx-c}}{2x^2} - \frac{d^2 abe^{-c} \operatorname{Ei}(1, dx)}{2} + \frac{db^2 e^{-c}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^2\*cosh(d\*x+c)/x^4,x)

[Out] 
$$\frac{1}{12} d^3 a^2 \exp(-c) \operatorname{Ei}(1, dx) - \frac{1}{12} d^2 a^2 \exp(-dx-c) / x - \frac{1}{6} a^2 \exp(-dx-c) / x^3 + \frac{1}{12} d a^2 \exp(-dx-c) / x^2 - \frac{1}{2} b^2 \exp(-dx-c) / x + \frac{1}{2} d a b \exp(-dx-c) / x - \frac{1}{2} a b \exp(-dx-c) / x^2 - \frac{1}{2} d^2 a b \exp(-c) \operatorname{Ei}(1, dx) + \frac{1}{2} d b^2 \exp(-c) \operatorname{Ei}(1, dx) - \frac{1}{6} a^2 / x^3 \exp(dx+c) - \frac{1}{12} d a^2 / x^2 \exp(dx+c) - \frac{1}{12} d^2 a^2 / x \exp(dx+c) - \frac{1}{2} d^2 a b \exp(c) \operatorname{Ei}(1, -dx) - \frac{1}{2} a b / x^2 \exp(dx+c) - \frac{1}{2} d a b / x \exp(dx+c) - \frac{1}{12} d^3 a^2 \exp(c) \operatorname{Ei}(1, -dx) - \frac{1}{2} b^2 / x \exp(dx+c) - \frac{1}{2} d b^2 \exp(c) \operatorname{Ei}(1, -dx)$$

**Maxima [A]** time = 1.43103, size = 158, normalized size = 0.92

$$\frac{1}{6} \left( a^2 d^2 e^{(-c)} \Gamma(-2, dx) - a^2 d^2 e^c \Gamma(-2, -dx) + 3 abde^{(-c)} \Gamma(-1, dx) + 3 abde^c \Gamma(-1, -dx) - 3 b^2 \operatorname{Ei}(-dx) e^{(-c)} + 3 b^2 \operatorname{Ei}(dx) e^c \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c)/x^4,x, algorithm="maxima")

[Out] 
$$\frac{1}{6} (a^2 d^2 e^{(-c)} \operatorname{gamma}(-2, dx) - a^2 d^2 e^c \operatorname{gamma}(-2, -dx) + 3 a b d e^{(-c)} \operatorname{gamma}(-1, dx) + 3 a b d e^c \operatorname{gamma}(-1, -dx) - 3 b^2 \operatorname{Ei}(-dx) e^{(-c)} + 3 b^2 \operatorname{Ei}(dx) e^c) * d - \frac{1}{3} (3 b^2 x^2 + 3 a b x + a^2) \cosh(dx + c) / x^3$$

---

**Fricas [A]** time = 2.01559, size = 431, normalized size = 2.51

$$\frac{2\left(6abx + (a^2d^2 + 6b^2)x^2 + 2a^2\right)\cosh(dx + c) - \left((a^2d^3 + 6abd^2 + 6b^2d)x^3\text{Ei}(dx) - (a^2d^3 - 6abd^2 + 6b^2d)x^3\text{Ei}(-d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c)/x^4,x, algorithm="fricas")

[Out] -1/12\*(2\*(6\*a\*b\*x + (a^2\*d^2 + 6\*b^2)\*x^2 + 2\*a^2)\*cosh(d\*x + c) - ((a^2\*d^3 + 6\*a\*b\*d^2 + 6\*b^2\*d)\*x^3\*Ei(d\*x) - (a^2\*d^3 - 6\*a\*b\*d^2 + 6\*b^2\*d)\*x^3\*Ei(-d\*x))\*cosh(c) + 2\*(6\*a\*b\*d\*x^2 + a^2\*d\*x)\*sinh(d\*x + c) - ((a^2\*d^3 + 6\*a\*b\*d^2 + 6\*b^2\*d)\*x^3\*Ei(d\*x) + (a^2\*d^3 - 6\*a\*b\*d^2 + 6\*b^2\*d)\*x^3\*Ei(-d\*x))\*sinh(c))/x^3

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx)^2 \cosh(c + dx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*cosh(d\*x+c)/x\*\*4,x)

[Out] Integral((a + b\*x)\*\*2\*cosh(c + d\*x)/x\*\*4, x)

---

**Giac [A]** time = 1.15162, size = 385, normalized size = 2.24

$$\frac{a^2d^3x^3\text{Ei}(-dx)e^{(-c)} - a^2d^3x^3\text{Ei}(dx)e^c - 6abd^2x^3\text{Ei}(-dx)e^{(-c)} - 6abd^2x^3\text{Ei}(dx)e^c + 6b^2dx^3\text{Ei}(-dx)e^{(-c)} - 6b^2dx^3\text{Ei}(dx)e^c}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c)/x^4,x, algorithm="giac")

[Out] -1/12\*(a^2\*d^3\*x^3\*Ei(-d\*x)\*e^(-c) - a^2\*d^3\*x^3\*Ei(d\*x)\*e^c - 6\*a\*b\*d^2\*x^3\*Ei(-d\*x)\*e^(-c) - 6\*a\*b\*d^2\*x^3\*Ei(d\*x)\*e^c + 6\*b^2\*d\*x^3\*Ei(-d\*x)\*e^(-c) - 6\*b^2\*d\*x^3\*Ei(d\*x)\*e^c + a^2\*d^2\*x^2\*e^(d\*x + c) + a^2\*d^2\*x^2\*e^(-d\*x

$$\begin{aligned} & -c) + 6*a*b*d*x^2*e^{(d*x + c)} - 6*a*b*d*x^2*e^{(-d*x - c)} + a^2*d*x*e^{(d*x} \\ & + c) + 6*b^2*x^2*e^{(d*x + c)} - a^2*d*x*e^{(-d*x - c)} + 6*b^2*x^2*e^{(-d*x - c} \\ & ) + 6*a*b*x*e^{(d*x + c)} + 6*a*b*x*e^{(-d*x - c)} + 2*a^2*e^{(d*x + c)} + 2*a^2* \\ & e^{(-d*x - c)})/x^3 \end{aligned}$$



$$3.17 \quad \int \frac{(a+bx)^2 \cosh(c+dx)}{x^5} dx$$

**Optimal.** Leaf size=248

$$\frac{1}{24}a^2d^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4 \sinh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{24x^2} - \frac{a^2d^3 \sinh(c+dx)}{24x} - \frac{a^2d \sinh(c+dx)}{12x^3} - \frac{a^2 \cos$$

[Out]  $-(a^2 \text{Cosh}[c + d*x])/(4*x^4) - (2*a*b*\text{Cosh}[c + d*x])/(3*x^3) - (b^2*\text{Cosh}[c + d*x])/(2*x^2) - (a^2*d^2*\text{Cosh}[c + d*x])/(24*x^2) - (a*b*d^2*\text{Cosh}[c + d*x])/(3*x) + (b^2*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (a^2*d^4*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/24 + (a*b*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/3 - (a^2*d*\text{Sinh}[c + d*x])/(12*x^3) - (a*b*d*\text{Sinh}[c + d*x])/(3*x^2) - (b^2*d*\text{Sinh}[c + d*x])/(2*x) - (a^2*d^3*\text{Sinh}[c + d*x])/(24*x) + (a*b*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/3 + (b^2*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2 + (a^2*d^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/24$

**Rubi [A]** time = 0.520699, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{1}{24}a^2d^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4 \sinh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{24x^2} - \frac{a^2d^3 \sinh(c+dx)}{24x} - \frac{a^2d \sinh(c+dx)}{12x^3} - \frac{a^2 \cos$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x)^2\*Cosh[c + d\*x])/x^5,x]

[Out]  $-(a^2 \text{Cosh}[c + d*x])/(4*x^4) - (2*a*b*\text{Cosh}[c + d*x])/(3*x^3) - (b^2*\text{Cosh}[c + d*x])/(2*x^2) - (a^2*d^2*\text{Cosh}[c + d*x])/(24*x^2) - (a*b*d^2*\text{Cosh}[c + d*x])/(3*x) + (b^2*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (a^2*d^4*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/24 + (a*b*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/3 - (a^2*d*\text{Sinh}[c + d*x])/(12*x^3) - (a*b*d*\text{Sinh}[c + d*x])/(3*x^2) - (b^2*d*\text{Sinh}[c + d*x])/(2*x) - (a^2*d^3*\text{Sinh}[c + d*x])/(24*x) + (a*b*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/3 + (b^2*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2 + (a^2*d^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/24$

**Rule 6742**

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5} dx &= \int \left( \frac{a^2 \cosh(c+dx)}{x^5} + \frac{2ab \cosh(c+dx)}{x^4} + \frac{b^2 \cosh(c+dx)}{x^3} \right) dx \\
&= a^2 \int \frac{\cosh(c+dx)}{x^5} dx + (2ab) \int \frac{\cosh(c+dx)}{x^4} dx + b^2 \int \frac{\cosh(c+dx)}{x^3} dx \\
&= -\frac{a^2 \cosh(c+dx)}{4x^4} - \frac{2ab \cosh(c+dx)}{3x^3} - \frac{b^2 \cosh(c+dx)}{2x^2} + \frac{1}{4} (a^2 d) \int \frac{\sinh(c+dx)}{x^4} dx + \dots \\
&= -\frac{a^2 \cosh(c+dx)}{4x^4} - \frac{2ab \cosh(c+dx)}{3x^3} - \frac{b^2 \cosh(c+dx)}{2x^2} - \frac{a^2 d \sinh(c+dx)}{12x^3} - \frac{abd \sinh(c+dx)}{3x^2} + \dots \\
&= -\frac{a^2 \cosh(c+dx)}{4x^4} - \frac{2ab \cosh(c+dx)}{3x^3} - \frac{b^2 \cosh(c+dx)}{2x^2} - \frac{a^2 d^2 \cosh(c+dx)}{24x^2} - \frac{abd^2 \cosh(c+dx)}{6x} + \dots \\
&= -\frac{a^2 \cosh(c+dx)}{4x^4} - \frac{2ab \cosh(c+dx)}{3x^3} - \frac{b^2 \cosh(c+dx)}{2x^2} - \frac{a^2 d^2 \cosh(c+dx)}{24x^2} - \frac{abd^2 \cosh(c+dx)}{6x} + \dots \\
&= -\frac{a^2 \cosh(c+dx)}{4x^4} - \frac{2ab \cosh(c+dx)}{3x^3} - \frac{b^2 \cosh(c+dx)}{2x^2} - \frac{a^2 d^2 \cosh(c+dx)}{24x^2} - \frac{abd^2 \cosh(c+dx)}{6x} + \dots \\
&= -\frac{a^2 \cosh(c+dx)}{4x^4} - \frac{2ab \cosh(c+dx)}{3x^3} - \frac{b^2 \cosh(c+dx)}{2x^2} - \frac{a^2 d^2 \cosh(c+dx)}{24x^2} - \frac{abd^2 \cosh(c+dx)}{6x} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.523918, size = 206, normalized size = 0.83

$$-\frac{d^2 x^4 \text{Chi}(dx) (\cosh(c) (a^2 d^2 + 12b^2) + 8abd \sinh(c)) - d^2 x^4 \text{Shi}(dx) (a^2 d^2 \sinh(c) + 8abd \cosh(c) + 12b^2 \sinh(c))}{24x^4} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)^2\*Cosh[c + d\*x])/x^5,x]

[Out]  $-(6a^2 \text{Cosh}[c + d*x] + 16a*b*x \text{Cosh}[c + d*x] + 12b^2*x^2 \text{Cosh}[c + d*x] + a^2*d^2*x^2 \text{Cosh}[c + d*x] + 8a*b*d^2*x^3 \text{Cosh}[c + d*x] - d^2*x^4 \text{CoshIntegral}[d*x] * ((12*b^2 + a^2*d^2) \text{Cosh}[c] + 8a*b*d \text{Sinh}[c]) + 2*a^2*d*x \text{Sinh}[c + d*x] + 8a*b*d*x^2 \text{Sinh}[c + d*x] + 12*b^2*d*x^3 \text{Sinh}[c + d*x] + a^2*d^3*x^3 \text{Sinh}[c + d*x] - d^2*x^4 * (8a*b*d \text{Cosh}[c] + 12*b^2 \text{Sinh}[c] + a^2*d^2 \text{SinhIntegral}[d*x])) / (24*x^4)$

**Maple [A]** time = 0.072, size = 396, normalized size = 1.6

$$-\frac{d^4 a^2 e^{-c} \text{Ei}(1, dx)}{48} + \frac{da^2 e^{-dx-c}}{24x^3} - \frac{a^2 e^{-dx-c}}{8x^4} + \frac{a^2 d^3 e^{-dx-c}}{48x} - \frac{a^2 d^2 e^{-dx-c}}{48x^2} + \frac{d^3 abe^{-c} \text{Ei}(1, dx)}{6} - \frac{abd^2 e^{-dx-c}}{6x} + \frac{bdae^{-dx-c}}{6x^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.



$x^4 \text{Ei}(dx) - (a^2 d^4 - 8 a b d^3 + 12 b^2 d^2) x^4 \text{Ei}(-dx) \sinh(c) / x^4$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx)^2 \cosh(c + dx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*cosh(d\*x+c)/x\*\*5,x)

[Out] Integral((a + b\*x)\*\*2\*cosh(c + d\*x)/x\*\*5, x)

**Giac [A]** time = 1.11518, size = 533, normalized size = 2.15

$a^2 d^4 x^4 \text{Ei}(-dx) e^{-c} + a^2 d^4 x^4 \text{Ei}(dx) e^c - 8 a b d^3 x^4 \text{Ei}(-dx) e^{-c} + 8 a b d^3 x^4 \text{Ei}(dx) e^c + 12 b^2 d^2 x^4 \text{Ei}(-dx) e^{-c} + 12 b^2 d^2$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*cosh(d\*x+c)/x^5,x, algorithm="giac")

[Out]  $1/48*(a^2*d^4*x^4*\text{Ei}(-d*x)*e^{-c} + a^2*d^4*x^4*\text{Ei}(d*x)*e^c - 8*a*b*d^3*x^4*\text{Ei}(-d*x)*e^{-c} + 8*a*b*d^3*x^4*\text{Ei}(d*x)*e^c + 12*b^2*d^2*x^4*\text{Ei}(-d*x)*e^{-c} + 12*b^2*d^2*x^4*\text{Ei}(d*x)*e^c - a^2*d^3*x^3*e^{(d*x + c)} + a^2*d^3*x^3*e^{(-d*x - c)} - 8*a*b*d^2*x^3*e^{(d*x + c)} - 8*a*b*d^2*x^3*e^{(-d*x - c)} - a^2*d^2*x^2*e^{(d*x + c)} - 12*b^2*d*x^3*e^{(d*x + c)} - a^2*d^2*x^2*e^{(-d*x - c)} + 12*b^2*d*x^3*e^{(-d*x - c)} - 8*a*b*d*x^2*e^{(d*x + c)} + 8*a*b*d*x^2*e^{(-d*x - c)} - 2*a^2*d*x*e^{(d*x + c)} - 12*b^2*x^2*e^{(d*x + c)} + 2*a^2*d*x*e^{(-d*x - c)} - 12*b^2*x^2*e^{(-d*x - c)} - 16*a*b*x*e^{(d*x + c)} - 16*a*b*x*e^{(-d*x - c)} - 6*a^2*e^{(d*x + c)} - 6*a^2*e^{(-d*x - c)})/x^4$

$$3.18 \quad \int \frac{x^4 \cosh(c+dx)}{a+bx} dx$$

**Optimal.** Leaf size=219

$$\frac{a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{a^2 \cosh(c+dx)}{b^3 d^2} + \frac{a^4 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{a^3 \sinh(c+dx)}{b^4 d} + \frac{a^2 x \sinh(c+dx)}{b^3 d}$$

[Out] (-6\*Cosh[c + d\*x])/(b\*d^4) - (a^2\*Cosh[c + d\*x])/(b^3\*d^2) + (2\*a\*x\*Cosh[c + d\*x])/(b^2\*d^2) - (3\*x^2\*Cosh[c + d\*x])/(b\*d^2) + (a^4\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b^5 - (2\*a\*Sinh[c + d\*x])/(b^2\*d^3) - (a^3\*Sinh[c + d\*x])/(b^4\*d) + (6\*x\*Sinh[c + d\*x])/(b\*d^3) + (a^2\*x\*Sinh[c + d\*x])/(b^3\*d) - (a\*x^2\*Sinh[c + d\*x])/(b^2\*d) + (x^3\*Sinh[c + d\*x])/(b\*d) + (a^4\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^5

**Rubi [A]** time = 0.484027, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {6742, 2637, 3296, 2638, 3303, 3298, 3301}

$$\frac{a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{a^2 \cosh(c+dx)}{b^3 d^2} + \frac{a^4 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{a^3 \sinh(c+dx)}{b^4 d} + \frac{a^2 x \sinh(c+dx)}{b^3 d}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*Cosh[c + d\*x])/(a + b\*x),x]

[Out] (-6\*Cosh[c + d\*x])/(b\*d^4) - (a^2\*Cosh[c + d\*x])/(b^3\*d^2) + (2\*a\*x\*Cosh[c + d\*x])/(b^2\*d^2) - (3\*x^2\*Cosh[c + d\*x])/(b\*d^2) + (a^4\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b^5 - (2\*a\*Sinh[c + d\*x])/(b^2\*d^3) - (a^3\*Sinh[c + d\*x])/(b^4\*d) + (6\*x\*Sinh[c + d\*x])/(b\*d^3) + (a^2\*x\*Sinh[c + d\*x])/(b^3\*d) - (a\*x^2\*Sinh[c + d\*x])/(b^2\*d) + (x^3\*Sinh[c + d\*x])/(b\*d) + (a^4\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^5

**Rule 6742**

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

**Rule 2637**

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^4 \cosh(c + dx)}{a + bx} dx &= \int \left( -\frac{a^3 \cosh(c + dx)}{b^4} + \frac{a^2 x \cosh(c + dx)}{b^3} - \frac{ax^2 \cosh(c + dx)}{b^2} + \frac{x^3 \cosh(c + dx)}{b} + \frac{a^4 \cosh(c + dx)}{b^4(a + bx)} \right) dx \\
&= -\frac{a^3 \int \cosh(c + dx) dx}{b^4} + \frac{a^4 \int \frac{\cosh(c+dx)}{a+bx} dx}{b^4} + \frac{a^2 \int x \cosh(c + dx) dx}{b^3} - \frac{a \int x^2 \cosh(c + dx) dx}{b^2} + \frac{a^4 \int \frac{\cosh(c+dx)}{a+bx} dx}{b^4} \\
&= -\frac{a^3 \sinh(c + dx)}{b^4 d} + \frac{a^2 x \sinh(c + dx)}{b^3 d} - \frac{ax^2 \sinh(c + dx)}{b^2 d} + \frac{x^3 \sinh(c + dx)}{bd} - \frac{a^2 \int \sinh(c + dx) dx}{b^3 d} \\
&= -\frac{a^2 \cosh(c + dx)}{b^3 d^2} + \frac{2ax \cosh(c + dx)}{b^2 d^2} - \frac{3x^2 \cosh(c + dx)}{bd^2} + \frac{a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5} - \frac{a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5} \\
&= -\frac{a^2 \cosh(c + dx)}{b^3 d^2} + \frac{2ax \cosh(c + dx)}{b^2 d^2} - \frac{3x^2 \cosh(c + dx)}{bd^2} + \frac{a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5} - \frac{2a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5} \\
&= -\frac{6 \cosh(c + dx)}{bd^4} - \frac{a^2 \cosh(c + dx)}{b^3 d^2} + \frac{2ax \cosh(c + dx)}{b^2 d^2} - \frac{3x^2 \cosh(c + dx)}{bd^2} + \frac{a^4 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5}
\end{aligned}$$

**Mathematica [A]** time = 0.631353, size = 159, normalized size = 0.73

$$\frac{-b \left( d \left( -a^2 b d^2 x + a^3 d^2 + ab^2 \left( d^2 x^2 + 2 \right) - b^3 x \left( d^2 x^2 + 6 \right) \right) \sinh(c + dx) + b \left( a^2 d^2 - 2abd^2 x + 3b^2 \left( d^2 x^2 + 2 \right) \right) \cosh(c + dx) \right)}{b^5 d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*Cosh[c + d\*x])/(a + b\*x),x]

[Out] (a^4\*d^4\*Cosh[c - (a\*d)/b]\*CoshIntegral[d\*(a/b + x)] - b\*(b\*(a^2\*d^2 - 2\*a\*b\*d^2\*x + 3\*b^2\*(2 + d^2\*x^2))\*Cosh[c + d\*x] + d\*(a^3\*d^2 - a^2\*b\*d^2\*x + a\*b^2\*(2 + d^2\*x^2) - b^3\*x\*(6 + d^2\*x^2))\*Sinh[c + d\*x]) + a^4\*d^4\*Sinh[c - (a\*d)/b]\*SinhIntegral[d\*(a/b + x)]/(b^5\*d^4)

**Maple [A]** time = 0.133, size = 442, normalized size = 2.

$$-\frac{a^4}{2b^5} e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) - \frac{3e^{-dx-c}x^2}{2bd^2} - \frac{e^{-dx-c}a^2}{2d^2b^3} - 3\frac{e^{-dx-c}}{bd^4} + \frac{e^{-dx-c}ax}{d^2b^2} - \frac{e^{-dx-c}x^3}{2bd} + \frac{e^{-dx-c}a^3}{2db^4} - 3\frac{e^{-dx-c}x}{d^3b} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*cosh(d\*x+c)/(b\*x+a),x)



[Out]  $-1/2/b^5*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)*a^4-3/2/d^2*\exp(-d*x-c)/b*x^2-1/2/d^2*\exp(-d*x-c)/b^3*a^2-3/d^4*\exp(-d*x-c)/b+1/d^2*\exp(-d*x-c)/b^2*a*x-1/2/d*\exp(-d*x-c)/b*x^3+1/2/d*\exp(-d*x-c)/b^4*a^3-3/d^3*\exp(-d*x-c)/b*x+1/d^3*\exp(-d*x-c)/b^2*a+1/2/d*\exp(-d*x-c)/b^2*a*x^2-1/2/d*\exp(-d*x-c)/b^3*a^2*x-3/d^4/b*\exp(d*x+c)-1/2/d/b^2*a*\exp(d*x+c)*x^2+1/d^2/b^2*a*\exp(d*x+c)*x-1/2/b^5*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)*a^4+1/2/d/b*\exp(d*x+c)*x^3-3/2/d^2/b*\exp(d*x+c)*x^2+3/d^3/b*\exp(d*x+c)*x-1/2/d/b^4*a^3*\exp(d*x+c)-1/d^3/b^2*a*\exp(d*x+c)-1/2/d^2/b^3*a^2*\exp(d*x+c)+1/2/d/b^3*a^2*\exp(d*x+c)*x$

**Maxima [A]** time = 1.44775, size = 590, normalized size = 2.69

$$-\frac{1}{24}d \left( \frac{12a^4 \left( \frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+ad)d}{b}\right) + \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+ad)d}{b}\right)}{b}}{b^4d} \right) - 12a^3 \left( \frac{(dx e^c - e^c) e^{(dx)}}{d^2} + \frac{(dx+1) e^{(-dx-c)}}{d^2} \right)}{b^4} + \frac{6a^2 \left( \frac{(d^2 x^2 e^c - 2 dx e^c + 2 e^c) e^{(dx)}}{d^3} + \frac{(d^2 x^2 - 2 dx + 2) e^{(-dx-c)}}{d^3} \right)}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")`

[Out]  $-1/24*d*(12*a^4*(e^{(-c+a*d/b)}*\exp\_integral\_e(1,(b*x+a)*d/b)/b + e^{(c-a*d/b)}*\exp\_integral\_e(1,-(b*x+a)*d/b)/b)/(b^4*d) - 12*a^3*((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x+1)*e^{(-d*x-c)}/d^2)/b^4 + 6*a^2*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2)*e^{(-d*x-c)}/d^3)/b^3 - 4*a*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^{(d*x)}/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^{(-d*x-c)}/d^4)/b^2 + 3*((d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*e^{(d*x)}/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*e^{(-d*x-c)}/d^5)/b + 24*a^4*cosh(d*x+c)*log(b*x+a)/(b^5*d) + 1/12*(12*a^4*log(b*x+a)/b^5 + (3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3*x)/b^4)*cosh(d*x+c)$

**Fricas [A]** time = 2.09321, size = 481, normalized size = 2.2

$$\frac{2(3b^4d^2x^2 - 2ab^3d^2x + a^2b^2d^2 + 6b^4) \cosh(dx+c) - \left(a^4d^4\text{Ei}\left(\frac{bdx+ad}{b}\right) + a^4d^4\text{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(-\frac{bc-ad}{b}\right) - 2(b^4d^3 - 4ab^3d^2x + 6a^2b^2d^2 + 6b^4) \cosh(dx+c)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*cosh(d\*x+c)/(b\*x+a),x, algorithm="fricas")

[Out] 
$$-1/2*(2*(3*b^4*d^2*x^2 - 2*a*b^3*d^2*x + a^2*b^2*d^2 + 6*b^4)*\cosh(d*x + c) - (a^4*d^4*Ei((b*d*x + a*d)/b) + a^4*d^4*Ei(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - 2*(b^4*d^3*x^3 - a*b^3*d^3*x^2 - a^3*b*d^3 - 2*a*b^3*d + (a^2*b^2*d^3 + 6*b^4*d)*x)*\sinh(d*x + c) + (a^4*d^4*Ei((b*d*x + a*d)/b) - a^4*d^4*Ei(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^5*d^4)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*cosh(d\*x+c)/(b\*x+a),x)

[Out] Integral(x\*\*4\*cosh(c + d\*x)/(a + b\*x), x)

**Giac [A]** time = 1.14898, size = 84, normalized size = 0.38

$$\frac{a^4 Ei\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + a^4 Ei\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)}}{2 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*cosh(d\*x+c)/(b\*x+a),x, algorithm="giac")

[Out] 
$$1/2*(a^4*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^4*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)})/b^5$$

$$3.19 \quad \int \frac{x^3 \cosh(c+dx)}{a+bx} dx$$

**Optimal.** Leaf size=150

$$-\frac{a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4} - \frac{a^3 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^4} + \frac{a^2 \sinh(c+dx)}{b^3 d} + \frac{a \cosh(c+dx)}{b^2 d^2} - \frac{ax \sinh(c+dx)}{b^2 d}$$

[Out] (a\*Cosh[c + d\*x])/(b^2\*d^2) - (2\*x\*Cosh[c + d\*x])/(b\*d^2) - (a^3\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b^4 + (2\*Sinh[c + d\*x])/(b\*d^3) + (a^2\*Sinh[c + d\*x])/(b^3\*d) - (a\*x\*Sinh[c + d\*x])/(b^2\*d) + (x^2\*Sinh[c + d\*x])/(b\*d) - (a^3\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^4

**Rubi [A]** time = 0.328539, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {6742, 2637, 3296, 2638, 3303, 3298, 3301}

$$-\frac{a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4} - \frac{a^3 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^4} + \frac{a^2 \sinh(c+dx)}{b^3 d} + \frac{a \cosh(c+dx)}{b^2 d^2} - \frac{ax \sinh(c+dx)}{b^2 d}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*Cosh[c + d\*x])/(a + b\*x),x]

[Out] (a\*Cosh[c + d\*x])/(b^2\*d^2) - (2\*x\*Cosh[c + d\*x])/(b\*d^2) - (a^3\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b^4 + (2\*Sinh[c + d\*x])/(b\*d^3) + (a^2\*Sinh[c + d\*x])/(b^3\*d) - (a\*x\*Sinh[c + d\*x])/(b^2\*d) + (x^2\*Sinh[c + d\*x])/(b\*d) - (a^3\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^4

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c + dx)}{a + bx} dx &= \int \left( \frac{a^2 \cosh(c + dx)}{b^3} - \frac{ax \cosh(c + dx)}{b^2} + \frac{x^2 \cosh(c + dx)}{b} - \frac{a^3 \cosh(c + dx)}{b^3(a + bx)} \right) dx \\
&= \frac{a^2 \int \cosh(c + dx) dx}{b^3} - \frac{a^3 \int \frac{\cosh(c+dx)}{a+bx} dx}{b^3} - \frac{a \int x \cosh(c + dx) dx}{b^2} + \frac{\int x^2 \cosh(c + dx) dx}{b} \\
&= \frac{a^2 \sinh(c + dx)}{b^3 d} - \frac{ax \sinh(c + dx)}{b^2 d} + \frac{x^2 \sinh(c + dx)}{bd} + \frac{a \int \sinh(c + dx) dx}{b^2 d} - \frac{2 \int x \sinh(c + dx) dx}{bd} \\
&= \frac{a \cosh(c + dx)}{b^2 d^2} - \frac{2x \cosh(c + dx)}{bd^2} - \frac{a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} + \frac{a^2 \sinh(c + dx)}{b^3 d} - \frac{ax \sinh(c + dx)}{bd} \\
&= \frac{a \cosh(c + dx)}{b^2 d^2} - \frac{2x \cosh(c + dx)}{bd^2} - \frac{a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} + \frac{2 \sinh(c + dx)}{bd^3} + \frac{a^2 \sinh(c + dx)}{bd}
\end{aligned}$$

**Mathematica [A]** time = 0.430323, size = 118, normalized size = 0.79

$$\frac{b \left( (a^2 d^2 - abd^2 x + b^2 (d^2 x^2 + 2)) \sinh(c + dx) + bd(a - 2bx) \cosh(c + dx) \right) - a^3 d^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d \left(\frac{a}{b} + x\right)\right) - a^3 d^3}{b^4 d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*Cosh[c + d\*x])/(a + b\*x), x]

[Out]  $(-(a^3 d^3 \text{Cosh}[c - (a*d)/b] * \text{CoshIntegral}[d*(a/b + x)]) + b*(b*d*(a - 2*b*x) * \text{Cosh}[c + d*x] + (a^2*d^2 - a*b*d^2*x + b^2*(2 + d^2*x^2)) * \text{Sinh}[c + d*x]) - a^3*d^3 * \text{Sinh}[c - (a*d)/b] * \text{SinhIntegral}[d*(a/b + x)]) / (b^4*d^3)$

**Maple [A]** time = 0.034, size = 292, normalized size = 2.

$$-\frac{e^{-dx-cx^2}}{2bd} + \frac{e^{-dx-c}ax}{2db^2} - \frac{e^{-dx-c}a^2}{2db^3} - \frac{e^{-dx-c}x}{bd^2} + \frac{e^{-dx-c}a}{2d^2b^2} - \frac{e^{-dx-c}}{d^3b} + \frac{a^3}{2b^4} e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) - \frac{e^{dx+c}x}{bd^2} + \frac{e^{dx+c}}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*cosh(d\*x+c)/(b\*x+a), x)

[Out]  $-1/2/d*\exp(-d*x-c)/b*x^2+1/2/d*\exp(-d*x-c)/b^2*a*x-1/2/d*\exp(-d*x-c)/b^3*a^2-1/d^2*\exp(-d*x-c)/b*x+1/2/d^2*\exp(-d*x-c)/b^2*a-1/d^3*\exp(-d*x-c)/b+1/2/b$

$$\begin{aligned} &^4 \exp((a*d-b*c)/b) * \text{Ei}(1, d*x+c+(a*d-b*c)/b) * a^3 - 1/d^2/b * \exp(d*x+c) * x + 1/2/d/ \\ &b * \exp(d*x+c) * x^2 + 1/d^3/b * \exp(d*x+c) + 1/2/b^4 * \exp(-(a*d-b*c)/b) * \text{Ei}(1, -d*x-c-( \\ &a*d-b*c)/b) * a^3 - 1/2/d/b^2 * a * \exp(d*x+c) * x + 1/2/d^2/b^2 * a * \exp(d*x+c) + 1/2/d/b^3 \\ &* a^2 * \exp(d*x+c) \end{aligned}$$

**Maxima [B]** time = 1.27553, size = 443, normalized size = 2.95

$$\frac{1}{12} d \left( \frac{6 a^3 \left( \frac{e^{(-c+\frac{ad}{b})} \text{Ei}\left(\frac{(bx+a)d}{b}\right) + e^{(c-\frac{ad}{b})} \text{Ei}\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{b^3 d} - \frac{6 a^2 \left( \frac{(dx e^c - e^c) e^{(dx)}}{d^2} + \frac{(dx+1) e^{(-dx-c)}}{d^2} \right)}{b^3} + \frac{3 a \left( \frac{(d^2 x^2 e^c - 2 dx e^c + 2 e^c) e^{(dx)}}{d^3} + \frac{(d^2 x^2 + 2 dx + 2) e^{(-dx-c)}}{d^3} \right)}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x+a),x, algorithm="maxima")

[Out]  $\frac{1}{12} d * (6 a^3 * (e^{(-c + a*d/b)} * \text{exp\_integral\_e}(1, (b*x + a)*d/b)/b + e^{(c - a*d/b)} * \text{exp\_integral\_e}(1, -(b*x + a)*d/b)/b) / (b^3*d) - 6 a^2 * ((d*x*e^c - e^c) * e^{(d*x)}/d^2 + (d*x + 1) * e^{(-d*x - c)}/d^2) / b^3 + 3 a * ((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c) * e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2) * e^{(-d*x - c)}/d^3) / b^2 - 2 * ((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c) * e^{(d*x)}/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6) * e^{(-d*x - c)}/d^4) / b + 12 a^3 * \cosh(d*x + c) * \log(b*x + a) / (b^4*d) - 1/6 * (6 a^3 * \log(b*x + a) / b^4 - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x) / b^3) * \cosh(d*x + c)$

**Fricas [A]** time = 2.09382, size = 392, normalized size = 2.61

$$\frac{2(2b^3dx - ab^2d) \cosh(dx + c) + \left(a^3d^3 \text{Ei}\left(\frac{bdx+ad}{b}\right) + a^3d^3 \text{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(-\frac{bc-ad}{b}\right) - 2(b^3d^2x^2 - ab^2d^2x + a^2bd^2 + 2a^2d^2x - ab^2d^2) \sinh(dx + c)}{2b^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x+a),x, algorithm="fricas")

[Out]  $-1/2 * (2 * (2*b^3*d*x - a*b^2*d) * \cosh(d*x + c) + (a^3*d^3 * \text{Ei}((b*d*x + a*d)/b) + a^3*d^3 * \text{Ei}(-(b*d*x + a*d)/b)) * \cosh(-(b*c - a*d)/b) - 2 * (b^3*d^2*x^2 - a*b^2*d^2*x + a^2*b*d^2 + 2*b^3) * \sinh(d*x + c) - (a^3*d^3 * \text{Ei}((b*d*x + a*d)/b) + a^3*d^3 * \text{Ei}(-(b*d*x + a*d)/b)) * \cosh(-(b*c - a*d)/b)$

$$- a^3 d^3 \operatorname{Ei}(- (b d x + a d) / b) * \sinh(- (b c - a d) / b) / (b^4 d^3)$$


---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*cosh(d\*x+c)/(b\*x+a),x)

[Out] Integral(x\*\*3\*cosh(c + d\*x)/(a + b\*x), x)

---

**Giac [A]** time = 1.1756, size = 84, normalized size = 0.56

$$\frac{a^3 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(\frac{c-ad}{b}\right)} + a^3 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)}}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x+a),x, algorithm="giac")

[Out] -1/2\*(a^3\*Ei((b\*d\*x + a\*d)/b)\*e^(c - a\*d/b) + a^3\*Ei(-(b\*d\*x + a\*d)/b)\*e^(-c + a\*d/b))/b^4

### 3.20 $\int \frac{x^2 \cosh(c+dx)}{a+bx} dx$

**Optimal.** Leaf size=100

$$\frac{a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} + \frac{a^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^3} - \frac{a \sinh(c+dx)}{b^2 d} - \frac{\cosh(c+dx)}{bd^2} + \frac{x \sinh(c+dx)}{bd}$$

[Out]  $-(\text{Cosh}[c + d*x]/(b*d^2)) + (a^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/b^3 - (a*\text{Sinh}[c + d*x])/(b^2*d) + (x*\text{Sinh}[c + d*x])/(b*d) + (a^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^3$

**Rubi [A]** time = 0.26226, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {6742, 2637, 3296, 2638, 3303, 3298, 3301}

$$\frac{a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} + \frac{a^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^3} - \frac{a \sinh(c+dx)}{b^2 d} - \frac{\cosh(c+dx)}{bd^2} + \frac{x \sinh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*\text{Cosh}[c + d*x])/(a + b*x), x]$

[Out]  $-(\text{Cosh}[c + d*x]/(b*d^2)) + (a^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/b^3 - (a*\text{Sinh}[c + d*x])/(b^2*d) + (x*\text{Sinh}[c + d*x])/(b*d) + (a^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^3$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \text{ :> With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] \text{ /; SumQ}[v]$   
]

#### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x\_)], x\_Symbol] \text{ :> Simp}[\text{Sin}[c + d*x]/d, x] \text{ /; FreeQ}\{c, d\}, x]$

#### Rule 3296

$\text{Int}(((c_.) + (d_.)*(x\_))^{(m_.)}*\sin[(e_.) + (f_.)*(x\_)], x\_Symbol] \text{ :> -Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[$



$e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}\{c, d\}, x]$

### Rule 3303

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{NeQ}[d*e - c*f, 0]$

### Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(I*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \ \text{EqQ}[d*e - c*f*fz*I, 0]$

### Rule 3301

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \ \text{EqQ}[d*(e - \text{Pi}/2) - c*f*fz*I, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{x^2 \cosh(c + dx)}{a + bx} dx &= \int \left( -\frac{a \cosh(c + dx)}{b^2} + \frac{x \cosh(c + dx)}{b} + \frac{a^2 \cosh(c + dx)}{b^2(a + bx)} \right) dx \\ &= -\frac{a \int \cosh(c + dx) dx}{b^2} + \frac{a^2 \int \frac{\cosh(c+dx)}{a+bx} dx}{b^2} + \frac{\int x \cosh(c + dx) dx}{b} \\ &= -\frac{a \sinh(c + dx)}{b^2 d} + \frac{x \sinh(c + dx)}{bd} - \frac{\int \sinh(c + dx) dx}{bd} + \frac{\left( a^2 \cosh\left( c - \frac{ad}{b} \right) \right) \int \frac{\cosh\left( \frac{ad}{b} + dx \right)}{a+bx} dx}{b^2} + \\ &= -\frac{\cosh(c + dx)}{bd^2} + \frac{a^2 \cosh\left( c - \frac{ad}{b} \right) \text{Chi}\left( \frac{ad}{b} + dx \right)}{b^3} - \frac{a \sinh(c + dx)}{b^2 d} + \frac{x \sinh(c + dx)}{bd} + \frac{a^2 \sinh(c + dx)}{bd^2} \end{aligned}$$

**Mathematica [A]** time = 0.283274, size = 89, normalized size = 0.89

$$\frac{a^2 d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + a^2 d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + b(d(bx - a) \sinh(c + dx) - b \cosh(c + dx))}{b^3 d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Cosh[c + d\*x])/(a + b\*x),x]

[Out] (a^2\*d^2\*Cosh[c - (a\*d)/b]\*CoshIntegral[d\*(a/b + x)] + b\*(-(b\*Cosh[c + d\*x]) + d\*(-a + b\*x)\*Sinh[c + d\*x]) + a^2\*d^2\*Sinh[c - (a\*d)/b]\*SinhIntegral[d\*(a/b + x)]/(b^3\*d^2)

**Maple [A]** time = 0.03, size = 184, normalized size = 1.8

$$-\frac{e^{-dx-c}x}{2bd} + \frac{e^{-dx-c}a}{2db^2} - \frac{e^{-dx-c}}{2bd^2} - \frac{a^2}{2b^3} e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) + \frac{e^{dx+c}x}{2bd} - \frac{e^{dx+c}}{2bd^2} - \frac{ae^{dx+c}}{2db^2} - \frac{a^2}{2b^3} e^{-\frac{da-cb}{b}} \text{Ei}\left(1, -dx - c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cosh(d\*x+c)/(b\*x+a),x)

[Out] -1/2/d\*exp(-d\*x-c)/b\*x+1/2/d\*exp(-d\*x-c)/b^2\*a-1/2/d^2\*exp(-d\*x-c)/b-1/2/b^3\*exp((a\*d-b\*c)/b)\*Ei(1,d\*x+c+(a\*d-b\*c)/b)\*a^2+1/2/d/b\*exp(d\*x+c)\*x-1/2/d^2/b\*exp(d\*x+c)-1/2/d/b^2\*a\*exp(d\*x+c)-1/2/b^3\*exp(-(a\*d-b\*c)/b)\*Ei(1,-d\*x-c-(a\*d-b\*c)/b)\*a^2

**Maxima [B]** time = 1.37828, size = 315, normalized size = 3.15

$$-\frac{1}{4}d \left( \frac{2a^2 \left( \frac{e^{\left(-c+\frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)} + \frac{e^{\left(c-\frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)} \right)}{b^2 d} - \frac{2a \left( \frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2} \right)}{b^2} + \frac{\left(d^2x^2e^c - 2dxe^c + 2e^c\right)e^{(dx)}}{d^3} + \frac{\left(d^2x^2 + 2dx + 2\right)e^{(-dx-c)}}{d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x+a),x, algorithm="maxima")

[Out]  $-1/4*d*(2*a^2*(e^{(-c + a*d/b)}*\exp\_integral\_e(1, (b*x + a)*d/b)/b + e^{(c - a*d/b)}*\exp\_integral\_e(1, -(b*x + a)*d/b)/b)/(b^2*d) - 2*a*((d*x*e^c - e^c)*e^{(d*x)/d^2} + (d*x + 1)*e^{(-d*x - c)/d^2})/b^2 + ((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^{(d*x)/d^3} + (d^2*x^2 + 2*d*x + 2)*e^{(-d*x - c)/d^3})/b + 4*a^2*\cosh(d*x + c)*\log(b*x + a)/(b^3*d) + 1/2*(2*a^2*\log(b*x + a)/b^3 + (b*x^2 - 2*a*x)/b^2)*\cosh(d*x + c)$

**Fricas [A]** time = 2.15575, size = 327, normalized size = 3.27

$$\frac{2b^2 \cosh(dx + c) - \left(a^2 d^2 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) + a^2 d^2 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(-\frac{bc-ad}{b}\right) - 2(b^2 dx - abd) \sinh(dx + c) + \left(a^2 d^2 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) - a^2 d^2 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \sinh\left(-\frac{bc-ad}{b}\right)}{2b^3 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")`

[Out]  $-1/2*(2*b^2*\cosh(d*x + c) - (a^2*d^2*\operatorname{Ei}((b*d*x + a*d)/b) + a^2*d^2*\operatorname{Ei}(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - 2*(b^2*d*x - a*b*d)*\sinh(d*x + c) + (a^2*d^2*\operatorname{Ei}((b*d*x + a*d)/b) - a^2*d^2*\operatorname{Ei}(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^3*d^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cosh(d*x+c)/(b*x+a),x)`

[Out] `Integral(x**2*cosh(c + d*x)/(a + b*x), x)`

**Giac [A]** time = 1.20668, size = 84, normalized size = 0.84

$$\frac{a^2 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + a^2 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(a^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b))/b^3
```

$$3.21 \quad \int \frac{x \cosh(c+dx)}{a+bx} dx$$

**Optimal.** Leaf size=68

$$-\frac{a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} - \frac{a \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{\sinh(c + dx)}{bd}$$

[Out]  $-\left(\frac{a \text{Cosh}\left[c - \frac{a*d}{b}\right] \text{CoshIntegral}\left[\frac{a*d}{b} + d*x\right]}{b^2}\right) + \frac{\text{Sinh}\left[c + d*x\right]}{b*d} - \left(\frac{a \text{Sinh}\left[c - \frac{a*d}{b}\right] \text{SinhIntegral}\left[\frac{a*d}{b} + d*x\right]}{b^2}\right)$

**Rubi [A]** time = 0.176147, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 2637, 3303, 3298, 3301}

$$-\frac{a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} - \frac{a \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{\sinh(c + dx)}{bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\frac{x \text{Cosh}\left[c + d*x\right]}{a + b*x}, x\right]$

[Out]  $-\left(\frac{a \text{Cosh}\left[c - \frac{a*d}{b}\right] \text{CoshIntegral}\left[\frac{a*d}{b} + d*x\right]}{b^2}\right) + \frac{\text{Sinh}\left[c + d*x\right]}{b*d} - \left(\frac{a \text{Sinh}\left[c - \frac{a*d}{b}\right] \text{SinhIntegral}\left[\frac{a*d}{b} + d*x\right]}{b^2}\right)$

### Rule 6742

$\text{Int}[u, x\_Symbol] \rightarrow \text{With}\left[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] \text{ ; SumQ}[v]\right]$

### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c\_.) + (d\_.)*(x\_)], x\_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] \text{ ; FreeQ}\{c, d\}, x]$

### Rule 3303

$\text{Int}[\sin[(e\_.) + (f\_.)*(x\_)]/((c\_.) + (d\_.)*(x\_)), x\_Symbol] \rightarrow \text{Dist}[\cos[(d*e - c*f)/d], \text{Int}[\sin[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\sin[(d*e - c*f)/d], \text{Int}[\cos[(c*f)/d + f*x]/(c + d*x), x], x] \text{ ; FreeQ}\{c, d, e, f\}, x \ \&\& \text{NeQ}[d*e - c*f, 0]$

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x \cosh(c + dx)}{a + bx} dx &= \int \left( \frac{\cosh(c + dx)}{b} - \frac{a \cosh(c + dx)}{b(a + bx)} \right) dx \\
&= \frac{\int \cosh(c + dx) dx}{b} - \frac{a \int \frac{\cosh(c + dx)}{a + bx} dx}{b} \\
&= \frac{\sinh(c + dx)}{bd} - \frac{\left( a \cosh\left(c - \frac{ad}{b}\right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a + bx} dx \right)}{b} - \frac{\left( a \sinh\left(c - \frac{ad}{b}\right) \int \frac{\sinh\left(\frac{ad}{b} + dx\right)}{a + bx} dx \right)}{b} \\
&= -\frac{a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^2} + \frac{\sinh(c + dx)}{bd} - \frac{a \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.135784, size = 64, normalized size = 0.94

$$\frac{-ad \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) - ad \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + b \sinh(c + dx)}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cosh[c + d*x])/(a + b*x), x]
```

```
[Out] (-(a*d*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)]) + b*Sinh[c + d*x] - a*d*
*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^2*d)
```

**Maple [A]** time = 0.023, size = 114, normalized size = 1.7

$$-\frac{e^{-dx-c}}{2bd} + \frac{a}{2b^2} e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx+c+\frac{da-cb}{b}\right) + \frac{e^{dx+c}}{2bd} + \frac{a}{2b^2} e^{-\frac{da-cb}{b}} \operatorname{Ei}\left(1, -dx-c-\frac{da-cb}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cosh(d*x+c)/(b*x+a),x)`

[Out]  $-1/2/d*\exp(-d*x-c)/b+1/2/b^2*\exp((a*d-b*c)/b)*\operatorname{Ei}(1,d*x+c+(a*d-b*c)/b)*a+1/2/d/b*\exp(d*x+c)+1/2/b^2*\exp(-(a*d-b*c)/b)*\operatorname{Ei}(1,-d*x-c-(a*d-b*c)/b)*a$

**Maxima [B]** time = 1.33287, size = 211, normalized size = 3.1

$$\frac{1}{2} d \left( \frac{a \left( \frac{e^{(-c+\frac{ad}{b})} \operatorname{Ei}\left(\frac{(bx+a)d}{b}\right)} + \frac{e^{(c-\frac{ad}{b})} \operatorname{Ei}\left(-\frac{(bx+a)d}{b}\right)} \right)}{bd} - \frac{\frac{(dx)e^c - e^c}{d^2} e^{(dx)}}{b} + \frac{(dx+1)e^{(-dx-c)}}{d^2} + \frac{2a \cosh(dx+c) \log(bx+a)}{b^2 d} \right) + \left( \frac{x}{b} - \frac{a \log(bx+a)}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")`

[Out]  $1/2*d*(a*(e^{(-c+a*d/b)}*\exp\_integral\_e(1,(b*x+a)*d/b)/b + e^{(c-a*d/b)}*\exp\_integral\_e(1,-(b*x+a)*d/b)/b)/(b*d) - ((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x+1)*e^{(-d*x-c)}/d^2)/b + 2*a*cosh(d*x+c)*\log(b*x+a)/(b^2*d) + (x/b - a*\log(b*x+a)/b^2)*cosh(d*x+c)$

**Fricas [A]** time = 1.9967, size = 251, normalized size = 3.69

$$\frac{\left( ad \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) + ad \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) \right) \cosh\left(-\frac{bc-ad}{b}\right) - 2b \sinh(dx+c) - \left( ad \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) - ad \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) \right) \sinh\left(-\frac{bc-ad}{b}\right)}{2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")`

[Out]  $-1/2*((a*d*Ei((b*d*x + a*d)/b) + a*d*Ei(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - 2*b*\sinh(d*x + c) - (a*d*Ei((b*d*x + a*d)/b) - a*d*Ei(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^2*d)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(d*x+c)/(b*x+a), x)`

[Out] `Integral(x*cosh(c + d*x)/(a + b*x), x)`

**Giac [A]** time = 1.18626, size = 78, normalized size = 1.15

$$\frac{aEi\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + aEi\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(d*x+c)/(b*x+a), x, algorithm="giac")`

[Out]  $-1/2*(a*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)})/b^2$



$$3.22 \quad \int \frac{\cosh(c+dx)}{a+bx} dx$$

**Optimal.** Leaf size=51

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b} + \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b}$$

[Out] (Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b + (Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b

**Rubi [A]** time = 0.0795504, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3303, 3298, 3301}

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b} + \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(a + b\*x), x]

[Out] (Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b + (Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}

}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\cosh(c + dx)}{a + bx} dx &= \cosh\left(c - \frac{ad}{b}\right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a + bx} dx + \sinh\left(c - \frac{ad}{b}\right) \int \frac{\sinh\left(\frac{ad}{b} + dx\right)}{a + bx} dx \\ &= \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b} + \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0611119, size = 49, normalized size = 0.96

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right) + \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(a + b\*x),x]

[Out] (Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x] + Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b

**Maple [A]** time = 0.019, size = 81, normalized size = 1.6

$$-\frac{1}{2b} e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) - \frac{1}{2b} e^{-\frac{da-cb}{b}} \text{Ei}\left(1, -dx - c - \frac{da-cb}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/(b\*x+a),x)

[Out] -1/2/b\*exp((a\*d-b\*c)/b)\*Ei(1,d\*x+c+(a\*d-b\*c)/b)-1/2/b\*exp(-(a\*d-b\*c)/b)\*Ei(1,-d\*x-c-(a\*d-b\*c)/b)

**Maxima [A]** time = 1.31556, size = 77, normalized size = 1.51

$$-\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{2b}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x+a),x, algorithm="maxima")

[Out]  $-1/2*e^{(-c + a*d/b)}*\exp\_integral\_e(1, (b*x + a)*d/b)/b - 1/2*e^{(c - a*d/b)}*\exp\_integral\_e(1, -(b*x + a)*d/b)/b$

**Fricas [A]** time = 2.08618, size = 193, normalized size = 3.78

$$\frac{\left(Ei\left(\frac{bdx+ad}{b}\right) + Ei\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(-\frac{bc-ad}{b}\right) - \left(Ei\left(\frac{bdx+ad}{b}\right) - Ei\left(-\frac{bdx+ad}{b}\right)\right) \sinh\left(-\frac{bc-ad}{b}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x+a),x, algorithm="fricas")

[Out]  $1/2*((Ei((b*d*x + a*d)/b) + Ei(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - (Ei((b*d*x + a*d)/b) - Ei(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/b$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x+a),x)

[Out] Integral(cosh(c + d\*x)/(a + b\*x), x)

**Giac [A]** time = 1.17093, size = 76, normalized size = 1.49

$$\frac{\operatorname{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(\frac{c-ad}{b}\right)} + \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x+a),x, algorithm="giac")

[Out] 1/2\*(Ei((b\*d\*x + a\*d)/b)\*e^(c - a\*d/b) + Ei(-(b\*d\*x + a\*d)/b)\*e^(-c + a\*d/b))/b

### 3.23 $\int \frac{\cosh(c+dx)}{x(a+bx)} dx$

**Optimal.** Leaf size=73

$$-\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a} - \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a} + \frac{\cosh(c) \text{Chi}(dx)}{a} + \frac{\sinh(c) \text{Shi}(dx)}{a}$$

[Out] (Cosh[c]\*CoshIntegral[d\*x])/a - (Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/a + (Sinh[c]\*SinhIntegral[d\*x])/a - (Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/a

**Rubi [A]** time = 0.258935, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {6742, 3303, 3298, 3301}

$$-\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a} - \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a} + \frac{\cosh(c) \text{Chi}(dx)}{a} + \frac{\sinh(c) \text{Shi}(dx)}{a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x\*(a + b\*x)),x]

[Out] (Cosh[c]\*CoshIntegral[d\*x])/a - (Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/a + (Sinh[c]\*SinhIntegral[d\*x])/a - (Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/a

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

#### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\cosh(c + dx)}{x(a + bx)} dx &= \int \left( \frac{\cosh(c + dx)}{ax} - \frac{b \cosh(c + dx)}{a(a + bx)} \right) dx \\ &= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{a+bx} dx}{a} \\ &= \frac{\cosh(c) \int \frac{\cosh(dx)}{x} dx}{a} - \frac{\left( b \cosh\left(c - \frac{ad}{b}\right) \right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{a} + \frac{\sinh(c) \int \frac{\sinh(dx)}{x} dx}{a} - \frac{\left( b \sinh\left(c - \frac{ad}{b}\right) \right) \int \frac{\sinh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{a} \\ &= \frac{\cosh(c) \text{Chi}(dx)}{a} - \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a} + \frac{\sinh(c) \text{Shi}(dx)}{a} - \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{a} \end{aligned}$$

**Mathematica [A]** time = 0.140752, size = 63, normalized size = 0.86

$$\frac{-\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) - \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + \cosh(c) \text{Chi}(dx) + \sinh(c) \text{Shi}(dx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(x*(a + b*x)), x]
```

```
[Out] (Cosh[c]*CoshIntegral[d*x] - Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + Sinh[c]*SinhIntegral[d*x] - Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/a
```

**Maple [A]** time = 0.031, size = 108, normalized size = 1.5

$$-\frac{e^{-c} \text{Ei}(1, dx)}{2a} + \frac{1}{2a} e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) - \frac{e^c \text{Ei}(1, -dx)}{2a} + \frac{1}{2a} e^{-\frac{da-cb}{b}} \text{Ei}\left(1, -dx - c - \frac{da-cb}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/x/(b*x+a),x)`

[Out]  $-1/2/a*\exp(-c)*Ei(1,d*x)+1/2/a*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/a*\exp(c)*Ei(1,-d*x)+1/2/a*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)$

**Maxima [B]** time = 1.43734, size = 209, normalized size = 2.86

$$\frac{1}{2}d \left( \frac{b \left( \frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{ad} + \frac{2 \cosh(dx+c) \log(bx+a)}{ad} - \frac{2 \cosh(dx+c) \log(x)}{ad} + \frac{Ei(-dx) e^{(-c)} + Ei(dx) e^c}{ad} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="maxima")`

[Out]  $1/2*d*(b*(e^{(-c+a*d/b)}*\exp\_integral\_e(1,(b*x+a)*d/b)/b + e^{(c-a*d/b)}*\exp\_integral\_e(1,-(b*x+a)*d/b)/b)/(a*d) + 2*\cosh(d*x+c)*\log(b*x+a)/(a*d) - 2*\cosh(d*x+c)*\log(x)/(a*d) + (Ei(-d*x)*e^{(-c)} + Ei(d*x)*e^c)/(a*d) - (\log(b*x+a)/a - \log(x)/a)*\cosh(d*x+c)$

**Fricas [A]** time = 2.06579, size = 277, normalized size = 3.79

$$\frac{(Ei(dx) + Ei(-dx)) \cosh(c) - \left( Ei\left(\frac{bdx+ad}{b}\right) + Ei\left(-\frac{bdx+ad}{b}\right) \right) \cosh\left(-\frac{bc-ad}{b}\right) + (Ei(dx) - Ei(-dx)) \sinh(c) + \left( Ei\left(\frac{bdx+ad}{b}\right) - Ei\left(-\frac{bdx+ad}{b}\right) \right) \sinh\left(-\frac{bc-ad}{b}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="fricas")`

[Out]  $1/2*((Ei(d*x) + Ei(-d*x))*\cosh(c) - (Ei((b*d*x+a*d)/b) + Ei(-(b*d*x+a*d)/b))*\cosh(-(b*c-a*d)/b) + (Ei(d*x) - Ei(-d*x))*\sinh(c) + (Ei((b*d*x+a*d)/b) - Ei(-(b*d*x+a*d)/b))*\sinh(-(b*c-a*d)/b))/a$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx)}{x(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x+a),x)

[Out] Integral(cosh(c + d\*x)/(x\*(a + b\*x)), x)

---

**Giac [A]** time = 1.15917, size = 101, normalized size = 1.38

$$\frac{\operatorname{Ei}(-dx)e^{(-c)} - \operatorname{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + \operatorname{Ei}(dx)e^c - \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x+a),x, algorithm="giac")

[Out] 1/2\*(Ei(-d\*x)\*e^(-c) - Ei((b\*d\*x + a\*d)/b)\*e^(c - a\*d/b) + Ei(d\*x)\*e^c - Ei(-(b\*d\*x + a\*d)/b)\*e^(-c + a\*d/b))/a



$$3.24 \quad \int \frac{\cosh(c+dx)}{x^2(a+bx)} dx$$

**Optimal.** Leaf size=113

$$-\frac{b \cosh(c)\text{Chi}(dx)}{a^2} + \frac{b \cosh\left(c - \frac{ad}{b}\right)\text{Chi}\left(xd + \frac{ad}{b}\right)}{a^2} - \frac{b \sinh(c)\text{Shi}(dx)}{a^2} + \frac{b \sinh\left(c - \frac{ad}{b}\right)\text{Shi}\left(xd + \frac{ad}{b}\right)}{a^2} + \frac{d \sinh(c)\text{Chi}(dx)}{a}$$

[Out] -(Cosh[c + d\*x]/(a\*x)) - (b\*Cosh[c]\*CoshIntegral[d\*x])/a^2 + (b\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/a^2 + (d\*CoshIntegral[d\*x]\*Sinh[c])/a + (d\*Cosh[c]\*SinhIntegral[d\*x])/a - (b\*Sinh[c]\*SinhIntegral[d\*x])/a^2 + (b\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/a^2

**Rubi [A]** time = 0.368679, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$-\frac{b \cosh(c)\text{Chi}(dx)}{a^2} + \frac{b \cosh\left(c - \frac{ad}{b}\right)\text{Chi}\left(xd + \frac{ad}{b}\right)}{a^2} - \frac{b \sinh(c)\text{Shi}(dx)}{a^2} + \frac{b \sinh\left(c - \frac{ad}{b}\right)\text{Shi}\left(xd + \frac{ad}{b}\right)}{a^2} + \frac{d \sinh(c)\text{Chi}(dx)}{a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x^2\*(a + b\*x)),x]

[Out] -(Cosh[c + d\*x]/(a\*x)) - (b\*Cosh[c]\*CoshIntegral[d\*x])/a^2 + (b\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/a^2 + (d\*CoshIntegral[d\*x]\*Sinh[c])/a + (d\*Cosh[c]\*SinhIntegral[d\*x])/a - (b\*Sinh[c]\*SinhIntegral[d\*x])/a^2 + (b\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/a^2

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 3297

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx)} dx &= \int \left( \frac{\cosh(c+dx)}{ax^2} - \frac{b \cosh(c+dx)}{a^2x} + \frac{b^2 \cosh(c+dx)}{a^2(a+bx)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{x} dx}{a^2} + \frac{b^2 \int \frac{\cosh(c+dx)}{a+bx} dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{ax} + \frac{d \int \frac{\sinh(c+dx)}{x} dx}{a} - \frac{(b \cosh(c)) \int \frac{\cosh(dx)}{x} dx}{a^2} + \frac{\left( b^2 \cosh\left(c - \frac{ad}{b}\right) \right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{ax} - \frac{b \cosh(c) \text{Chi}(dx)}{a^2} + \frac{b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^2} - \frac{b \sinh(c) \text{Shi}(dx)}{a^2} + \frac{b \sinh\left(\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{a^2} \\
&= -\frac{\cosh(c+dx)}{ax} - \frac{b \cosh(c) \text{Chi}(dx)}{a^2} + \frac{b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^2} + \frac{d \text{Chi}(dx) \sinh(c)}{a} + \frac{d \cosh\left(\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{a}
\end{aligned}$$

**Mathematica [A]** time = 0.380891, size = 101, normalized size = 0.89

$$\frac{bx \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + \text{Chi}(dx)(adx \sinh(c) - bx \cosh(c)) + bx \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + adx \cosh(c) \text{Shi}(dx)}{a^2x}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x^2\*(a + b\*x)),x]

[Out]  $(-(a*\text{Cosh}[c + d*x]) + b*x*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[d*(a/b + x)] + \text{CoshIntegral}[d*x]*(-b*x*\text{Cosh}[c]) + a*d*x*\text{Sinh}[c]) + a*d*x*\text{Cosh}[c]*\text{SinhIntegral}[d*x] - b*x*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + b*x*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[d*(a/b + x)])/(a^2*x)$

**Maple [A]** time = 0.048, size = 172, normalized size = 1.5

$$-\frac{e^{-dx-c}}{2ax} + \frac{de^{-c}\text{Ei}(1,dx)}{2a} + \frac{be^{-c}\text{Ei}(1,dx)}{2a^2} - \frac{b}{2a^2}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) + \frac{be^c\text{Ei}(1,-dx)}{2a^2} - \frac{e^{dx+c}}{2ax} - \frac{de^c\text{Ei}(1,-dx)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/x^2/(b\*x+a),x)

[Out]  $-1/2*\exp(-d*x-c)/a/x+1/2*d/a*\exp(-c)*\text{Ei}(1,d*x)+1/2/a^2*\exp(-c)*\text{Ei}(1,d*x)*b-1/2*b/a^2*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)+1/2*b/a^2*\exp(c)*\text{Ei}(1,-d*x)-1/2/a/x*\exp(d*x+c)-1/2*d/a*\exp(c)*\text{Ei}(1,-d*x)-1/2/a^2*b*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)$

**Maxima [A]** time = 1.45567, size = 259, normalized size = 2.29

$$-\frac{1}{2}d\left(\frac{\text{Ei}(-dx)e^{(-c)} - \text{Ei}(dx)e^c}{a} + \frac{b^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}\text{Ei}\left(\frac{(bx+a)d}{b}\right) + e^{\left(c-\frac{ad}{b}\right)}\text{Ei}\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{a^2d} + \frac{2b\cosh(dx+c)\log(bx+a)}{a^2d} - \frac{2b\cosh(dx+c)\log(bx+a)}{a^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x+a),x, algorithm="maxima")

[Out]  $-1/2*d*((\text{Ei}(-d*x)*e^{(-c)} - \text{Ei}(d*x)*e^c)/a + b^2*(e^{(-c + a*d/b)}*\text{exp\_integral}_e(1, (b*x + a)*d/b)/b + e^{(c - a*d/b)}*\text{exp\_integral}_e(1, -(b*x + a)*d/b)/b)/(a^2*d) + 2*b*cosh(d*x + c)*log(b*x + a)/(a^2*d) - 2*b*cosh(d*x + c)*log(x)/(a^2*d) + (\text{Ei}(-d*x)*e^{(-c)} + \text{Ei}(d*x)*e^c)*b/(a^2*d) + (b*log(b*x + a)/a$

$$x^2 - b \log(x)/a^2 - 1/(a*x)) * \cosh(d*x + c)$$

**Fricas [A]** time = 2.04736, size = 400, normalized size = 3.54

$$\frac{2 a \cosh(dx + c) - ((ad - b)x \operatorname{Ei}(dx) - (ad + b)x \operatorname{Ei}(-dx)) \cosh(c) - \left( bx \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) + bx \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) \right) \cosh\left(-\frac{bc-ad}{b}\right) -}{2 a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x+a),x, algorithm="fricas")

[Out]  $-\frac{1}{2} * (2*a*\cosh(d*x + c) - ((a*d - b)*x*\operatorname{Ei}(d*x) - (a*d + b)*x*\operatorname{Ei}(-d*x))*\cosh(c) - (b*x*\operatorname{Ei}((b*d*x + a*d)/b) + b*x*\operatorname{Ei}(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - ((a*d - b)*x*\operatorname{Ei}(d*x) + (a*d + b)*x*\operatorname{Ei}(-d*x))*\sinh(c) + (b*x*\operatorname{Ei}((b*d*x + a*d)/b) - b*x*\operatorname{Ei}(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b)) / (a^2*x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx)}{x^2(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x\*\*2/(b\*x+a),x)

[Out] Integral(cosh(c + d\*x)/(x\*\*2\*(a + b\*x)), x)

**Giac [A]** time = 1.17191, size = 174, normalized size = 1.54

$$\frac{adx \operatorname{Ei}(-dx) e^{(-c)} - adx \operatorname{Ei}(dx) e^c + bx \operatorname{Ei}(-dx) e^{(-c)} - bx \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(\frac{c-ad}{b}\right)} + bx \operatorname{Ei}(dx) e^c - bx \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} + ae}{2 a^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x+a),x, algorithm="giac")

```
[Out] -1/2*(a*d*x*Ei(-d*x)*e^(-c) - a*d*x*Ei(d*x)*e^c + b*x*Ei(-d*x)*e^(-c) - b*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + b*x*Ei(d*x)*e^c - b*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a*e^(d*x + c) + a*e^(-d*x - c))/(a^2*x)
```

### 3.25 $\int \frac{\cosh(c+dx)}{x^3(a+bx)} dx$

**Optimal.** Leaf size=190

$$\frac{b^2 \cosh(c) \operatorname{Chi}(dx)}{a^3} - \frac{b^2 \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} + \frac{b^2 \sinh(c) \operatorname{Shi}(dx)}{a^3} - \frac{b^2 \sinh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{bd \sinh(c)}{a^2}$$

[Out]  $-\operatorname{Cosh}[c + d*x]/(2*a*x^2) + (b*\operatorname{Cosh}[c + d*x])/(a^2*x) + (b^2*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x])/a^3 + (d^2*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x])/(2*a) - (b^2*\operatorname{Cosh}[c - (a*d)/b]*\operatorname{CoshIntegral}[(a*d)/b + d*x])/a^3 - (b*d*\operatorname{CoshIntegral}[d*x]*\operatorname{Sinh}[c])/a^2 - (d*\operatorname{Sinh}[c + d*x])/(2*a*x) - (b*d*\operatorname{Cosh}[c]*\operatorname{SinhIntegral}[d*x])/a^2 + (b^2*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x])/a^3 + (d^2*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x])/(2*a) - (b^2*\operatorname{Sinh}[c - (a*d)/b]*\operatorname{SinhIntegral}[(a*d)/b + d*x])/a^3$

**Rubi [A]** time = 0.485121, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{b^2 \cosh(c) \operatorname{Chi}(dx)}{a^3} - \frac{b^2 \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} + \frac{b^2 \sinh(c) \operatorname{Shi}(dx)}{a^3} - \frac{b^2 \sinh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{bd \sinh(c)}{a^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Cosh}[c + d*x]/(x^3*(a + b*x)), x]$

[Out]  $-\operatorname{Cosh}[c + d*x]/(2*a*x^2) + (b*\operatorname{Cosh}[c + d*x])/(a^2*x) + (b^2*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x])/a^3 + (d^2*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x])/(2*a) - (b^2*\operatorname{Cosh}[c - (a*d)/b]*\operatorname{CoshIntegral}[(a*d)/b + d*x])/a^3 - (b*d*\operatorname{CoshIntegral}[d*x]*\operatorname{Sinh}[c])/a^2 - (d*\operatorname{Sinh}[c + d*x])/(2*a*x) - (b*d*\operatorname{Cosh}[c]*\operatorname{SinhIntegral}[d*x])/a^2 + (b^2*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x])/a^3 + (d^2*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x])/(2*a) - (b^2*\operatorname{Sinh}[c - (a*d)/b]*\operatorname{SinhIntegral}[(a*d)/b + d*x])/a^3$

#### Rule 6742

$\operatorname{Int}[u_, x\_Symbol] \rightarrow \operatorname{With}[\{v = \operatorname{ExpandIntegrand}[u, x]\}, \operatorname{Int}[v, x] /; \operatorname{SumQ}[v]]$

#### Rule 3297

$\operatorname{Int}[(c + d*x)^m * \sin[e + f*x], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{m+1} * \operatorname{Sin}[e + f*x] / (d*(m+1)), x] - \operatorname{Dist}[f / (d*(m+1)), \operatorname{Int}[(c + d*x)^m * \sin[e + f*x], x]]$

+ d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(c + dx)}{x^3(a + bx)} dx &= \int \left( \frac{\cosh(c + dx)}{ax^3} - \frac{b \cosh(c + dx)}{a^2x^2} + \frac{b^2 \cosh(c + dx)}{a^3x} - \frac{b^3 \cosh(c + dx)}{a^3(a + bx)} \right) dx \\
 &= \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{x^2} dx}{a^2} + \frac{b^2 \int \frac{\cosh(c+dx)}{x} dx}{a^3} - \frac{b^3 \int \frac{\cosh(c+dx)}{a+bx} dx}{a^3} \\
 &= -\frac{\cosh(c + dx)}{2ax^2} + \frac{b \cosh(c + dx)}{a^2x} + \frac{d \int \frac{\sinh(c+dx)}{x^2} dx}{2a} - \frac{(bd) \int \frac{\sinh(c+dx)}{x} dx}{a^2} + \frac{(b^2 \cosh(c)) \int \frac{\cosh(dx)}{a+bx} dx}{a^3} \\
 &= -\frac{\cosh(c + dx)}{2ax^2} + \frac{b \cosh(c + dx)}{a^2x} + \frac{b^2 \cosh(c) \text{Chi}(dx)}{a^3} - \frac{b^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} - \frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} \\
 &= -\frac{\cosh(c + dx)}{2ax^2} + \frac{b \cosh(c + dx)}{a^2x} + \frac{b^2 \cosh(c) \text{Chi}(dx)}{a^3} - \frac{b^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} - \frac{bd \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} \\
 &= -\frac{\cosh(c + dx)}{2ax^2} + \frac{b \cosh(c + dx)}{a^2x} + \frac{b^2 \cosh(c) \text{Chi}(dx)}{a^3} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{2a} - \frac{b^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.487479, size = 178, normalized size = 0.94

$$x^2 \text{Chi}(dx) \left( \cosh(c) (a^2 d^2 + 2b^2) - 2abd \sinh(c) \right) + a^2 d^2 x^2 \sinh(c) \text{Shi}(dx) - a^2 dx \sinh(c + dx) + a^2 (-\cosh(c + dx)) - 2b$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x^3\*(a + b\*x)), x]

[Out] 
$$\frac{-(a^2 \text{Cosh}[c + d*x]) + 2*a*b*x*\text{Cosh}[c + d*x] - 2*b^2*x^2*\text{Cosh}[c - (a*d)/b] * \text{CoshIntegral}[d*(a/b + x)] + x^2*\text{CoshIntegral}[d*x]*((2*b^2 + a^2*d^2)*\text{Cosh}[c] - 2*a*b*d*\text{Sinh}[c]) - a^2*d*x*\text{Sinh}[c + d*x] - 2*a*b*d*x^2*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + 2*b^2*x^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + a^2*d^2*x^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] - 2*b^2*x^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[d*(a/b + x)]}{(2*a^3*x^2)}$$

**Maple [A]** time = 0.05, size = 281, normalized size = 1.5

$$\frac{de^{-dx-c}}{4ax} + \frac{e^{-dx-c}b}{2a^2x} - \frac{e^{-dx-c}}{4ax^2} - \frac{d^2e^{-c}\text{Ei}(1, dx)}{4a} - \frac{dbe^{-c}\text{Ei}(1, dx)}{2a^2} - \frac{b^2e^{-c}\text{Ei}(1, dx)}{2a^3} + \frac{b^2}{2a^3} e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/x^3/(b\*x+a), x)

[Out] 
$$\frac{1}{4}d*\exp(-d*x-c)/a/x + \frac{1}{2}*\exp(-d*x-c)/a^2/x*b - \frac{1}{4}*\exp(-d*x-c)/a/x^2 - \frac{1}{4}d^2/a*\exp(-c)*\text{Ei}(1, d*x) - \frac{1}{2}d/a^2*\exp(-c)*\text{Ei}(1, d*x)*b - \frac{1}{2}a^3*\exp(-c)*\text{Ei}(1, d*x)*b^2 + \frac{1}{2}b^2/a^3*\exp((a*d-b*c)/b)*\text{Ei}(1, d*x+c+(a*d-b*c)/b) - \frac{1}{2}b^2/a^3*\exp(c)*\text{Ei}(1, -d*x) - \frac{1}{4}a/x^2*\exp(d*x+c) - \frac{1}{4}d/a/x*\exp(d*x+c) - \frac{1}{4}d^2/a*\exp(c)*\text{Ei}(1, -d*x) + \frac{1}{2}a^2*b/x*\exp(d*x+c) + \frac{1}{2}d/a^2*b*\exp(c)*\text{Ei}(1, -d*x) + \frac{1}{2}a^3*b^2*\exp(-(a*d-b*c)/b)*\text{Ei}(1, -d*x-c-(a*d-b*c)/b)$$

**Maxima [A]** time = 1.62637, size = 327, normalized size = 1.72

$$\frac{1}{4}d \left( \frac{de^{(-c)}\Gamma(-1, dx) + de^c\Gamma(-1, -dx)}{a} + \frac{2(\text{Ei}(-dx)e^{(-c)} - \text{Ei}(dx)e^c)b}{a^2} + \frac{2b^3 \left( \frac{e^{(-c+\frac{ad}{b})}E_1\left(\frac{(bx+a)d}{b}\right) + e^{(c-\frac{ad}{b})}E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{a^3d} + \frac{4b^2}{4b^2} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="maxima")`

[Out]  $\frac{1}{4}d*((d*e^{-c}*\gamma(-1, d*x) + d*e^c*\gamma(-1, -d*x))/a + 2*(Ei(-d*x)*e^{-c} - Ei(d*x)*e^c)*b/a^2 + 2*b^3*(e^{-c + a*d/b}*\exp\_integral\_e(1, (b*x + a)*d/b)/b + e^{c - a*d/b}*\exp\_integral\_e(1, -(b*x + a)*d/b)/b)/(a^3*d) + 4*b^2*\cosh(d*x + c)*\log(b*x + a)/(a^3*d) - 4*b^2*\cosh(d*x + c)*\log(x)/(a^3*d) + 2*(Ei(-d*x)*e^{-c} + Ei(d*x)*e^c)*b^2/(a^3*d)) - 1/2*(2*b^2*\log(b*x + a)/a^3 - 2*b^2*\log(x)/a^3 - (2*b*x - a)/(a^2*x^2))*\cosh(d*x + c)$

**Fricas [A]** time = 2.08548, size = 591, normalized size = 3.11

$$2a^2dx \sinh(dx + c) - 2(2abx - a^2) \cosh(dx + c) - ((a^2d^2 - 2abd + 2b^2)x^2Ei(dx) + (a^2d^2 + 2abd + 2b^2)x^2Ei(-dx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="fricas")`

[Out]  $-1/4*(2*a^2*d*x*\sinh(d*x + c) - 2*(2*a*b*x - a^2)*\cosh(d*x + c) - ((a^2*d^2 - 2*a*b*d + 2*b^2)*x^2*Ei(d*x) + (a^2*d^2 + 2*a*b*d + 2*b^2)*x^2*Ei(-d*x))*\cosh(c) + 2*(b^2*x^2*Ei((b*d*x + a*d)/b) + b^2*x^2*Ei(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - ((a^2*d^2 - 2*a*b*d + 2*b^2)*x^2*Ei(d*x) - (a^2*d^2 + 2*a*b*d + 2*b^2)*x^2*Ei(-d*x))*\sinh(c) - 2*(b^2*x^2*Ei((b*d*x + a*d)/b) - b^2*x^2*Ei(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(a^3*x^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx)}{x^3(a + bx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x**3/(b*x+a),x)`

[Out] `Integral(cosh(c + d*x)/(x**3*(a + b*x)), x)`

---

**Giac [A]** time = 1.20454, size = 335, normalized size = 1.76

$$a^2 d^2 x^2 \operatorname{Ei}(-dx) e^{(-c)} + a^2 d^2 x^2 \operatorname{Ei}(dx) e^c + 2 abdx^2 \operatorname{Ei}(-dx) e^{(-c)} - 2 abdx^2 \operatorname{Ei}(dx) e^c + 2 b^2 x^2 \operatorname{Ei}(-dx) e^{(-c)} - 2 b^2 x^2 \operatorname{Ei}\left(\frac{bdx+a}{b}\right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^3/(b\*x+a),x, algorithm="giac")

[Out]  $\frac{1}{4} * (a^2 * d^2 * x^2 * \operatorname{Ei}(-d * x) * e^{(-c)} + a^2 * d^2 * x^2 * \operatorname{Ei}(d * x) * e^c + 2 * a * b * d * x^2 * \operatorname{Ei}(-d * x) * e^{(-c)} - 2 * a * b * d * x^2 * \operatorname{Ei}(d * x) * e^c + 2 * b^2 * x^2 * \operatorname{Ei}(-d * x) * e^{(-c)} - 2 * b^2 * x^2 * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d / b)} + 2 * b^2 * x^2 * \operatorname{Ei}(d * x) * e^c - 2 * b^2 * x^2 * \operatorname{Ei}(-(b * d * x + a * d) / b) * e^{(-c + a * d / b)} - a^2 * d * x * e^{(d * x + c)} + a^2 * d * x * e^{(-d * x - c)} + 2 * a * b * x * e^{(d * x + c)} + 2 * a * b * x * e^{(-d * x - c)} - a^2 * e^{(d * x + c)} - a^2 * e^{(-d * x - c)}) / (a^3 * x^2)$

$$3.26 \quad \int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} dx$$

**Optimal.** Leaf size=231

$$\frac{a^4 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^6} - \frac{4a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{4a^3 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^5} + \frac{a^4 d \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^6}$$

[Out] (2\*a\*Cosh[c + d\*x])/(b^3\*d^2) - (2\*x\*Cosh[c + d\*x])/(b^2\*d^2) - (a^4\*Cosh[c + d\*x])/(b^5\*(a + b\*x)) - (4\*a^3\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b^5 + (a^4\*d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/b^6 + (2\*Sinh[c + d\*x])/(b^2\*d^3) + (3\*a^2\*Sinh[c + d\*x])/(b^4\*d) - (2\*a\*x\*Sinh[c + d\*x])/(b^3\*d) + (x^2\*Sinh[c + d\*x])/(b^2\*d) + (a^4\*d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^6 - (4\*a^3\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^5

**Rubi [A]** time = 0.536144, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {6742, 2637, 3296, 2638, 3297, 3303, 3298, 3301}

$$\frac{a^4 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^6} - \frac{4a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{4a^3 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^5} + \frac{a^4 d \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^6}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*Cosh[c + d\*x])/(a + b\*x)^2,x]

[Out] (2\*a\*Cosh[c + d\*x])/(b^3\*d^2) - (2\*x\*Cosh[c + d\*x])/(b^2\*d^2) - (a^4\*Cosh[c + d\*x])/(b^5\*(a + b\*x)) - (4\*a^3\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b^5 + (a^4\*d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/b^6 + (2\*Sinh[c + d\*x])/(b^2\*d^3) + (3\*a^2\*Sinh[c + d\*x])/(b^4\*d) - (2\*a\*x\*Sinh[c + d\*x])/(b^3\*d) + (x^2\*Sinh[c + d\*x])/(b^2\*d) + (a^4\*d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^6 - (4\*a^3\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^5

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} dx &= \int \left( \frac{3a^2 \cosh(c+dx)}{b^4} - \frac{2ax \cosh(c+dx)}{b^3} + \frac{x^2 \cosh(c+dx)}{b^2} + \frac{a^4 \cosh(c+dx)}{b^4(a+bx)^2} - \frac{4a^3 \cosh(c+dx)}{b^4(a+bx)} \right) dx \\
&= \frac{(3a^2) \int \cosh(c+dx) dx}{b^4} - \frac{(4a^3) \int \frac{\cosh(c+dx)}{a+bx} dx}{b^4} + \frac{a^4 \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b^4} - \frac{(2a) \int x \cosh(c+dx) dx}{b^3} \\
&= -\frac{a^4 \cosh(c+dx)}{b^5(a+bx)} + \frac{3a^2 \sinh(c+dx)}{b^4 d} - \frac{2ax \sinh(c+dx)}{b^3 d} + \frac{x^2 \sinh(c+dx)}{b^2 d} + \frac{(2a) \int \sinh(c+dx) dx}{b^3 d} \\
&= \frac{2a \cosh(c+dx)}{b^3 d^2} - \frac{2x \cosh(c+dx)}{b^2 d^2} - \frac{a^4 \cosh(c+dx)}{b^5(a+bx)} - \frac{4a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5} + \frac{3a^4 \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5} \\
&= \frac{2a \cosh(c+dx)}{b^3 d^2} - \frac{2x \cosh(c+dx)}{b^2 d^2} - \frac{a^4 \cosh(c+dx)}{b^5(a+bx)} - \frac{4a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5} + \frac{3a^4 \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5}
\end{aligned}$$

**Mathematica [A]** time = 1.20078, size = 173, normalized size = 0.75

$$\frac{b^2(3a^2d^2 - 2abd^2x + b^2(d^2x^2 + 2)) \sinh(c+dx)}{d^3} - \frac{b(-2a^2b^2 + a^4d^2 + 2b^4x^2) \cosh(c+dx)}{d^2(a+bx)} + a^3 \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) \left(ad \sinh\left(c - \frac{ad}{b}\right) - 4b \cosh\left(c - \frac{ad}{b}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*Cosh[c + d\*x])/(a + b\*x)^2,x]

[Out] (-((b\*(-2\*a^2\*b^2 + a^4\*d^2 + 2\*b^4\*x^2)\*Cosh[c + d\*x])/(d^2\*(a + b\*x))) + a^3\*CoshIntegral[d\*(a/b + x)]\*(-4\*b\*Cosh[c - (a\*d)/b] + a\*d\*Sinh[c - (a\*d)/b]) + (b^2\*(3\*a^2\*d^2 - 2\*a\*b\*d^2\*x + b^2\*(2 + d^2\*x^2))\*Sinh[c + d\*x])/d^3 + a^3\*(a\*d\*Cosh[c - (a\*d)/b] - 4\*b\*Sinh[c - (a\*d)/b])\*SinhIntegral[d\*(a/b + x)]/b^6)

**Maple [A]** time = 0.122, size = 431, normalized size = 1.9

$$-\frac{e^{-dx-c}}{d^3 b^2} + \frac{da^4}{2b^6} e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) + 2 \frac{a^3}{b^5} e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) - \frac{e^{-dx-c} x}{b^2 d^2} + \frac{e^{-dx-c} a}{d^2 b^3} - \frac{e^{-dx-c} x^2}{2 db^2} - \frac{3e^{-dx-c}}{2 db^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*cosh(d\*x+c)/(b\*x+a)^2,x)

[Out]  $-1/d^3 \exp(-d*x-c)/b^2 + 1/2*d/b^6 \exp((a*d-b*c)/b) * Ei(1, d*x+c+(a*d-b*c)/b) * a^4 + 2/b^5 \exp((a*d-b*c)/b) * Ei(1, d*x+c+(a*d-b*c)/b) * a^3 - 1/d^2 \exp(-d*x-c)/b^2 * x + 1/d^2 \exp(-d*x-c)/b^3 * a - 1/2/d \exp(-d*x-c)/b^2 * x^2 - 3/2/d \exp(-d*x-c)/b^4 * a^2 - 1/2*d \exp(-d*x-c)/b^5 / (b*d*x+a*d) * a^4 + 1/d \exp(-d*x-c)/b^3 * a * x - 1/d/b^3 * a * \exp(d*x+c) * x + 3/2/d/b^4 * a^2 \exp(d*x+c) + 1/d^2/b^3 * a * \exp(d*x+c) + 1/d^3/b^2 \exp(d*x+c) - 1/2*d/b^6 \exp(d*x+c) / (1/b*d*a+d*x) * a^4 - 1/2*d/b^6 \exp(-(a*d-b*c)/b) * Ei(1, -d*x-c-(a*d-b*c)/b) * a^4 + 2/b^5 \exp(-(a*d-b*c)/b) * Ei(1, -d*x-c-(a*d-b*c)/b) * a^3 + 1/2/d/b^2 \exp(d*x+c) * x^2 - 1/d^2/b^2 \exp(d*x+c) * x$

**Maxima [A]** time = 1.51568, size = 548, normalized size = 2.37

$$\frac{1}{6} \left( 3 a^4 \left( \frac{e^{\left(-c+\frac{ad}{b}\right)} E_1\left(\frac{(bx+ad)d}{b}\right)} - \frac{e^{\left(c-\frac{ad}{b}\right)} E_1\left(-\frac{(bx+ad)d}{b}\right)}{b^6} \right) + \frac{12 a^3 \left( \frac{e^{\left(-c+\frac{ad}{b}\right)} E_1\left(\frac{(bx+ad)d}{b}\right)} + \frac{e^{\left(c-\frac{ad}{b}\right)} E_1\left(-\frac{(bx+ad)d}{b}\right)}{b} \right)}{b^4 d} - \frac{9 a^2 \left( \frac{(dx e^c - e^c) e^{(dx)}}{d^2} + \frac{(dx)}{b^4} \right)}{b^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*cosh(d\*x+c)/(b\*x+a)^2,x, algorithm="maxima")

[Out]  $1/6*(3*a^4*(e^{(-c+a*d/b)}*\exp\_integral\_e(1, (b*x+a)*d/b)/b^6 - e^{(c-a*d/b)}*\exp\_integral\_e(1, -(b*x+a)*d/b)/b^6) + 12*a^3*(e^{(-c+a*d/b)}*\exp\_integral\_e(1, (b*x+a)*d/b)/b + e^{(c-a*d/b)}*\exp\_integral\_e(1, -(b*x+a)*d/b)/b)/(b^4*d) - 9*a^2*((d*x*e^c - e^c)*e^{(d*x)}/d^2 + (d*x+1)*e^{(-d*x-c)}/d^2)/b^4 + 3*a*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^{(d*x)}/d^3 + (d^2*x^2 + 2*d*x + 2)*e^{(-d*x-c)}/d^3)/b^3 - ((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^{(d*x)}/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^{(-d*x-c)}/d^4)/b^2 + 24*a^3*cosh(d*x+c)*log(b*x+a)/(b^5*d)*d - 1/3*(3*a^4/(b^6*x + a*b^5) + 12*a^3*log(b*x+a)/b^5 - (b^2*x^3 - 3*a*b*x^2 + 9*a^2*x)/b^4)*cosh(d*x+c)$

**Fricas [A]** time = 2.07989, size = 740, normalized size = 3.2

$$\frac{2(a^4 b d^3 + 2 b^5 d x^2 - 2 a^2 b^3 d) \cosh(dx + c) - \left( (a^5 d^4 - 4 a^4 b d^3 + (a^4 b d^4 - 4 a^3 b^2 d^3) x \right) Ei\left(\frac{bdx+ad}{b}\right) - (a^5 d^4 + 4 a^4 b d^3 + ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*cosh(d\*x+c)/(b\*x+a)^2,x, algorithm="fricas")

[Out] 
$$-1/2*(2*(a^4*b*d^3 + 2*b^5*d*x^2 - 2*a^2*b^3*d)*\cosh(d*x + c) - ((a^5*d^4 - 4*a^4*b*d^3 + (a^4*b*d^4 - 4*a^3*b^2*d^3)*x)*\text{Ei}((b*d*x + a*d)/b) - (a^5*d^4 + 4*a^4*b*d^3 + (a^4*b*d^4 + 4*a^3*b^2*d^3)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - 2*(b^5*d^2*x^3 - a*b^4*d^2*x^2 + 3*a^3*b^2*d^2 + 2*a*b^4 + (a^2*b^3*d^2 + 2*b^5)*x)*\sinh(d*x + c) + ((a^5*d^4 - 4*a^4*b*d^3 + (a^4*b*d^4 - 4*a^3*b^2*d^3)*x)*\text{Ei}((b*d*x + a*d)/b) + (a^5*d^4 + 4*a^4*b*d^3 + (a^4*b*d^4 + 4*a^3*b^2*d^3)*x)*\text{Ei}(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^7*d^3*x + a*b^6*d^3)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*cosh(d\*x+c)/(b\*x+a)\*\*2,x)

[Out] Integral(x\*\*4\*cosh(c + d\*x)/(a + b\*x)\*\*2, x)

**Giac [A]** time = 1.18744, size = 393, normalized size = 1.7

$$\frac{a^4 b d x \text{Ei}\left(\frac{b d x + a d}{b}\right) e^{\left(\frac{c - a d}{b}\right)} - a^4 b d x \text{Ei}\left(-\frac{b d x + a d}{b}\right) e^{\left(-\frac{c + a d}{b}\right)} + a^5 d \text{Ei}\left(\frac{b d x + a d}{b}\right) e^{\left(\frac{c - a d}{b}\right)} - 4 a^3 b^2 x \text{Ei}\left(\frac{b d x + a d}{b}\right) e^{\left(\frac{c - a d}{b}\right)} - a^5 d \text{Ei}\left(-\frac{b d x + a d}{b}\right) e^{\left(-\frac{c + a d}{b}\right)}}{2(b^7 d^3 x + a b^6 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*cosh(d\*x+c)/(b\*x+a)^2,x, algorithm="giac")

[Out] 
$$1/2*(a^4*b*d*x*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - a^4*b*d*x*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + a^5*d*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 4*a^3*b^2*x*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - a^5*d*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 4*a^3*b^2*x*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 4*a^4*b*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 4*a^4*b*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^4*b*e^{(d*x + c)} - a^4*b*e^{(-d*x - c)})/(b^7*x + a*b^6)$$

$$3.27 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx)^2} dx$$

**Optimal.** Leaf size=182

$$-\frac{a^3 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} + \frac{3a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4} + \frac{3a^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^4} - \frac{a^3 d \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5}$$

[Out]  $-(\text{Cosh}[c + d*x]/(b^2*d^2)) + (a^3*\text{Cosh}[c + d*x])/(b^4*(a + b*x)) + (3*a^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/b^4 - (a^3*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/b^5 - (2*a*\text{Sinh}[c + d*x])/(b^3*d) + (x*\text{Sinh}[c + d*x])/(b^2*d) - (a^3*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^5 + (3*a^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^4$

**Rubi [A]** time = 0.422891, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {6742, 2637, 3296, 2638, 3297, 3303, 3298, 3301}

$$-\frac{a^3 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} + \frac{3a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4} + \frac{3a^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^4} - \frac{a^3 d \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*\text{Cosh}[c + d*x])/(a + b*x)^2, x]$

[Out]  $-(\text{Cosh}[c + d*x]/(b^2*d^2)) + (a^3*\text{Cosh}[c + d*x])/(b^4*(a + b*x)) + (3*a^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/b^4 - (a^3*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/b^5 - (2*a*\text{Sinh}[c + d*x])/(b^3*d) + (x*\text{Sinh}[c + d*x])/(b^2*d) - (a^3*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^5 + (3*a^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/b^4$

**Rule 6742**

$\text{Int}[u_, x\_Symbol] \text{ :> With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] \text{ /; SumQ}[v]]$

**Rule 2637**

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x\_Symbol] \text{ :> Simp}[\text{Sin}[c + d*x]/d, x] \text{ /; FreeQ}[\{c, d\}, x]$



Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2} dx &= \int \left( -\frac{2a \cosh(c+dx)}{b^3} + \frac{x \cosh(c+dx)}{b^2} - \frac{a^3 \cosh(c+dx)}{b^3(a+bx)^2} + \frac{3a^2 \cosh(c+dx)}{b^3(a+bx)} \right) dx \\
&= -\frac{(2a) \int \cosh(c+dx) dx}{b^3} + \frac{(3a^2) \int \frac{\cosh(c+dx)}{a+bx} dx}{b^3} - \frac{a^3 \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b^3} + \frac{\int x \cosh(c+dx) dx}{b^2} \\
&= \frac{a^3 \cosh(c+dx)}{b^4(a+bx)} - \frac{2a \sinh(c+dx)}{b^3 d} + \frac{x \sinh(c+dx)}{b^2 d} - \frac{\int \sinh(c+dx) dx}{b^2 d} - \frac{(a^3 d) \int \frac{\sinh(c+dx)}{a+bx} dx}{b^4} \\
&= -\frac{\cosh(c+dx)}{b^2 d^2} + \frac{a^3 \cosh(c+dx)}{b^4(a+bx)} + \frac{3a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} - \frac{2a \sinh(c+dx)}{b^3 d} + \frac{x \sinh(c+dx)}{b^2 d} \\
&= -\frac{\cosh(c+dx)}{b^2 d^2} + \frac{a^3 \cosh(c+dx)}{b^4(a+bx)} + \frac{3a^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} - \frac{a^3 d \text{Chi}\left(\frac{ad}{b} + dx\right) \sinh\left(\frac{ad}{b} + dx\right)}{b^5}
\end{aligned}$$

**Mathematica [A]** time = 0.914422, size = 156, normalized size = 0.86

$$\frac{b((a^3 d^2 - ab^2 - b^3 x) \cosh(c+dx) + bd(-2a^2 - abx + b^2 x^2) \sinh(c+dx))}{d^2(a+bx)} + a^2 \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) \left(3b \cosh\left(c - \frac{ad}{b}\right) - ad \sinh\left(c - \frac{ad}{b}\right)\right) - a^2 \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) \sinh\left(\frac{ad}{b} + dx\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*Cosh[c + d\*x])/(a + b\*x)^2,x]

[Out] (a^2\*CoshIntegral[d\*(a/b + x)]\*(3\*b\*Cosh[c - (a\*d)/b] - a\*d\*Sinh[c - (a\*d)/b]) + (b\*((-(a\*b^2) + a^3\*d^2 - b^3\*x)\*Cosh[c + d\*x] + b\*d\*(-2\*a^2 - a\*b\*x + b^2\*x^2)\*Sinh[c + d\*x]))/(d^2\*(a + b\*x)) - a^2\*(a\*d\*Cosh[c - (a\*d)/b] - 3\*b\*Sinh[c - (a\*d)/b])\*SinhIntegral[d\*(a/b + x)]/b^5

**Maple [A]** time = 0.046, size = 325, normalized size = 1.8

$$\frac{de^{-dx-c}a^3}{2b^4(bdx+da)} - \frac{da^3}{2b^5}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) - \frac{3a^2}{2b^4}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) - \frac{e^{-dx-c}x}{2db^2} + \frac{e^{-dx-c}a}{db^3} - \frac{e^{-dx-c}}{2d^2b^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*cosh(d\*x+c)/(b\*x+a)^2,x)

[Out]  $\frac{1}{2}d \exp(-dx-c)/b^4/(b dx+a)d \cdot a^3 - \frac{1}{2}d/b^5 \exp((a d-b c)/b) \operatorname{Ei}(1, dx+c+(a d-b c)/b) \cdot a^3 - \frac{3}{2}/b^4 \exp((a d-b c)/b) \operatorname{Ei}(1, dx+c+(a d-b c)/b) \cdot a^2 - \frac{1}{2}/d \exp(-dx-c)/b^2 x + 1/d \exp(-dx-c)/b^3 a - \frac{1}{2}/d^2 \exp(-dx-c)/b^2 + 1/2/d/b^2 \exp(dx+c) \cdot x + 1/2d/b^5 \exp(dx+c)/(1/b d+a+d x) \cdot a^3 + 1/2d/b^5 \exp(-(a d-b c)/b) \operatorname{Ei}(1, -dx-c-(a d-b c)/b) \cdot a^3 - 1/d/b^3 a \exp(dx+c) - 1/2/d^2/b^2 \exp(dx+c) - \frac{3}{2}/b^4 \exp(-(a d-b c)/b) \operatorname{Ei}(1, -dx-c-(a d-b c)/b) \cdot a^2$

**Maxima [A]** time = 1.3432, size = 421, normalized size = 2.31

$$-\frac{1}{4} \left[ 2a^3 \left( \frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+a)d}{b}\right)}{b^5} - \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+a)d}{b}\right)}{b^5} \right) + \frac{6a^2 \left( \frac{e^{(-c+\frac{ad}{b})} E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{(c-\frac{ad}{b})} E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{b^3 d} - \frac{4a \left( \frac{(dx e^c - e^c) e^{(dx)}}{d^2} + \frac{(a d - b c) e^{(dx+c)}}{d^2} \right)}{b^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(dx+c)/(b\*x+a)^2,x, algorithm="maxima")

[Out]  $-\frac{1}{4} \cdot (2a^3 \cdot (e^{(-c+a d/b)} \operatorname{exp\_integral\_e}(1, (b x+a) d/b) / b^5 - e^{(c-a d/b)} \operatorname{exp\_integral\_e}(1, -(b x+a) d/b) / b^5) + 6a^2 \cdot (e^{(-c+a d/b)} \operatorname{exp\_integral\_e}(1, (b x+a) d/b) / b + e^{(c-a d/b)} \operatorname{exp\_integral\_e}(1, -(b x+a) d/b) / b) / (b^3 d) - 4a \cdot ((d x \cdot e^c - e^c) \cdot e^{(d x)} / d^2 + (d x + 1) \cdot e^{(-d x - c)} / d^2) / b^3 + ((d^2 x^2 \cdot e^c - 2d x \cdot e^c + 2e^c) \cdot e^{(d x)} / d^3 + (d^2 x^2 + 2d x + 2) \cdot e^{(-d x - c)} / d^3) / b^2 + 12a^2 \cdot \cosh(d x + c) \cdot \log(b x + a) / (b^4 d) \cdot d + 1/2 \cdot (2a^3 / (b^5 x + a b^4) + 6a^2 \cdot \log(b x + a) / b^4 + (b x^2 - 4a x) / b^3) \cdot \cosh(d x + c)$

**Fricas [A]** time = 2.03256, size = 663, normalized size = 3.64

$$\frac{2(a^3 b d^2 - b^4 x - a b^3) \cosh(dx+c) - \left( (a^4 d^3 - 3a^3 b d^2 + (a^3 b d^3 - 3a^2 b^2 d^2) x \right) \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) - (a^4 d^3 + 3a^3 b d^2 + (a^3 b d^3 + (a^2 b^2 d^2) x) \cosh(dx+c)}{b^4 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(dx+c)/(b\*x+a)^2,x, algorithm="fricas")

```
[Out] 1/2*(2*(a^3*b*d^2 - b^4*x - a*b^3)*cosh(d*x + c) - ((a^4*d^3 - 3*a^3*b*d^2
+ (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) - (a^4*d^3 + 3*a^3*b*d
^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)
/b) + 2*(b^4*d*x^2 - a*b^3*d*x - 2*a^2*b^2*d)*sinh(d*x + c) + ((a^4*d^3 - 3
*a^3*b*d^2 + (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^3
+ 3*a^3*b*d^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-
(b*c - a*d)/b))/(b^6*d^2*x + a*b^5*d^2)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(d*x+c)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*cosh(c + d*x)/(a + b*x)**2, x)
```

**Giac [A]** time = 1.18621, size = 393, normalized size = 2.16

$$\frac{a^3 b d x \operatorname{Ei}\left(\frac{b d x+a d}{b}\right) e^{\left(\frac{c-a d}{b}\right)} - a^3 b d x \operatorname{Ei}\left(-\frac{b d x+a d}{b}\right) e^{\left(-\frac{c+a d}{b}\right)} + a^4 d \operatorname{Ei}\left(\frac{b d x+a d}{b}\right) e^{\left(\frac{c-a d}{b}\right)} - 3 a^2 b^2 x \operatorname{Ei}\left(\frac{b d x+a d}{b}\right) e^{\left(\frac{c-a d}{b}\right)} - a^4 d \operatorname{Ei}\left(-\frac{b d x+a d}{b}\right) e^{\left(-\frac{c+a d}{b}\right)}}{2\left(b^6 d^2 x^2 + a b^5 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^3*b*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a^3*b*d*x*Ei(-(b*d*x +
a*d)/b)*e^(-c + a*d/b) + a^4*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 3*a^2*b^
2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - a^4*d*Ei(-(b*d*x + a*d)/b)*e^(-c +
a*d/b) - 3*a^2*b^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 3*a^3*b*Ei((b*d*
x + a*d)/b)*e^(c - a*d/b) - 3*a^3*b*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a
^3*b*e^(d*x + c) - a^3*b*e^(-d*x - c))/(b^6*x + a*b^5)
```

$$3.28 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx)^2} dx$$

**Optimal.** Leaf size=147

$$\frac{a^2 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4} + \frac{a^2 d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^4} - \frac{a^2 \cosh(c+dx)}{b^3(a+bx)} - \frac{2a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3}$$

[Out] -((a^2\*Cosh[c + d\*x])/(b^3\*(a + b\*x))) - (2\*a\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b^3 + (a^2\*d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/b^4 + Sinh[c + d\*x]/(b^2\*d) + (a^2\*d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^4 - (2\*a\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^3

---

**Rubi [A]** time = 0.374579, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {6742, 2637, 3297, 3303, 3298, 3301}

$$\frac{a^2 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^4} + \frac{a^2 d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^4} - \frac{a^2 \cosh(c+dx)}{b^3(a+bx)} - \frac{2a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*Cosh[c + d\*x])/(a + b\*x)^2,x]

[Out] -((a^2\*Cosh[c + d\*x])/(b^3\*(a + b\*x))) - (2\*a\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b^3 + (a^2\*d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/b^4 + Sinh[c + d\*x]/(b^2\*d) + (a^2\*d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^4 - (2\*a\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^3

**Rule 6742**

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

**Rule 2637**

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

**Rule 3297**

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{x^2 \cosh(c + dx)}{(a + bx)^2} dx &= \int \left( \frac{\cosh(c + dx)}{b^2} + \frac{a^2 \cosh(c + dx)}{b^2(a + bx)^2} - \frac{2a \cosh(c + dx)}{b^2(a + bx)} \right) dx \\
 &= \frac{\int \cosh(c + dx) dx}{b^2} - \frac{(2a) \int \frac{\cosh(c+dx)}{a+bx} dx}{b^2} + \frac{a^2 \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b^2} \\
 &= -\frac{a^2 \cosh(c + dx)}{b^3(a + bx)} + \frac{\sinh(c + dx)}{b^2 d} + \frac{(a^2 d) \int \frac{\sinh(c+dx)}{a+bx} dx}{b^3} - \frac{\left(2a \cosh\left(c - \frac{ad}{b}\right)\right) \int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx} dx}{b^2} \\
 &= -\frac{a^2 \cosh(c + dx)}{b^3(a + bx)} - \frac{2a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} + \frac{\sinh(c + dx)}{b^2 d} - \frac{2a \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^3} \\
 &= -\frac{a^2 \cosh(c + dx)}{b^3(a + bx)} - \frac{2a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} + \frac{a^2 d \text{Chi}\left(\frac{ad}{b} + dx\right) \sinh\left(c - \frac{ad}{b}\right)}{b^4} + \frac{\sinh(c + dx)}{b^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.692046, size = 115, normalized size = 0.78

$$\frac{b \left( \frac{b \sinh(c+dx)}{d} - \frac{a^2 \cosh(c+dx)}{a+bx} \right) + a \operatorname{Chi} \left( d \left( \frac{a}{b} + x \right) \right) \left( ad \sinh \left( c - \frac{ad}{b} \right) - 2b \cosh \left( c - \frac{ad}{b} \right) \right) + a \operatorname{Shi} \left( d \left( \frac{a}{b} + x \right) \right) \left( ad \cosh \left( c - \frac{ad}{b} \right) - 2b \sinh \left( c - \frac{ad}{b} \right) \right)}{b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Cosh[c + d\*x])/(a + b\*x)^2,x]

[Out] (a\*CoshIntegral[d\*(a/b + x)]\*(-2\*b\*Cosh[c - (a\*d)/b] + a\*d\*Sinh[c - (a\*d)/b]) + b\*(-((a^2\*Cosh[c + d\*x])/(a + b\*x)) + (b\*Sinh[c + d\*x])/d) + a\*(a\*d\*Cosh[c - (a\*d)/b] - 2\*b\*Sinh[c - (a\*d)/b])\*SinhIntegral[d\*(a/b + x)]/b^4

**Maple [A]** time = 0.039, size = 254, normalized size = 1.7

$$-\frac{e^{-dx-c}}{2db^2} - \frac{de^{-dx-c}a^2}{2b^3(bdx+da)} + \frac{da^2}{2b^4} e^{\frac{da-cb}{b}} \operatorname{Ei} \left( 1, dx + c + \frac{da-cb}{b} \right) + \frac{a}{b^3} e^{\frac{da-cb}{b}} \operatorname{Ei} \left( 1, dx + c + \frac{da-cb}{b} \right) + \frac{e^{dx+c}}{2db^2} - \frac{de^{dx+c}a^2}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cosh(d\*x+c)/(b\*x+a)^2,x)

[Out] -1/2/d\*exp(-d\*x-c)/b^2-1/2\*d\*exp(-d\*x-c)/b^3/(b\*d\*x+a\*d)\*a^2+1/2\*d/b^4\*exp((a\*d-b\*c)/b)\*Ei(1,d\*x+c+(a\*d-b\*c)/b)\*a^2+1/b^3\*exp((a\*d-b\*c)/b)\*Ei(1,d\*x+c+(a\*d-b\*c)/b)\*a+1/2/d/b^2\*exp(d\*x+c)-1/2\*d/b^4\*exp(d\*x+c)/(1/b\*d\*a+d\*x)\*a^2-1/2\*d/b^4\*exp(-(a\*d-b\*c)/b)\*Ei(1,-d\*x-c-(a\*d-b\*c)/b)\*a^2+1/b^3\*exp(-(a\*d-b\*c)/b)\*Ei(1,-d\*x-c-(a\*d-b\*c)/b)\*a

**Maxima [A]** time = 1.34762, size = 319, normalized size = 2.17

$$\frac{1}{2} \left( a^2 \left( \frac{e^{\left(-c+\frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)} - \frac{e^{\left(c-\frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)}{b^4} \right) + \frac{2a \left( \frac{e^{\left(-c+\frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)} + \frac{e^{\left(c-\frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{b^2 d} - \frac{(dx e^c - e^c) e^{(dx)}}{d^2} + \frac{(dx+1) e^{(-dx)}}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x+a)^2,x, algorithm="maxima")

[Out] 1/2\*(a^2\*(e^(-c + a\*d/b)\*exp\_integral\_e(1, (b\*x + a)\*d/b)/b^4 - e^(c - a\*d/b)\*exp\_integral\_e(1, -(b\*x + a)\*d/b)/b^4) + 2\*a\*(e^(-c + a\*d/b)\*exp\_integral\_e(1, (b\*x + a)\*d/b)/b + e^(c - a\*d/b)\*exp\_integral\_e(1, -(b\*x + a)\*d/b)/b)/(b^2\*d) - ((d\*x\*e^c - e^c)\*e^(d\*x)/d^2 + (d\*x + 1)\*e^(-d\*x - c)/d^2)/b^2 + 4\*a\*cosh(d\*x + c)\*log(b\*x + a)/(b^3\*d))\*d - (a^2/(b^4\*x + a\*b^3) - x/b^2 + 2\*a\*log(b\*x + a)/b^3)\*cosh(d\*x + c)

**Fricas [A]** time = 1.99239, size = 570, normalized size = 3.88

$$2 a^2 b d \cosh (d x+c)-\left(\left(a^3 d^2-2 a^2 b d+\left(a^2 b d^2-2 a b^2 d\right) x\right) \operatorname{Ei}\left(\frac{b d x+a d}{b}\right)-\left(a^3 d^2+2 a^2 b d+\left(a^2 b d^2+2 a b^2 d\right) x\right) \operatorname{Ei}\left(-\frac{b d x+a d}{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x+a)^2,x, algorithm="fricas")

[Out] -1/2\*(2\*a^2\*b\*d\*cosh(d\*x + c) - ((a^3\*d^2 - 2\*a^2\*b\*d + (a^2\*b\*d^2 - 2\*a\*b^2\*d)\*x)\*Ei((b\*d\*x + a\*d)/b) - (a^3\*d^2 + 2\*a^2\*b\*d + (a^2\*b\*d^2 + 2\*a\*b^2\*d)\*x)\*Ei(-(b\*d\*x + a\*d)/b))\*cosh(-(b\*c - a\*d)/b) - 2\*(b^3\*x + a\*b^2)\*sinh(d\*x + c) + ((a^3\*d^2 - 2\*a^2\*b\*d + (a^2\*b\*d^2 - 2\*a\*b^2\*d)\*x)\*Ei((b\*d\*x + a\*d)/b) + (a^3\*d^2 + 2\*a^2\*b\*d + (a^2\*b\*d^2 + 2\*a\*b^2\*d)\*x)\*Ei(-(b\*d\*x + a\*d)/b))\*sinh(-(b\*c - a\*d)/b))/(b^5\*d\*x + a\*b^4\*d)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh (c+d x)}{(a+b x)^2} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cosh(d\*x+c)/(b\*x+a)\*\*2,x)

[Out] Integral(x\*\*2\*cosh(c + d\*x)/(a + b\*x)\*\*2, x)



**Giac [A]** time = 1.22872, size = 387, normalized size = 2.63

$$\frac{a^2 b d x \operatorname{Ei}\left(\frac{b d x+a d}{b}\right) e^{\left(\frac{c-a d}{b}\right)} - a^2 b d x \operatorname{Ei}\left(-\frac{b d x+a d}{b}\right) e^{\left(-c+\frac{a d}{b}\right)} + a^3 d \operatorname{Ei}\left(\frac{b d x+a d}{b}\right) e^{\left(\frac{c-a d}{b}\right)} - 2 a b^2 x \operatorname{Ei}\left(\frac{b d x+a d}{b}\right) e^{\left(\frac{c-a d}{b}\right)} - a^3 d \operatorname{Ei}\left(-\frac{b d x+a d}{b}\right) e^{\left(-c+\frac{a d}{b}\right)}}{2\left(b^5 x^2 + a^2 b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x+a)^2,x, algorithm="giac")

[Out] 1/2\*(a^2\*b\*d\*x\*Ei((b\*d\*x + a\*d)/b)\*e^(c - a\*d/b) - a^2\*b\*d\*x\*Ei(-(b\*d\*x + a\*d)/b)\*e^(-c + a\*d/b) + a^3\*d\*Ei((b\*d\*x + a\*d)/b)\*e^(c - a\*d/b) - 2\*a\*b^2\*x\*Ei((b\*d\*x + a\*d)/b)\*e^(c - a\*d/b) - a^3\*d\*Ei(-(b\*d\*x + a\*d)/b)\*e^(-c + a\*d/b) - 2\*a\*b^2\*x\*Ei(-(b\*d\*x + a\*d)/b)\*e^(-c + a\*d/b) - 2\*a^2\*b\*Ei((b\*d\*x + a\*d)/b)\*e^(c - a\*d/b) - 2\*a^2\*b\*Ei(-(b\*d\*x + a\*d)/b)\*e^(-c + a\*d/b) - a^2\*b\*e^(d\*x + c) - a^2\*b\*e^(-d\*x - c))/(b^5\*x + a\*b^4)

$$3.29 \quad \int \frac{x \cosh(c+dx)}{(a+bx)^2} dx$$

**Optimal.** Leaf size=125

$$-\frac{ad \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2} - \frac{ad \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^3}$$

[Out] (a\*Cosh[c + d\*x])/(b^2\*(a + b\*x)) + (Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b^2 - (a\*d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/b^3 - (a\*d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^3 + (Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^2

**Rubi [A]** time = 0.297248, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$-\frac{ad \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2} - \frac{ad \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(x\*Cosh[c + d\*x])/(a + b\*x)^2,x]

[Out] (a\*Cosh[c + d\*x])/(b^2\*(a + b\*x)) + (Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/b^2 - (a\*d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/b^3 - (a\*d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^3 + (Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^2

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x \cosh(c + dx)}{(a + bx)^2} dx &= \int \left( -\frac{a \cosh(c + dx)}{b(a + bx)^2} + \frac{\cosh(c + dx)}{b(a + bx)} \right) dx \\
&= \frac{\int \frac{\cosh(c + dx)}{a + bx} dx}{b} - \frac{a \int \frac{\cosh(c + dx)}{(a + bx)^2} dx}{b} \\
&= \frac{a \cosh(c + dx)}{b^2(a + bx)} - \frac{(ad) \int \frac{\sinh(c + dx)}{a + bx} dx}{b^2} + \frac{\cosh\left(c - \frac{ad}{b}\right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a + bx} dx}{b} + \frac{\sinh\left(c - \frac{ad}{b}\right) \int \frac{\sinh\left(\frac{ad}{b} + dx\right)}{a + bx} dx}{b} \\
&= \frac{a \cosh(c + dx)}{b^2(a + bx)} + \frac{\cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(\frac{ad}{b} + dx\right)}{b^2} + \frac{\sinh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(\frac{ad}{b} + dx\right)}{b^2} - \frac{\left(ad \cosh\left(c - \frac{ad}{b}\right)\right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a + bx} dx}{b^2} \\
&= \frac{a \cosh(c + dx)}{b^2(a + bx)} + \frac{\cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(\frac{ad}{b} + dx\right)}{b^2} - \frac{ad \operatorname{Chi}\left(\frac{ad}{b} + dx\right) \sinh\left(c - \frac{ad}{b}\right)}{b^3} - \frac{ad \cosh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(\frac{ad}{b} + dx\right)}{b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.41379, size = 97, normalized size = 0.78

$$\frac{\operatorname{Chi}\left(d\left(\frac{a}{b} + x\right)\right) \left(b \cosh\left(c - \frac{ad}{b}\right) - ad \sinh\left(c - \frac{ad}{b}\right)\right) + \operatorname{Shi}\left(d\left(\frac{a}{b} + x\right)\right) \left(b \sinh\left(c - \frac{ad}{b}\right) - ad \cosh\left(c - \frac{ad}{b}\right)\right) + \frac{ab \cosh(c + dx)}{a + bx}}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*Cosh[c + d\*x])/(a + b\*x)^2,x]

[Out] ((a\*b\*Cosh[c + d\*x])/(a + b\*x) + CoshIntegral[d\*(a/b + x)]\*(b\*Cosh[c - (a\*d)/b] - a\*d\*Sinh[c - (a\*d)/b]) + (-(a\*d\*Cosh[c - (a\*d)/b]) + b\*Sinh[c - (a\*d)/b])\*SinhIntegral[d\*(a/b + x)]/b^3

**Maple [A]** time = 0.027, size = 215, normalized size = 1.7

$$\frac{de^{-dx-c}a}{2b^2(bdx+da)} - \frac{da}{2b^3}e^{\frac{da-cb}{b}}\text{Ei}\left(1, dx+c+\frac{da-cb}{b}\right) - \frac{1}{2b^2}e^{\frac{da-cb}{b}}\text{Ei}\left(1, dx+c+\frac{da-cb}{b}\right) + \frac{de^{dx+c}a}{2b^3}\left(\frac{da}{b}+dx\right)^{-1} + \frac{da}{2b^3}e^{\frac{da-cb}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cosh(d\*x+c)/(b\*x+a)^2,x)

[Out] 1/2\*d\*exp(-d\*x-c)/b^2/(b\*d\*x+a\*d)\*a-1/2\*d/b^3\*exp((a\*d-b\*c)/b)\*Ei(1,d\*x+c+(a\*d-b\*c)/b)\*a-1/2/b^2\*exp((a\*d-b\*c)/b)\*Ei(1,d\*x+c+(a\*d-b\*c)/b)+1/2\*d/b^3\*exp(d\*x+c)/(1/b\*d\*a+d\*x)\*a+1/2\*d/b^3\*exp(-(a\*d-b\*c)/b)\*Ei(1,-d\*x-c-(a\*d-b\*c)/b)\*a-1/2/b^2\*exp(-(a\*d-b\*c)/b)\*Ei(1,-d\*x-c-(a\*d-b\*c)/b)

**Maxima [A]** time = 1.36264, size = 240, normalized size = 1.92

$$-\frac{1}{2}\left(a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^3}\right) + \frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{bd} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{bd} + \frac{2\cosh(dx+c)\log(bx+a)}{b^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x+a)^2,x, algorithm="maxima")

[Out] -1/2\*(a\*(e^(-c + a\*d/b)\*exp\_integral\_e(1, (b\*x + a)\*d/b)/b^3 - e^(c - a\*d/b)\*exp\_integral\_e(1, -(b\*x + a)\*d/b)/b^3) + (e^(-c + a\*d/b)\*exp\_integral\_e(1, (b\*x + a)\*d/b)/b + e^(c - a\*d/b)\*exp\_integral\_e(1, -(b\*x + a)\*d/b)/b)/(b\*d) + 2\*cosh(d\*x + c)\*log(b\*x + a)/(b^2\*d)\*d + (a/(b^3\*x + a\*b^2) + log(b\*x + a)/b^2)\*cosh(d\*x + c)

**Fricas [A]** time = 2.03375, size = 414, normalized size = 3.31

$$\frac{2ab \cosh(dx + c) - \left( (a^2d - ab + (abd - b^2)x) \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) - (a^2d + ab + (abd + b^2)x) \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) \right) \cosh\left(-\frac{bc-ad}{b}\right) + \left( (a^2d - ab + (abd - b^2)x) \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) + (a^2d + ab + (abd + b^2)x) \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) \right) \sinh\left(-\frac{bc-ad}{b}\right)}{2(b^4x + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x+a)^2,x, algorithm="fricas")

[Out]  $\frac{1}{2} * (2 * a * b * \cosh(d * x + c) - ((a^2 * d - a * b + (a * b * d - b^2) * x) * \operatorname{Ei}((b * d * x + a * d) / b) - (a^2 * d + a * b + (a * b * d + b^2) * x) * \operatorname{Ei}(-(b * d * x + a * d) / b)) * \cosh(-(b * c - a * d) / b) + ((a^2 * d - a * b + (a * b * d - b^2) * x) * \operatorname{Ei}((b * d * x + a * d) / b) + (a^2 * d + a * b + (a * b * d + b^2) * x) * \operatorname{Ei}(-(b * d * x + a * d) / b)) * \sinh(-(b * c - a * d) / b)) / (b^4 * x + a * b^3)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(c + dx)}{(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x+a)\*\*2,x)

[Out] Integral(x\*cosh(c + d\*x)/(a + b\*x)\*\*2, x)

**Giac [B]** time = 1.24044, size = 373, normalized size = 2.98

$$\frac{abdxEi\left(\frac{bdx+ad}{b}\right)e^{\left(\frac{c-ad}{b}\right)} - abdxEi\left(-\frac{bdx+ad}{b}\right)e^{\left(-\frac{c+ad}{b}\right)} + a^2dEi\left(\frac{bdx+ad}{b}\right)e^{\left(\frac{c-ad}{b}\right)} - b^2xEi\left(\frac{bdx+ad}{b}\right)e^{\left(\frac{c-ad}{b}\right)} - a^2dEi\left(-\frac{bdx+ad}{b}\right)e^{\left(-\frac{c+ad}{b}\right)}}{2(b^3x + ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x+a)^2,x, algorithm="giac")

[Out]  $-1/2 * (a * b * d * x * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} - a * b * d * x * \operatorname{Ei}(-(b * d * x + a * d) / b) * e^{(-c + a * d) / b} + a^2 * d * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} - b^2 * x * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} - a^2 * d * \operatorname{Ei}(-\frac{bdx+ad}{b}) * e^{(-\frac{c+ad}{b})} + b^2 * x * \operatorname{Ei}(-\frac{bdx+ad}{b}) * e^{(-\frac{c+ad}{b})}) / (b^3 * x + a * b^2)$

$$\frac{d*x + a*d}{b} * e^{(c - a*d/b)} - a^2*d*Ei(-(b*d*x + a*d)/b) * e^{(-c + a*d/b)} - b^2*x*Ei(-(b*d*x + a*d)/b) * e^{(-c + a*d/b)} - a*b*Ei((b*d*x + a*d)/b) * e^{(c - a*d/b)} - a*b*Ei(-(b*d*x + a*d)/b) * e^{(-c + a*d/b)} - a*b*e^{(d*x + c)} - a*b*e^{(-d*x - c)} / ((b^3*x + a*b^2)*b)$$

### 3.30 $\int \frac{\cosh(c+dx)}{(a+bx)^2} dx$

**Optimal.** Leaf size=71

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2} - \frac{\cosh(c + dx)}{b(a + bx)}$$

[Out] -(Cosh[c + d\*x]/(b\*(a + b\*x))) + (d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/b^2 + (d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^2

**Rubi [A]** time = 0.109993, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3297, 3303, 3298, 3301}

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^2} + \frac{d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^2} - \frac{\cosh(c + dx)}{b(a + bx)}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(a + b\*x)^2,x]

[Out] -(Cosh[c + d\*x]/(b\*(a + b\*x))) + (d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/b^2 + (d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^2

#### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

#### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

#### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\cosh(c+dx)}{(a+bx)^2} dx &= -\frac{\cosh(c+dx)}{b(a+bx)} + \frac{d \int \frac{\sinh(c+dx)}{a+bx} dx}{b} \\ &= -\frac{\cosh(c+dx)}{b(a+bx)} + \frac{\left(d \cosh\left(c - \frac{ad}{b}\right)\right) \int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx} dx}{b} + \frac{\left(d \sinh\left(c - \frac{ad}{b}\right)\right) \int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx} dx}{b} \\ &= -\frac{\cosh(c+dx)}{b(a+bx)} + \frac{d \operatorname{Chi}\left(\frac{ad}{b}+dx\right) \sinh\left(c - \frac{ad}{b}\right)}{b^2} + \frac{d \cosh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^2} \end{aligned}$$

**Mathematica [A]** time = 0.201944, size = 65, normalized size = 0.92

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + d \cosh\left(c - \frac{ad}{b}\right) \operatorname{Shi}\left(d\left(\frac{a}{b} + x\right)\right) - \frac{b \cosh(c+dx)}{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x)^2, x]
```

```
[Out] (-((b*Cosh[c + d*x])/(a + b*x)) + d*CoshIntegral[d*(a/b + x)]*Sinh[c - (a*d)/b] + d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/b^2
```

**Maple [A]** time = 0.025, size = 132, normalized size = 1.9

$$-\frac{de^{-dx-c}}{2b(bdx+da)} + \frac{d}{2b^2} e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx+c+\frac{da-cb}{b}\right) - \frac{de^{dx+c}}{2b^2} \left(\frac{da}{b}+dx\right)^{-1} - \frac{d}{2b^2} e^{-\frac{da-cb}{b}} \operatorname{Ei}\left(1, -dx-c-\frac{da-cb}{b}\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/(b*x+a)^2,x)`

[Out]  $-1/2*d*\exp(-d*x-c)/b/(b*d*x+a*d)+1/2*d/b^2*\exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2*d/b^2*\exp(d*x+c)/(1/b*d*a+d*x)-1/2*d/b^2*\exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)$

**Maxima [A]** time = 1.20947, size = 109, normalized size = 1.54

$$d \left( \frac{e^{\left(-c+\frac{ad}{b}\right)} E_1\left(\frac{(bx+a)d}{b}\right)}{b} - \frac{e^{\left(c-\frac{ad}{b}\right)} E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right) - \frac{\cosh(dx+c)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")`

[Out]  $1/2*d*(e^{-c+a*d/b}*\exp\_integral\_e(1,(b*x+a)*d/b)/b - e^{c-a*d/b}*\exp\_integral\_e(1,-(b*x+a)*d/b)/b) - \cosh(d*x+c)/((b*x+a)*b)$

**Fricas [B]** time = 2.03666, size = 316, normalized size = 4.45

$$\frac{2b \cosh(dx+c) - \left( (bdx+ad)Ei\left(\frac{bdx+ad}{b}\right) - (bdx+ad)Ei\left(-\frac{bdx+ad}{b}\right) \right) \cosh\left(-\frac{bc-ad}{b}\right) + \left( (bdx+ad)Ei\left(\frac{bdx+ad}{b}\right) + (bdx+ad)Ei\left(-\frac{bdx+ad}{b}\right) \right) \sinh\left(-\frac{bc-ad}{b}\right)}{2(b^3x+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")`

[Out]  $-1/2*(2*b*\cosh(d*x+c) - ((b*d*x+a*d)*Ei((b*d*x+a*d)/b) - (b*d*x+a*d)*Ei(-(b*d*x+a*d)/b))*\cosh(-(b*c-a*d)/b) + ((b*d*x+a*d)*Ei((b*d*x+a*d)/b) + (b*d*x+a*d)*Ei(-(b*d*x+a*d)/b))*\sinh(-(b*c-a*d)/b)/(b^3*x+a*b^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x+a)\*\*2,x)

[Out] Timed out

**Giac [B]** time = 1.21064, size = 201, normalized size = 2.83

$$\frac{bdx\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(\frac{c-ad}{b}\right)} - bdx\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-\frac{c+ad}{b}\right)} + ad\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(\frac{c-ad}{b}\right)} - ad\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-\frac{c+ad}{b}\right)} - be^{(dx+c)} - be^{(-dx-c)}}{2(b^3x + ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x+a)^2,x, algorithm="giac")

[Out] 1/2\*(b\*d\*x\*Ei((b\*d\*x + a\*d)/b)\*e^(c - a\*d/b) - b\*d\*x\*Ei(-(b\*d\*x + a\*d)/b)\*e^(-c + a\*d/b) + a\*d\*Ei((b\*d\*x + a\*d)/b)\*e^(c - a\*d/b) - a\*d\*Ei(-(b\*d\*x + a\*d)/b)\*e^(-c + a\*d/b) - b\*e^(d\*x + c) - b\*e^(-d\*x - c))/(b^3\*x + a\*b^2)

$$3.31 \quad \int \frac{\cosh(c+dx)}{x(a+bx)^2} dx$$

**Optimal.** Leaf size=150

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^2} - \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^2} + \frac{\cosh(c) \text{Chi}(dx)}{a^2} + \frac{\sinh(c) \text{Shi}(dx)}{a^2} - \frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{ab}$$

[Out] Cosh[c + d\*x]/(a\*(a + b\*x)) + (Cosh[c]\*CoshIntegral[d\*x])/a^2 - (Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/a^2 - (d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/(a\*b) + (Sinh[c]\*SinhIntegral[d\*x])/a^2 - (d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/(a\*b) - (Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/a^2

**Rubi [A]** time = 0.400274, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6742, 3303, 3298, 3301, 3297}

$$\frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^2} - \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^2} + \frac{\cosh(c) \text{Chi}(dx)}{a^2} + \frac{\sinh(c) \text{Shi}(dx)}{a^2} - \frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x\*(a + b\*x)^2), x]

[Out] Cosh[c + d\*x]/(a\*(a + b\*x)) + (Cosh[c]\*CoshIntegral[d\*x])/a^2 - (Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/a^2 - (d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/(a\*b) + (Sinh[c]\*SinhIntegral[d\*x])/a^2 - (d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/(a\*b) - (Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/a^2

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x] /; FreeQ[{c, d, e, f}, x] &&

NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x]/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(c + dx)}{x(a + bx)^2} dx &= \int \left( \frac{\cosh(c + dx)}{a^2 x} - \frac{b \cosh(c + dx)}{a(a + bx)^2} - \frac{b \cosh(c + dx)}{a^2(a + bx)} \right) dx \\
 &= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{a+bx} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{a} \\
 &= \frac{\cosh(c + dx)}{a(a + bx)} - \frac{d \int \frac{\sinh(c+dx)}{a+bx} dx}{a} + \frac{\cosh(c) \int \frac{\cosh(dx)}{x} dx}{a^2} - \frac{\left( b \cosh\left(c - \frac{ad}{b}\right) \right) \int \frac{\cosh\left(\frac{ad}{b} + dx\right)}{a+bx} dx}{a^2} + \frac{\sinh\left(c - \frac{ad}{b}\right)}{a} \\
 &= \frac{\cosh(c + dx)}{a(a + bx)} + \frac{\cosh(c) \text{Chi}(dx)}{a^2} - \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^2} + \frac{\sinh(c) \text{Shi}(dx)}{a^2} - \frac{\sinh\left(c - \frac{ad}{b}\right)}{a} \\
 &= \frac{\cosh(c + dx)}{a(a + bx)} + \frac{\cosh(c) \text{Chi}(dx)}{a^2} - \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^2} - \frac{d \text{Chi}\left(\frac{ad}{b} + dx\right) \sinh\left(c - \frac{ad}{b}\right)}{ab} + \frac{\sinh\left(c - \frac{ad}{b}\right)}{a}
 \end{aligned}$$

**Mathematica [A]** time = 1.05038, size = 241, normalized size = 1.61

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$$a^2 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{d(a+bx)}{b}\right) + a^2 d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{d(a+bx)}{b}\right) + b^2 x \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + ab dx \sinh\left(c - \frac{ad}{b}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x\*(a + b\*x)^2), x]

[Out] -(((a\*b\*Cosh[c + d\*x]) - b\*(a + b\*x)\*Cosh[c]\*CoshIntegral[d\*x] + b\*(a + b\*x)\*Cosh[c - (a\*d)/b]\*CoshIntegral[d\*(a/b + x)] + a^2\*d\*CoshIntegral[(d\*(a + b\*x))/b]\*Sinh[c - (a\*d)/b] + a\*b\*d\*x\*CoshIntegral[(d\*(a + b\*x))/b]\*Sinh[c - (a\*d)/b] - a\*b\*Sinh[c]\*SinhIntegral[d\*x] - b^2\*x\*Sinh[c]\*SinhIntegral[d\*x] + a\*b\*Sinh[c - (a\*d)/b]\*SinhIntegral[d\*(a/b + x)] + b^2\*x\*Sinh[c - (a\*d)/b]\*SinhIntegral[d\*(a/b + x)] + a^2\*d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(d\*(a + b\*x))/b] + a\*b\*d\*x\*Cosh[c - (a\*d)/b]\*SinhIntegral[(d\*(a + b\*x))/b])/(a^2\*b\*(a + b\*x))

**Maple [A]** time = 0.05, size = 254, normalized size = 1.7

$$\frac{e^{-dx-c}d}{2a((dx+c)b+da-cb)} - \frac{e^{-c}\text{Ei}(1,dx)}{2a^2} - \frac{d}{2ab}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) + \frac{1}{2a^2}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) - \frac{e^c}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/x/(b\*x+a)^2,x)

[Out] 1/2\*exp(-d\*x-c)\*d/a/((d\*x+c)\*b+d\*a-c\*b)-1/2/a^2\*exp(-c)\*Ei(1,d\*x)-1/2/a/b\*exp((a\*d-b\*c)/b)\*Ei(1,d\*x+c+(a\*d-b\*c)/b)\*d+1/2/a^2\*exp((a\*d-b\*c)/b)\*Ei(1,d\*x+c+(a\*d-b\*c)/b)-1/2/a^2\*exp(c)\*Ei(1,-d\*x)+1/2/b\*d/a\*exp(d\*x+c)/(1/b\*d\*a+d\*x)+1/2/b\*d/a\*exp(-(a\*d-b\*c)/b)\*Ei(1,-d\*x-c-(a\*d-b\*c)/b)+1/2/a^2\*exp(-(a\*d-b\*c)/b)\*Ei(1,-d\*x-c-(a\*d-b\*c)/b)

**Maxima [A]** time = 1.43188, size = 306, normalized size = 2.04

$$-\frac{1}{2}d \left[ \frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{ab} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{ab} - \frac{b \left( \frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b} \right)}{a^2d} - \frac{2 \cosh(dx+c) \log(bx+a)}{a^2d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x+a)^2,x, algorithm="maxima")

[Out] 
$$-1/2*d*(e^{(-c + a*d/b)}*exp\_integral\_e(1, (b*x + a)*d/b)/(a*b) - e^{(c - a*d/b)}*exp\_integral\_e(1, -(b*x + a)*d/b)/(a*b) - b*(e^{(-c + a*d/b)}*exp\_integral\_e(1, (b*x + a)*d/b)/b + e^{(c - a*d/b)}*exp\_integral\_e(1, -(b*x + a)*d/b)/b)/(a^2*d) - 2*cosh(d*x + c)*log(b*x + a)/(a^2*d) + 2*cosh(d*x + c)*log(x)/(a^2*d) - (Ei(-d*x)*e^{(-c)} + Ei(d*x)*e^c)/(a^2*d)) + (1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2)*cosh(d*x + c)$$

**Fricas [A]** time = 2.15082, size = 579, normalized size = 3.86

$$2ab \cosh(dx + c) + ((b^2x + ab)Ei(dx) + (b^2x + ab)Ei(-dx)) \cosh(c) - \left( (a^2d + ab + (abd + b^2)x)Ei\left(\frac{bdx+ad}{b}\right) - (a^2d - ab + (abd + b^2)x)Ei\left(\frac{bdx-ad}{b}\right) \right) \cosh\left(\frac{bdx+ad}{b}\right) - (a^2d - ab + (abd + b^2)x)Ei\left(\frac{bdx-ad}{b}\right) - (a^2d + ab + (abd + b^2)x)Ei\left(\frac{bdx+ad}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x+a)^2,x, algorithm="fricas")

[Out] 
$$1/2*(2*a*b*cosh(d*x + c) + ((b^2*x + a*b)*Ei(d*x) + (b^2*x + a*b)*Ei(-d*x))*cosh(c) - ((a^2*d + a*b + (a*b*d + b^2)*x)*Ei((b*d*x + a*d)/b) - (a^2*d - a*b + (a*b*d - b^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + ((b^2*x + a*b)*Ei(d*x) - (b^2*x + a*b)*Ei(-d*x))*sinh(c) + ((a^2*d + a*b + (a*b*d + b^2)*x)*Ei((b*d*x + a*d)/b) + (a^2*d - a*b + (a*b*d - b^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^2*b^2*x + a^3*b)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx)}{x(a + bx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x+a)\*\*2,x)

[Out] Integral(cosh(c + d\*x)/(x\*(a + b\*x)\*\*2), x)

**Giac [B]** time = 1.21112, size = 439, normalized size = 2.93

$$\left( abdx \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} - abdx \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} - b^2 x \operatorname{Ei}(-dx) e^{(-c)} + a^2 d \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + b^2 x \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} \right)$$


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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x+a)^2,x, algorithm="giac")

[Out]  $-1/2*(a*b*d*x*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - a*b*d*x*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - b^2*x*\operatorname{Ei}(-d*x)*e^{(-c)} + a^2*d*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + b^2*x*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - b^2*x*\operatorname{Ei}(d*x)*e^c - a^2*d*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + b^2*x*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a*b*\operatorname{Ei}(-d*x)*e^{(-c)} + a*b*\operatorname{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - a*b*\operatorname{Ei}(d*x)*e^c + a*b*\operatorname{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a*b*e^{(d*x + c)} - a*b*e^{(-d*x - c)}*b/(a^2*b^3*x + a^3*b^2)$

$$3.32 \quad \int \frac{\cosh(c+dx)}{x^2(a+bx)^2} dx$$

**Optimal.** Leaf size=186

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^2} - \frac{2b \cosh(c) \text{Chi}(dx)}{a^3} + \frac{2b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{2b \sinh(c) \text{Shi}(dx)}{a^3} + \frac{2b \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^3}$$

[Out]  $-(\text{Cosh}[c + d*x]/(a^2*x)) - (b*\text{Cosh}[c + d*x])/(a^2*(a + b*x)) - (2*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/a^3 + (2*b*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/a^3 + (d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a^2 + (d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/a^2 + (d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a^2 - (2*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/a^3 + (d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^2 + (2*b*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^3$

**Rubi [A]** time = 0.501074, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^2} - \frac{2b \cosh(c) \text{Chi}(dx)}{a^3} + \frac{2b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{2b \sinh(c) \text{Shi}(dx)}{a^3} + \frac{2b \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x^2\*(a + b\*x)^2), x]

[Out]  $-(\text{Cosh}[c + d*x]/(a^2*x)) - (b*\text{Cosh}[c + d*x])/(a^2*(a + b*x)) - (2*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/a^3 + (2*b*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/a^3 + (d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a^2 + (d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/a^2 + (d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a^2 - (2*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/a^3 + (d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^2 + (2*b*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^3$

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c



+ d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(c+dx)}{x^2(a+bx)^2} dx &= \int \left( \frac{\cosh(c+dx)}{a^2x^2} - \frac{2b \cosh(c+dx)}{a^3x} + \frac{b^2 \cosh(c+dx)}{a^2(a+bx)^2} + \frac{2b^2 \cosh(c+dx)}{a^3(a+bx)} \right) dx \\
 &= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a^2} - \frac{(2b) \int \frac{\cosh(c+dx)}{x} dx}{a^3} + \frac{(2b^2) \int \frac{\cosh(c+dx)}{a+bx} dx}{a^3} + \frac{b^2 \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{a^2} \\
 &= -\frac{\cosh(c+dx)}{a^2x} - \frac{b \cosh(c+dx)}{a^2(a+bx)} + \frac{d \int \frac{\sinh(c+dx)}{x} dx}{a^2} + \frac{(bd) \int \frac{\sinh(c+dx)}{a+bx} dx}{a^2} - \frac{(2b \cosh(c)) \int \frac{\cosh(dx)}{x} dx}{a^3} \\
 &= -\frac{\cosh(c+dx)}{a^2x} - \frac{b \cosh(c+dx)}{a^2(a+bx)} - \frac{2b \cosh(c) \text{Chi}(dx)}{a^3} + \frac{2b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} - \frac{2b \sinh(c) \text{Chi}(dx)}{a^3} \\
 &= -\frac{\cosh(c+dx)}{a^2x} - \frac{b \cosh(c+dx)}{a^2(a+bx)} - \frac{2b \cosh(c) \text{Chi}(dx)}{a^3} + \frac{2b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} + \frac{d \text{Chi}(dx)}{a^2}
 \end{aligned}$$

**Mathematica [A]** time = 1.41446, size = 183, normalized size = 0.98

$$ad \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + 2b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + 2b \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) + ad \cosh\left(c - \frac{ad}{b}\right)$$


---

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x^2\*(a + b\*x)^2), x]

[Out] 
$$\left(-\left(\frac{a(a + 2bx) \cosh[c] \cosh[dx]}{x(a + bx)}\right) - 2b \cosh[c] \text{CoshIntegral}[dx] + 2b \cosh\left[c - \frac{ad}{b}\right] \text{CoshIntegral}\left[d\left(\frac{a}{b} + x\right)\right] + a d \text{CoshIntegral}[dx] \sinh[c] + a d \text{CoshIntegral}\left[d\left(\frac{a}{b} + x\right)\right] \sinh\left[c - \frac{ad}{b}\right] - \frac{a(a + 2bx) \sinh[c] \sinh[dx]}{x(a + bx)} + a d \cosh[c] \text{SinhIntegral}[dx] - 2b \sinh[c] \text{SinhIntegral}[dx] + a d \cosh\left[c - \frac{ad}{b}\right] \text{SinhIntegral}\left[d\left(\frac{a}{b} + x\right)\right] + 2b \sinh\left[c - \frac{ad}{b}\right] \text{SinhIntegral}\left[d\left(\frac{a}{b} + x\right)\right]\right) / a^3$$

---

**Maple [A]** time = 0.058, size = 312, normalized size = 1.7

$$-\frac{de^{-dx-cb}}{a^2(bdx+da)} - \frac{de^{-dx-c}}{2ax(bdx+da)} + \frac{de^{-c}\text{Ei}(1,dx)}{2a^2} + \frac{be^{-c}\text{Ei}(1,dx)}{a^3} + \frac{d}{2a^2} e^{\frac{da-cb}{b}} \text{Ei}\left(1,dx+c+\frac{da-cb}{b}\right) - \frac{b}{a^3} e^{\frac{da-cb}{b}} \text{Ei}\left(1,\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/x^2/(b\*x+a)^2, x)

[Out] 
$$-d \exp(-dx-c) / a^2 / (b dx + a) * b - 1/2 * d \exp(-dx-c) / a / x / (b dx + a) + 1/2 * d / a^2 * \exp(-c) * \text{Ei}(1, dx) + 1/a^3 * \exp(-c) * \text{Ei}(1, dx) * b + 1/2 * d / a^2 * \exp((ad-bc)/b) * \text{Ei}(1, dx+c+(ad-bc)/b) - 1/a^3 * \exp((ad-bc)/b) * \text{Ei}(1, dx+c+(ad-bc)/b) * b + 1/a^3 * b * \exp(c) * \text{Ei}(1, -dx) - 1/2 * a^2 / x * \exp(dx+c) - 1/2 * d / a^2 * \exp(c) * \text{Ei}(1, -dx) - 1/2 * d / a^2 * \exp(dx+c) / (1/b * d * a + dx) - 1/2 * d / a^2 * \exp(-(ad-bc)/b) * \text{Ei}(1, -dx-c-(ad-bc)/b) - b / a^3 * \exp(-(ad-bc)/b) * \text{Ei}(1, -dx-c-(ad-bc)/b)$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx+a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x+a)^2,x, algorithm="maxima")

[Out] integrate(cosh(d\*x + c)/((b\*x + a)^2\*x^2), x)

**Fricas [A]** time = 2.11014, size = 802, normalized size = 4.31

$$2 \left( 2 abx + a^2 \right) \cosh(dx + c) - \left( (abd - 2b^2)x^2 + (a^2d - 2ab)x \right) \text{Ei}(dx) - \left( (abd + 2b^2)x^2 + (a^2d + 2ab)x \right) \text{Ei}(-dx) + c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x+a)^2,x, algorithm="fricas")

[Out] 
$$-1/2 * (2 * (2 * a * b * x + a^2) * \cosh(d * x + c) - ((a * b * d - 2 * b^2) * x^2 + (a^2 * d - 2 * a * b) * x) * \text{Ei}(d * x) - ((a * b * d + 2 * b^2) * x^2 + (a^2 * d + 2 * a * b) * x) * \text{Ei}(-d * x)) * \cosh(c) - (((a * b * d + 2 * b^2) * x^2 + (a^2 * d + 2 * a * b) * x) * \text{Ei}((b * d * x + a * d) / b) - ((a * b * d - 2 * b^2) * x^2 + (a^2 * d - 2 * a * b) * x) * \text{Ei}(-(b * d * x + a * d) / b)) * \cosh(-(b * c - a * d) / b) - (((a * b * d - 2 * b^2) * x^2 + (a^2 * d - 2 * a * b) * x) * \text{Ei}(d * x) + ((a * b * d + 2 * b^2) * x^2 + (a^2 * d + 2 * a * b) * x) * \text{Ei}(-d * x)) * \sinh(c) + (((a * b * d + 2 * b^2) * x^2 + (a^2 * d + 2 * a * b) * x) * \text{Ei}((b * d * x + a * d) / b) + ((a * b * d - 2 * b^2) * x^2 + (a^2 * d - 2 * a * b) * x) * \text{Ei}(-(b * d * x + a * d) / b)) * \sinh(-(b * c - a * d) / b)) / (a^3 * b * x^2 + a^4 * x)$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x\*\*2/(b\*x+a)\*\*2,x)

[Out] Exception raised: ValueError

**Giac [B]** time = 1.28598, size = 576, normalized size = 3.1

$$abdx^2 \text{Ei}(-dx) e^{(-c)} - abdx^2 \text{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} - abdx^2 \text{Ei}(dx) e^c + abdx^2 \text{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} + a^2 dx \text{Ei}(-dx) e^{(-c)} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x+a)^2,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/2*(a*b*d*x^2*Ei(-d*x)*e^{-c} - a*b*d*x^2*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} \\ & - a*b*d*x^2*Ei(d*x)*e^c + a*b*d*x^2*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} \\ & + a^2*d*x*Ei(-d*x)*e^{-c} + 2*b^2*x^2*Ei(-d*x)*e^{-c} - a^2*d*x*Ei((b*d*x + \\ & a*d)/b)*e^{c - a*d/b} - 2*b^2*x^2*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} - a^2* \\ & d*x*Ei(d*x)*e^c + 2*b^2*x^2*Ei(d*x)*e^c + a^2*d*x*Ei(-(b*d*x + a*d)/b)*e^{-c \\ & + a*d/b} - 2*b^2*x^2*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} + 2*a*b*x*Ei(-d* \\ & x)*e^{-c} - 2*a*b*x*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} + 2*a*b*x*Ei(d*x)*e^c \\ & - 2*a*b*x*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} + 2*a*b*x*e^{d*x + c} + 2*a* \\ & b*x*e^{-d*x - c} + a^2*e^{d*x + c} + a^2*e^{-d*x - c})/(a^3*b*x^2 + a^4*x) \end{aligned}$$

$$3.33 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} dx$$

**Optimal.** Leaf size=264

$$-\frac{a^3 d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^6} + \frac{3a^2 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{a^3 d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^6} + \frac{3a^2 d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^5}$$

[Out]  $(a^3 \text{Cosh}[c + d*x]) / (2*b^4*(a + b*x)^2) - (3*a^2*\text{Cosh}[c + d*x]) / (b^4*(a + b*x)) - (3*a*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x]) / b^4 - (a^3*d^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x]) / (2*b^6) + (3*a^2*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b]) / b^5 + \text{Sinh}[c + d*x] / (b^3*d) + (a^3*d*\text{Sinh}[c + d*x]) / (2*b^5*(a + b*x)) + (3*a^2*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / b^5 - (3*a*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / b^4 - (a^3*d^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / (2*b^6)$

**Rubi [A]** time = 0.618072, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {6742, 2637, 3297, 3303, 3298, 3301}

$$-\frac{a^3 d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^6} + \frac{3a^2 d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^5} - \frac{a^3 d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^6} + \frac{3a^2 d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{b^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*\text{Cosh}[c + d*x]) / (a + b*x)^3, x]$

[Out]  $(a^3 \text{Cosh}[c + d*x]) / (2*b^4*(a + b*x)^2) - (3*a^2*\text{Cosh}[c + d*x]) / (b^4*(a + b*x)) - (3*a*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x]) / b^4 - (a^3*d^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x]) / (2*b^6) + (3*a^2*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b]) / b^5 + \text{Sinh}[c + d*x] / (b^3*d) + (a^3*d*\text{Sinh}[c + d*x]) / (2*b^5*(a + b*x)) + (3*a^2*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / b^5 - (3*a*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / b^4 - (a^3*d^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / (2*b^6)$

**Rule 6742**

$\text{Int}[u_, x\_Symbol] := \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

**Rule 2637**

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} dx &= \int \left( \frac{\cosh(c+dx)}{b^3} - \frac{a^3 \cosh(c+dx)}{b^3(a+bx)^3} + \frac{3a^2 \cosh(c+dx)}{b^3(a+bx)^2} - \frac{3a \cosh(c+dx)}{b^3(a+bx)} \right) dx \\
&= \frac{\int \cosh(c+dx) dx}{b^3} - \frac{(3a) \int \frac{\cosh(c+dx)}{a+bx} dx}{b^3} + \frac{(3a^2) \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b^3} - \frac{a^3 \int \frac{\cosh(c+dx)}{(a+bx)^3} dx}{b^3} \\
&= \frac{a^3 \cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2 \cosh(c+dx)}{b^4(a+bx)} + \frac{\sinh(c+dx)}{b^3 d} + \frac{(3a^2 d) \int \frac{\sinh(c+dx)}{a+bx} dx}{b^4} - \frac{(a^3 d) \int \frac{\sinh(c+dx)}{(a+bx)^2} dx}{2b^4} \\
&= \frac{a^3 \cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2 \cosh(c+dx)}{b^4(a+bx)} - \frac{3a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} + \frac{\sinh(c+dx)}{b^3 d} + \frac{a^3 d \text{Shi}\left(\frac{ad}{b} + dx\right)}{2b^4} \\
&= \frac{a^3 \cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2 \cosh(c+dx)}{b^4(a+bx)} - \frac{3a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} + \frac{3a^2 d \text{Chi}\left(\frac{ad}{b} + dx\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^5} \\
&= \frac{a^3 \cosh(c+dx)}{2b^4(a+bx)^2} - \frac{3a^2 \cosh(c+dx)}{b^4(a+bx)} - \frac{3a \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^4} - \frac{a^3 d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{2b^6}
\end{aligned}$$

**Mathematica [A]** time = 0.999999, size = 236, normalized size = 0.89

$$ad(a+bx)^2 \left( \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) \left( (a^2 d^2 + 6b^2) \cosh\left(c - \frac{ad}{b}\right) - 6abd \sinh\left(c - \frac{ad}{b}\right) \right) + \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) \left( (a^2 d^2 + 6b^2) \sinh\left(c - \frac{ad}{b}\right) - 6abd \cosh\left(c - \frac{ad}{b}\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*Cosh[c + d\*x])/(a + b\*x)^3,x]

[Out]  $-(b \cosh[d*x] * (a^2 * b * d * (5*a + 6*b*x) * \cosh[c] - (a + b*x) * (2*a*b^2 + a^3*d^2 + 2*b^3*x) * \sinh[c])) - b * ((a + b*x) * (2*a*b^2 + a^3*d^2 + 2*b^3*x) * \cosh[c] - a^2 * b * d * (5*a + 6*b*x) * \sinh[c]) * \sinh[d*x] + a * d * (a + b*x)^2 * (\text{CoshIntegral}[d * (a/b + x)] * ((6*b^2 + a^2*d^2) * \cosh[c - (a*d)/b] - 6*a*b*d * \sinh[c - (a*d)/b]) + (-6*a*b*d * \cosh[c - (a*d)/b] + (6*b^2 + a^2*d^2) * \sinh[c - (a*d)/b]) * \text{ShiIntegral}[d * (a/b + x)]) / (2*b^6*d*(a + b*x)^2)$

**Maple [B]** time = 0.117, size = 571, normalized size = 2.2

$$-\frac{e^{-dx-c}}{2db^3} + \frac{d^2 a^3}{4b^6} e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) + \frac{3da^2}{2b^5} e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right) + \frac{3a}{2b^4} e^{\frac{da-cb}{b}} \text{Ei}\left(1, dx + c + \frac{da-cb}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cosh(d*x+c)/(b*x+a)^3,x)`

[Out] 
$$-1/2/d*\exp(-d*x-c)/b^3+1/4*d^2/b^6*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)*a^3+3/2*d/b^5*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)*a^2+3/2/b^4*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)*a-5/4*d^2*\exp(-d*x-c)/b^4/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^3-3/2*d^2*\exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2*x-1/4*d^3*\exp(-d*x-c)/b^4/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^3*x-1/4*d^3*\exp(-d*x-c)/b^5/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^4-3/2*d/b^5*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)*a^2+3/2/b^4*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)*a+1/2/d/b^3*\exp(d*x+c)+1/4*d^2/b^6*\exp(d*x+c)/(1/b*d*a+d*x)*a^3+1/4*d^2/b^6*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)*a^3-3/2*d/b^5*\exp(d*x+c)/(1/b*d*a+d*x)*a^2+1/4*d^2/b^6*\exp(d*x+c)/(1/b*d*a+d*x)^2*a^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3}{2} a^2 d \int \frac{x e^{(dx+c)}}{b^5 d^2 x^4 + 4 a b^4 d^2 x^3 + 6 a^2 b^3 d^2 x^2 + 4 a^3 b^2 d^2 x + a^4 b d^2} dx - \frac{3}{2} a^2 d \int \frac{x}{b^5 d^2 x^4 e^{(dx+c)} + 4 a b^4 d^2 x^3 e^{(dx+c)} + 6 a^2 b^3 d^2 x^2 e^{(dx+c)} + 4 a^3 b^2 d^2 x e^{(dx+c)} + a^4 b d^2 e^{(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")`

[Out] 
$$3/2*a^2*d*\text{integrate}(x*e^{(d*x+c)}/(b^5*d^2*x^4+4*a*b^4*d^2*x^3+6*a^2*b^3*d^2*x^2+4*a^3*b^2*d^2*x+a^4*b*d^2),x)-3/2*a^2*d*\text{integrate}(x/(b^5*d^2*x^4*e^{(d*x+c)}+4*a*b^4*d^2*x^3*e^{(d*x+c)}+6*a^2*b^3*d^2*x^2*e^{(d*x+c)}+4*a^3*b^2*d^2*x*e^{(d*x+c)}+a^4*b*d^2*e^{(d*x+c)}),x)-3*a*b*\text{integrate}(x*e^{(d*x+c)}/(b^5*d^2*x^4+4*a*b^4*d^2*x^3+6*a^2*b^3*d^2*x^2+4*a^3*b^2*d^2*x+a^4*b*d^2),x)-3*a*b*\text{integrate}(x/(b^5*d^2*x^4*e^{(d*x+c)}+4*a*b^4*d^2*x^3*e^{(d*x+c)}+6*a^2*b^3*d^2*x^2*e^{(d*x+c)}+4*a^3*b^2*d^2*x*e^{(d*x+c)}+a^4*b*d^2*e^{(d*x+c)}),x)+1/2*((b*d*x^3*e^{(2*c)}-3*a*x*e^{(2*c)})*e^{(d*x)}-(b*d*x^3+3*a*x)*e^{(-d*x)})/(b^4*d^2*x^3*e^c+3*a*b^3*d^2*x^2*e^c+3*a^2*b^2*d^2*x*e^c+a^3*b*d^2*e^c)-3/2*a^2*e^{(-c+a*d/b)}*\text{exp\_integral\_e}(4,(b*x+a)*d/b)/((b*x+a)^3*b^2*d^2)-3/2*a^2*e^{(c-a*d/b)}*\text{exp\_integral\_e}(4,-(b*x+a)*d/b)/((b*x+a)^3*b^2*d^2)$$



**Fricas [B]** time = 2.50905, size = 1142, normalized size = 4.33

$$\frac{2(6a^2b^3dx + 5a^3b^2d) \cosh(dx + c) + \left( (a^5d^3 - 6a^4bd^2 + 6a^3b^2d + (a^3b^2d^3 - 6a^2b^3d^2 + 6ab^4d)x^2 + 2(a^4bd^3 - 6a^3b^2d^2 + 6a^2b^3d^2 + 6ab^4d)x + 2(a^4bd^3 - 6a^3b^2d^2 + 6a^2b^3d^2 + 6ab^4d) \right) \operatorname{Ei}\left(\frac{b^2d^2x + a^2d^2}{b^2d^2}\right) + \left( (a^5d^3 - 6a^4bd^2 + 6a^3b^2d + (a^3b^2d^3 - 6a^2b^3d^2 + 6ab^4d)x^2 + 2(a^4bd^3 - 6a^3b^2d^2 + 6a^2b^3d^2 + 6ab^4d) \right) \operatorname{Ei}\left(-\frac{b^2d^2x + a^2d^2}{b^2d^2}\right) \cosh\left(-\frac{b^2d^2x + a^2d^2}{b^2d^2}\right) - 2(a^4bd^3 - 6a^3b^2d^2 + 6a^2b^3d^2 + 6ab^4d)x \sinh(dx + c) - \left( (a^5d^3 - 6a^4bd^2 + 6a^3b^2d + (a^3b^2d^3 - 6a^2b^3d^2 + 6ab^4d)x^2 + 2(a^4bd^3 - 6a^3b^2d^2 + 6a^2b^3d^2 + 6ab^4d) \right) \operatorname{Ei}\left(\frac{b^2d^2x + a^2d^2}{b^2d^2}\right) - \left( (a^5d^3 - 6a^4bd^2 + 6a^3b^2d + (a^3b^2d^3 - 6a^2b^3d^2 + 6ab^4d)x^2 + 2(a^4bd^3 - 6a^3b^2d^2 + 6a^2b^3d^2 + 6ab^4d) \right) \operatorname{Ei}\left(-\frac{b^2d^2x + a^2d^2}{b^2d^2}\right) \sinh\left(-\frac{b^2d^2x + a^2d^2}{b^2d^2}\right)}{(b^8d^2x^2 + 2a^7bd^2x + a^6d^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x+a)^3,x, algorithm="fricas")

[Out] 
$$-1/4*(2*(6*a^2*b^3*d*x + 5*a^3*b^2*d)*\cosh(d*x + c) + ((a^5*d^3 - 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*\operatorname{Ei}((b*d*x + a*d)/b) + (a^5*d^3 + 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 + 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*\operatorname{Ei}(-(b*d*x + a*d)/b))*\cosh(-(b*c - a*d)/b) - 2*(a^4*b*d^2 + 2*b^5*x^2 + 2*a^2*b^3 + (a^3*b^2*d^2 + 4*a*b^4)*x)*\sinh(d*x + c) - ((a^5*d^3 - 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*\operatorname{Ei}((b*d*x + a*d)/b) - (a^5*d^3 + 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 + 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*\operatorname{Ei}(-(b*d*x + a*d)/b))*\sinh(-(b*c - a*d)/b))/(b^8*d*x^2 + 2*a*b^7*d*x + a^2*b^6*d)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*cosh(d\*x+c)/(b\*x+a)\*\*3,x)

[Out] Integral(x\*\*3\*cosh(c + d\*x)/(a + b\*x)\*\*3, x)

**Giac [B]** time = 1.16071, size = 1019, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x+a)^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/4*(a^3*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^3*b^2*d^2*x^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*a^4*b*d^2*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 6*a^2*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^4*b*d^2*x*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 6*a^2*b^3*d*x^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + a^5*d^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 12*a^3*b^2*d*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 6*a*b^4*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^5*d^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 12*a^3*b^2*d*x*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 6*a*b^4*x^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^3*b^2*d*x*e^{(d*x + c)} + a^3*b^2*d*x*e^{(-d*x - c)} - 6*a^4*b*d*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 12*a^2*b^3*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 6*a^4*b*d*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 12*a^2*b^3*x*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^4*b*d*e^{(d*x + c)} + 6*a^2*b^3*x*e^{(d*x + c)} + a^4*b*d*e^{(-d*x - c)} + 6*a^2*b^3*x*e^{(-d*x - c)} + 6*a^3*b^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 6*a^3*b^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 5*a^3*b^2*e^{(d*x + c)} + 5*a^3*b^2*e^{(-d*x - c)})/(b^8*x^2 + 2*a*b^7*x + a^2*b^6) \end{aligned}$$

$$3.34 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} dx$$

**Optimal.** Leaf size=241

$$\frac{a^2 d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^5} + \frac{a^2 d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^5} - \frac{a^2 d \sinh(c+dx)}{2b^4(a+bx)} - \frac{a^2 \cosh(c+dx)}{2b^3(a+bx)^2} - \frac{2ad \sinh(c+dx)}{2b^3(a+bx)^2}$$

[Out]  $-(a^2 \text{Cosh}[c + d*x]) / (2*b^3*(a + b*x)^2) + (2*a*\text{Cosh}[c + d*x]) / (b^3*(a + b*x)) + (\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x]) / b^3 + (a^2*d^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x]) / (2*b^5) - (2*a*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b]) / b^4 - (a^2*d*\text{Sinh}[c + d*x]) / (2*b^4*(a + b*x)) - (2*a*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / b^4 + (\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / b^3 + (a^2*d^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / (2*b^5)$

**Rubi [A]** time = 0.530363, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{a^2 d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^5} + \frac{a^2 d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^5} - \frac{a^2 d \sinh(c+dx)}{2b^4(a+bx)} - \frac{a^2 \cosh(c+dx)}{2b^3(a+bx)^2} - \frac{2ad \sinh(c+dx)}{2b^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*\text{Cosh}[c + d*x]) / (a + b*x)^3, x]$

[Out]  $-(a^2*\text{Cosh}[c + d*x]) / (2*b^3*(a + b*x)^2) + (2*a*\text{Cosh}[c + d*x]) / (b^3*(a + b*x)) + (\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x]) / b^3 + (a^2*d^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x]) / (2*b^5) - (2*a*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b]) / b^4 - (a^2*d*\text{Sinh}[c + d*x]) / (2*b^4*(a + b*x)) - (2*a*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / b^4 + (\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / b^3 + (a^2*d^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x]) / (2*b^5)$

**Rule 6742**

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

**Rule 3297**

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} dx &= \int \left( \frac{a^2 \cosh(c+dx)}{b^2(a+bx)^3} - \frac{2a \cosh(c+dx)}{b^2(a+bx)^2} + \frac{\cosh(c+dx)}{b^2(a+bx)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{a+bx} dx}{b^2} - \frac{(2a) \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{b^2} + \frac{a^2 \int \frac{\cosh(c+dx)}{(a+bx)^3} dx}{b^2} \\
&= -\frac{a^2 \cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a \cosh(c+dx)}{b^3(a+bx)} - \frac{(2ad) \int \frac{\sinh(c+dx)}{a+bx} dx}{b^3} + \frac{(a^2d) \int \frac{\sinh(c+dx)}{(a+bx)^2} dx}{2b^3} + \frac{\cosh(c+dx)}{b^2(a+bx)} \\
&= -\frac{a^2 \cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a \cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} - \frac{a^2d \sinh(c+dx)}{2b^4(a+bx)} + \frac{\sinh(c+dx)}{b^2(a+bx)} \\
&= -\frac{a^2 \cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a \cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} - \frac{2ad \text{Chi}\left(\frac{ad}{b} + dx\right) \sinh(c+dx)}{b^4} + \frac{\sinh(c+dx)}{b^2(a+bx)} \\
&= -\frac{a^2 \cosh(c+dx)}{2b^3(a+bx)^2} + \frac{2a \cosh(c+dx)}{b^3(a+bx)} + \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^3} + \frac{a^2d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{2b^5}
\end{aligned}$$

**Mathematica [A]** time = 0.928054, size = 153, normalized size = 0.63

$$\frac{\text{Chi}\left(d\left(\frac{a}{b} + x\right)\right)\left(\left(a^2d^2 + 2b^2\right)\cosh\left(c - \frac{ad}{b}\right) - 4abd \sinh\left(c - \frac{ad}{b}\right)\right) + \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right)\left(\left(a^2d^2 + 2b^2\right)\sinh\left(c - \frac{ad}{b}\right) - 4abd \cosh\left(c - \frac{ad}{b}\right)\right)}{2b^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Cosh[c + d\*x])/(a + b\*x)^3, x]

[Out] (CoshIntegral[d\*(a/b + x)]\*((2\*b^2 + a^2\*d^2)\*Cosh[c - (a\*d)/b] - 4\*a\*b\*d\*Sinh[c - (a\*d)/b]) - (a\*b\*(-(b\*(3\*a + 4\*b\*x)\*Cosh[c + d\*x]) + a\*d\*(a + b\*x)\*Sinh[c + d\*x]))/(a + b\*x)^2 + (-4\*a\*b\*d\*Cosh[c - (a\*d)/b] + (2\*b^2 + a^2\*d^2)\*Sinh[c - (a\*d)/b])\*SinhIntegral[d\*(a/b + x)]/(2\*b^5)

**Maple [B]** time = 0.047, size = 527, normalized size = 2.2

$$\frac{d^3 e^{-dx-c} a^2 x}{4b^3 (b^2 d^2 x^2 + 2abd^2 x + a^2 d^2)} + \frac{d^3 e^{-dx-c} a^3}{4b^4 (b^2 d^2 x^2 + 2abd^2 x + a^2 d^2)} + \frac{d^2 e^{-dx-c} a x}{b^2 (b^2 d^2 x^2 + 2abd^2 x + a^2 d^2)} + \frac{3d^2 e^{-dx-c} a^2}{4b^3 (b^2 d^2 x^2 + 2abd^2 x + a^2 d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cosh(d*x+c)/(b*x+a)^3,x)`

[Out]  $\frac{1}{4}d^3\exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2*x+1/4*d^3*\exp(-d*x-c)/b^4/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^3+d^2*\exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a*x+3/4*d^2*\exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2-1/4*d^2/b^5*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)*a^2-d/b^4*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)*a-1/2/b^3*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)-1/2/b^3*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)-1/4*d^2/b^5*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)*a^2-1/4*d^2/b^5*\exp(d*x+c)/(1/b*d*a+d*x)^2*a^2-1/4*d^2/b^5*\exp(d*x+c)/(1/b*d*a+d*x)*a^2+d/b^4*\exp(d*x+c)/(1/b*d*a+d*x)*a+d/b^4*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)*a$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{3}{2}ad \int \frac{xe^{(dx+c)}}{b^4d^2x^4 + 4ab^3d^2x^3 + 6a^2b^2d^2x^2 + 4a^3bd^2x + a^4d^2} dx + \frac{3}{2}ad \int \frac{x}{b^4d^2x^4e^{(dx+c)} + 4ab^3d^2x^3e^{(dx+c)} + 6a^2b^2d^2x^2e^{(dx+c)} + 4a^3bd^2xe^{(dx+c)} + a^4d^2e^{(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")`

[Out]  $-3/2*a*d*\text{integrate}(x*e^{(d*x+c)}/(b^4*d^2*x^4+4*a*b^3*d^2*x^3+6*a^2*b^2*d^2*x^2+4*a^3*b*d^2*x+a^4*d^2),x)+3/2*a*d*\text{integrate}(x/(b^4*d^2*x^4*e^{(d*x+c)}+4*a*b^3*d^2*x^3*e^{(d*x+c)}+6*a^2*b^2*d^2*x^2*e^{(d*x+c)}+4*a^3*b*d^2*x*e^{(d*x+c)}+a^4*d^2*e^{(d*x+c)}),x)+b*\text{integrate}(x*e^{(d*x+c)}/(b^4*d^2*x^4+4*a*b^3*d^2*x^3+6*a^2*b^2*d^2*x^2+4*a^3*b*d^2*x+a^4*d^2),x)+b*\text{integrate}(x/(b^4*d^2*x^4*e^{(d*x+c)}+4*a*b^3*d^2*x^3*e^{(d*x+c)}+6*a^2*b^2*d^2*x^2*e^{(d*x+c)}+4*a^3*b*d^2*x*e^{(d*x+c)}+a^4*d^2*e^{(d*x+c)}),x)+1/2*((d*x^2*e^{(2*c)}+x*e^{(2*c)})*e^{(d*x)}-(d*x^2-x)*e^{(-d*x)})/(b^3*d^2*x^3*e^c+3*a*b^2*d^2*x^2*e^c+3*a^2*b*d^2*x*e^c+a^3*d^2*e^c)+1/2*a*e^{(-c+a*d/b)}*\text{exp\_integral\_e}(4,(b*x+a)*d/b)/((b*x+a)^3*b*d^2)+1/2*a*e^{(c-a*d/b)}*\text{exp\_integral\_e}(4,-(b*x+a)*d/b)/((b*x+a)^3*b*d^2)$

**Fricas [A]** time = 2.47278, size = 973, normalized size = 4.04

$$2(4ab^3x+3a^2b^2)\cosh(dx+c)+\left((a^4d^2-4a^3bd+2a^2b^2+(a^2b^2d^2-4ab^3d+2b^4)x^2+2(a^3bd^2-4a^2b^2d+2ab^3)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")`

[Out]  $\frac{1}{4} * (2 * (4 * a * b^3 * x + 3 * a^2 * b^2) * \cosh(d * x + c) + ((a^4 * d^2 - 4 * a^3 * b * d + 2 * a^2 * b^2 + (a^2 * b^2 * d^2 - 4 * a * b^3 * d + 2 * b^4) * x^2 + 2 * (a^3 * b * d^2 - 4 * a^2 * b^2 * d + 2 * a * b^3) * x) * \operatorname{Ei}((b * d * x + a * d) / b) + (a^4 * d^2 + 4 * a^3 * b * d + 2 * a^2 * b^2 + (a^2 * b^2 * d^2 + 4 * a * b^3 * d + 2 * b^4) * x^2 + 2 * (a^3 * b * d^2 + 4 * a^2 * b^2 * d + 2 * a * b^3) * x) * \operatorname{Ei}(-(b * d * x + a * d) / b)) * \cosh(-(b * c - a * d) / b) - 2 * (a^2 * b^2 * d * x + a^3 * b * d) * \sinh(d * x + c) - ((a^4 * d^2 - 4 * a^3 * b * d + 2 * a^2 * b^2 + (a^2 * b^2 * d^2 - 4 * a * b^3 * d + 2 * b^4) * x^2 + 2 * (a^3 * b * d^2 - 4 * a^2 * b^2 * d + 2 * a * b^3) * x) * \operatorname{Ei}((b * d * x + a * d) / b) - (a^4 * d^2 + 4 * a^3 * b * d + 2 * a^2 * b^2 + (a^2 * b^2 * d^2 + 4 * a * b^3 * d + 2 * b^4) * x^2 + 2 * (a^3 * b * d^2 + 4 * a^2 * b^2 * d + 2 * a * b^3) * x) * \operatorname{Ei}(-(b * d * x + a * d) / b)) * \sinh(-(b * c - a * d) / b)) / (b^7 * x^2 + 2 * a * b^6 * x + a^2 * b^5)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*cosh(d*x+c)/(b*x+a)**3,x)`

[Out] `Integral(x**2*cosh(c + d*x)/(a + b*x)**3, x)`

**Giac [B]** time = 1.18058, size = 1000, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")`

[Out]  $\frac{1}{4} * (a^2 * b^2 * d^2 * x^2 * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} + a^2 * b^2 * d^2 * x^2 * \operatorname{Ei}(-(b * d * x + a * d) / b) * e^{(-c + a * d) / b} + 2 * a^3 * b * d^2 * x * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} - 4 * a * b^3 * d * x^2 * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} + 2 * a^3 * b * d^2 * x * \operatorname{Ei}(-(b * d * x + a * d) / b) * e^{(-c + a * d) / b} + 4 * a * b^3 * d * x^2 * \operatorname{Ei}(-(b * d * x + a * d) / b) * e^{(-c + a * d) / b} + a^4 * d^2 * \operatorname{Ei}((b * d * x + a * d) / b) * e^{(c - a * d) / b} - 8 * a^2 * b^2 * d * x * \operatorname{Ei}(-(b * d * x + a * d) / b) * e^{(-c + a * d) / b})$

$$\begin{aligned}
& ((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*b^4*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} \\
& + a^4*d^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 8*a^2*b^2*d*x*Ei(-(b*d*x \\
& + a*d)/b)*e^{(-c + a*d/b)} + 2*b^4*x^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - \\
& a^2*b^2*d*x*e^{(d*x + c)} + a^2*b^2*d*x*e^{(-d*x - c)} - 4*a^3*b*d*Ei((b*d*x + \\
& a*d)/b)*e^{(c - a*d/b)} + 4*a*b^3*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 4*a^3 \\
& *b*d*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 4*a*b^3*x*Ei(-(b*d*x + a*d)/b)* \\
& e^{(-c + a*d/b)} - a^3*b*d*e^{(d*x + c)} + 4*a*b^3*x*e^{(d*x + c)} + a^3*b*d*e^{(- \\
& d*x - c)} + 4*a*b^3*x*e^{(-d*x - c)} + 2*a^2*b^2*Ei((b*d*x + a*d)/b)*e^{(c - a* \\
& d/b)} + 2*a^2*b^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 3*a^2*b^2*e^{(d*x + c)} \\
& + 3*a^2*b^2*e^{(-d*x - c)})/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
\end{aligned}$$



$$3.35 \quad \int \frac{x \cosh(c+dx)}{(a+bx)^3} dx$$

**Optimal.** Leaf size=178

$$-\frac{ad^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^4} + \frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} - \frac{ad^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^4} + \frac{d \cosh\left(c - \frac{ad}{b}\right)}{b^3}$$

[Out] (a\*Cosh[c + d\*x])/(2\*b^2\*(a + b\*x)^2) - Cosh[c + d\*x]/(b^2\*(a + b\*x)) - (a\*d^2\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/(2\*b^4) + (d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/b^3 + (a\*d\*Sinh[c + d\*x])/(2\*b^3\*(a + b\*x)) + (d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^3 - (a\*d^2\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/(2\*b^4)

**Rubi [A]** time = 0.369989, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$-\frac{ad^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^4} + \frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{b^3} - \frac{ad^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^4} + \frac{d \cosh\left(c - \frac{ad}{b}\right)}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(x\*Cosh[c + d\*x])/(a + b\*x)^3, x]

[Out] (a\*Cosh[c + d\*x])/(2\*b^2\*(a + b\*x)^2) - Cosh[c + d\*x]/(b^2\*(a + b\*x)) - (a\*d^2\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/(2\*b^4) + (d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/b^3 + (a\*d\*Sinh[c + d\*x])/(2\*b^3\*(a + b\*x)) + (d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/b^3 - (a\*d^2\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/(2\*b^4)

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x \cosh(c + dx)}{(a + bx)^3} dx &= \int \left( -\frac{a \cosh(c + dx)}{b(a + bx)^3} + \frac{\cosh(c + dx)}{b(a + bx)^2} \right) dx \\
&= \frac{\int \frac{\cosh(c + dx)}{(a + bx)^2} dx}{b} - \frac{a \int \frac{\cosh(c + dx)}{(a + bx)^3} dx}{b} \\
&= \frac{a \cosh(c + dx)}{2b^2(a + bx)^2} - \frac{\cosh(c + dx)}{b^2(a + bx)} + \frac{d \int \frac{\sinh(c + dx)}{a + bx} dx}{b^2} - \frac{(ad) \int \frac{\sinh(c + dx)}{(a + bx)^2} dx}{2b^2} \\
&= \frac{a \cosh(c + dx)}{2b^2(a + bx)^2} - \frac{\cosh(c + dx)}{b^2(a + bx)} + \frac{ad \sinh(c + dx)}{2b^3(a + bx)} - \frac{(ad^2) \int \frac{\cosh(c + dx)}{a + bx} dx}{2b^3} + \frac{\left( d \cosh\left(c - \frac{ad}{b}\right) \right) \int \frac{\sinh(c + dx)}{a + bx} dx}{b^2} \\
&= \frac{a \cosh(c + dx)}{2b^2(a + bx)^2} - \frac{\cosh(c + dx)}{b^2(a + bx)} + \frac{d \operatorname{Chi}\left(\frac{ad}{b} + dx\right) \sinh\left(c - \frac{ad}{b}\right)}{b^3} + \frac{ad \sinh(c + dx)}{2b^3(a + bx)} + \frac{d \cosh\left(c - \frac{ad}{b}\right) \int \frac{\sinh(c + dx)}{a + bx} dx}{b^2} \\
&= \frac{a \cosh(c + dx)}{2b^2(a + bx)^2} - \frac{\cosh(c + dx)}{b^2(a + bx)} - \frac{ad^2 \cosh\left(c - \frac{ad}{b}\right) \operatorname{Chi}\left(\frac{ad}{b} + dx\right)}{2b^4} + \frac{d \operatorname{Chi}\left(\frac{ad}{b} + dx\right) \sinh\left(c - \frac{ad}{b}\right) \int \frac{\sinh(c + dx)}{a + bx} dx}{b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.578165, size = 158, normalized size = 0.89

$$\frac{d(a+bx)^2 \left( \text{Chi} \left( d \left( \frac{a}{b} + x \right) \right) \left( ad \cosh \left( c - \frac{ad}{b} \right) - 2b \sinh \left( c - \frac{ad}{b} \right) \right) + \text{Shi} \left( d \left( \frac{a}{b} + x \right) \right) \left( ad \sinh \left( c - \frac{ad}{b} \right) - 2b \cosh \left( c - \frac{ad}{b} \right) \right) \right)}{2b^4(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*Cosh[c + d\*x])/(a + b\*x)^3,x]

[Out]  $-(b*\text{Cosh}[d*x])*(b*(a + 2*b*x)*\text{Cosh}[c] - a*d*(a + b*x)*\text{Sinh}[c]) - b*(a*d*(a + b*x)*\text{Cosh}[c] - b*(a + 2*b*x)*\text{Sinh}[c])*\text{Sinh}[d*x] + d*(a + b*x)^2*(\text{CoshIntegral}[d*(a/b + x)]*(a*d*\text{Cosh}[c - (a*d)/b] - 2*b*\text{Sinh}[c - (a*d)/b]) + (-2*b*\text{Cosh}[c - (a*d)/b] + a*d*\text{Sinh}[c - (a*d)/b])*\text{SinhIntegral}[d*(a/b + x)])/(2*b^4*(a + b*x)^2)$

**Maple [B]** time = 0.036, size = 435, normalized size = 2.4

$$\frac{d^3 e^{-dx-c} ax}{4 b^2 (b^2 d^2 x^2 + 2 a b d^2 x + a^2 d^2)} - \frac{d^3 e^{-dx-c} a^2}{4 b^3 (b^2 d^2 x^2 + 2 a b d^2 x + a^2 d^2)} - \frac{d^2 e^{-dx-c} x}{2 b (b^2 d^2 x^2 + 2 a b d^2 x + a^2 d^2)} - \frac{d^2 e^{-dx-c}}{4 b^2 (b^2 d^2 x^2 + 2 a b d^2 x + a^2 d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cosh(d\*x+c)/(b\*x+a)^3,x)

[Out]  $-1/4*d^3*\exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a*x-1/4*d^3*\exp(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2-1/2*d^2*\exp(-d*x-c)/b/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*x-1/4*d^2*\exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a+1/4*d^2/b^4*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)*a+1/2*d/b^3*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)+1/4*d^2/b^4*\exp(d*x+c)/(1/b*d*a+d*x)^2*a+1/4*d^2/b^4*\exp(d*x+c)/(1/b*d*a+d*x)*a+1/4*d^2/b^4*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)*a-1/2*d/b^3*\exp(d*x+c)/(1/b*d*a+d*x)-1/2*d/b^3*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$b \int \frac{x e^{(dx+c)}}{b^4 dx^4 + 4 a b^3 dx^3 + 6 a^2 b^2 dx^2 + 4 a^3 b dx + a^4 d} dx - b \int \frac{x}{b^4 dx^4 e^{(dx+c)} + 4 a b^3 dx^3 e^{(dx+c)} + 6 a^2 b^2 dx^2 e^{(dx+c)} + 4 a^3 b dx e^{(dx+c)} + a^4 d e^{(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x+a)^3,x, algorithm="maxima")

[Out] b\*integrate(x\*e^(d\*x + c)/(b^4\*d\*x^4 + 4\*a\*b^3\*d\*x^3 + 6\*a^2\*b^2\*d\*x^2 + 4\*a^3\*b\*d\*x + a^4\*d), x) - b\*integrate(x/(b^4\*d\*x^4\*e^(d\*x + c) + 4\*a\*b^3\*d\*x^3\*e^(d\*x + c) + 6\*a^2\*b^2\*d\*x^2\*e^(d\*x + c) + 4\*a^3\*b\*d\*x\*e^(d\*x + c) + a^4\*d\*e^(d\*x + c)), x) + 1/2\*(x\*e^(d\*x + 2\*c) - x\*e^(-d\*x))/(b^3\*d\*x^3\*e^c + 3\*a\*b^2\*d\*x^2\*e^c + 3\*a^2\*b\*d\*x\*e^c + a^3\*d\*e^c) - 1/2\*a\*e^(-c + a\*d/b)\*exp\_integral\_e(4, (b\*x + a)\*d/b)/((b\*x + a)^3\*b\*d) + 1/2\*a\*e^(c - a\*d/b)\*exp\_integral\_e(4, -(b\*x + a)\*d/b)/((b\*x + a)^3\*b\*d)

**Fricas [B]** time = 2.23575, size = 770, normalized size = 4.33

$$2(2b^3x + ab^2) \cosh(dx + c) + \left( (a^3d^2 - 2a^2bd + (ab^2d^2 - 2b^3d)x^2 + 2(a^2bd^2 - 2ab^2d)x \right) \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) + (a^3d^2 + 2a^2bd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x+a)^3,x, algorithm="fricas")

[Out] -1/4\*(2\*(2\*b^3\*x + a\*b^2)\*cosh(d\*x + c) + ((a^3\*d^2 - 2\*a^2\*b\*d + (a\*b^2\*d^2 - 2\*b^3\*d)\*x^2 + 2\*(a^2\*b\*d^2 - 2\*a\*b^2\*d)\*x)\*Ei((b\*d\*x + a\*d)/b) + (a^3\*d^2 + 2\*a^2\*b\*d + (a\*b^2\*d^2 + 2\*b^3\*d)\*x^2 + 2\*(a^2\*b\*d^2 + 2\*a\*b^2\*d)\*x)\*Ei(-(b\*d\*x + a\*d)/b))\*cosh(-(b\*c - a\*d)/b) - 2\*(a\*b^2\*d\*x + a^2\*b\*d)\*sinh(d\*x + c) - ((a^3\*d^2 - 2\*a^2\*b\*d + (a\*b^2\*d^2 - 2\*b^3\*d)\*x^2 + 2\*(a^2\*b\*d^2 - 2\*a\*b^2\*d)\*x)\*Ei((b\*d\*x + a\*d)/b) - (a^3\*d^2 + 2\*a^2\*b\*d + (a\*b^2\*d^2 + 2\*b^3\*d)\*x^2 + 2\*(a^2\*b\*d^2 + 2\*a\*b^2\*d)\*x)\*Ei(-(b\*d\*x + a\*d)/b))\*sinh(-(b\*c - a\*d)/b))/(b^6\*x^2 + 2\*a\*b^5\*x + a^2\*b^4)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x+a)\*\*3,x)

[Out] Timed out

**Giac [B]** time = 1.23384, size = 714, normalized size = 4.01

$$ab^2d^2x^2\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + ab^2d^2x^2\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + 2a^2bd^2x\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - 2b^3dx^2\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x+a)^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/4*(a*b^2*d^2*x^2*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a*b^2*d^2*x^2*\text{Ei}(- \\ & (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*a^2*b*d^2*x*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a \\ & *d/b)} - 2*b^3*d*x^2*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^2*b*d^2*x*\text{Ei}(- \\ & (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*b^3*d*x^2*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a \\ & *d/b)} + a^3*d^2*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 4*a*b^2*d*x*\text{Ei}((b*d*x + \\ & a*d)/b)*e^{(c - a*d/b)} + a^3*d^2*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 4*a* \\ & b^2*d*x*\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a*b^2*d*x*e^{(d*x + c)} + a*b^2 \\ & *d*x*e^{(-d*x - c)} - 2*a^2*b*d*\text{Ei}((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^2*b*d \\ & *\text{Ei}(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^2*b*d*e^{(d*x + c)} + 2*b^3*x*e^{(d*x \\ & + c)} + a^2*b*d*e^{(-d*x - c)} + 2*b^3*x*e^{(-d*x - c)} + a*b^2*e^{(d*x + c)} + a \\ & *b^2*e^{(-d*x - c)})/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) \end{aligned}$$

$$3.36 \quad \int \frac{\cosh(c+dx)}{(a+bx)^3} dx$$

**Optimal.** Leaf size=104

$$\frac{d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^3} + \frac{d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^3} - \frac{d \sinh(c+dx)}{2b^2(a+bx)} - \frac{\cosh(c+dx)}{2b(a+bx)^2}$$

[Out] -Cosh[c + d\*x]/(2\*b\*(a + b\*x)^2) + (d^2\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/(2\*b^3) - (d\*Sinh[c + d\*x])/(2\*b^2\*(a + b\*x)) + (d^2\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/(2\*b^3)

**Rubi [A]** time = 0.146297, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3297, 3303, 3298, 3301}

$$\frac{d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2b^3} + \frac{d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{2b^3} - \frac{d \sinh(c+dx)}{2b^2(a+bx)} - \frac{\cosh(c+dx)}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(a + b\*x)^3,x]

[Out] -Cosh[c + d\*x]/(2\*b\*(a + b\*x)^2) + (d^2\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/(2\*b^3) - (d\*Sinh[c + d\*x])/(2\*b^2\*(a + b\*x)) + (d^2\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/(2\*b^3)

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c+dx)}{(a+bx)^3} dx &= -\frac{\cosh(c+dx)}{2b(a+bx)^2} + \frac{d \int \frac{\sinh(c+dx)}{(a+bx)^2} dx}{2b} \\ &= -\frac{\cosh(c+dx)}{2b(a+bx)^2} - \frac{d \sinh(c+dx)}{2b^2(a+bx)} + \frac{d^2 \int \frac{\cosh(c+dx)}{a+bx} dx}{2b^2} \\ &= -\frac{\cosh(c+dx)}{2b(a+bx)^2} - \frac{d \sinh(c+dx)}{2b^2(a+bx)} + \frac{\left(d^2 \cosh\left(c - \frac{ad}{b}\right)\right) \int \frac{\cosh\left(\frac{ad}{b}+dx\right)}{a+bx} dx}{2b^2} + \frac{\left(d^2 \sinh\left(c - \frac{ad}{b}\right)\right) \int \frac{\sinh\left(\frac{ad}{b}+dx\right)}{a+bx} dx}{2b^2} \\ &= -\frac{\cosh(c+dx)}{2b(a+bx)^2} + \frac{d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{2b^3} - \frac{d \sinh(c+dx)}{2b^2(a+bx)} + \frac{d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{2b^3} \end{aligned}$$

**Mathematica [A]** time = 0.467275, size = 88, normalized size = 0.85

$$\frac{d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b} + x\right)\right) + d^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(d\left(\frac{a}{b} + x\right)\right) - \frac{b(d(a+bx) \sinh(c+dx) + b \cosh(c+dx))}{(a+bx)^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x)^3, x]
```

```
[Out] (d^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] - (b*(b*Cosh[c + d*x] + d*(a + b*x)*Sinh[c + d*x]))/(a + b*x)^2 + d^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)]/(2*b^3)
```

**Maple [B]** time = 0.028, size = 276, normalized size = 2.7

$$\frac{d^3 e^{-dx-c} x}{4b(b^2 d^2 x^2 + 2abd^2 x + a^2 d^2)} + \frac{d^3 e^{-dx-c} a}{4b^2(b^2 d^2 x^2 + 2abd^2 x + a^2 d^2)} - \frac{d^2 e^{-dx-c}}{4b(b^2 d^2 x^2 + 2abd^2 x + a^2 d^2)} - \frac{d^2}{4b^3} e^{\frac{da-cb}{b}} \operatorname{Ei}\left(1, dx + c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/(b*x+a)^3,x)`

[Out]  $\frac{1}{4}d^3 \exp(-dx-c)/b / (b^2 d^2 x^2 + 2abd^2 x + a^2 d^2) * x + \frac{1}{4}d^3 \exp(-dx-c)/b^2 / (b^2 d^2 x^2 + 2abd^2 x + a^2 d^2) * a - \frac{1}{4}d^2 \exp(-dx-c)/b / (b^2 d^2 x^2 + 2abd^2 x + a^2 d^2) - \frac{1}{4}d^2/b^3 \exp((a*d-b*c)/b) * \operatorname{Ei}(1, dx+c+(a*d-b*c)/b) - \frac{1}{4}d^2/b^3 \exp(dx+c) / (1/b*d*a+dx)^2 - \frac{1}{4}d^2/b^3 \exp(dx+c) / (1/b*d*a+dx) - \frac{1}{4}d^2/b^3 \exp(-(a*d-b*c)/b) * \operatorname{Ei}(1, -dx-c-(a*d-b*c)/b)$

**Maxima [A]** time = 1.18995, size = 128, normalized size = 1.23

$$\frac{d \left( \frac{e^{\left(-c + \frac{ad}{b}\right)} \operatorname{E}_2\left(\frac{(bx+a)d}{b}\right)}{(bx+a)b} - \frac{e^{\left(c - \frac{ad}{b}\right)} \operatorname{E}_2\left(-\frac{(bx+a)d}{b}\right)}{(bx+a)b} \right)}{4b} - \frac{\cosh(dx+c)}{2(bx+a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")`

[Out]  $\frac{1}{4}d * (e^{(-c + a*d/b)} \exp\_integral\_e(2, (b*x + a)*d/b) / ((b*x + a)*b) - e^{(c - a*d/b)} \exp\_integral\_e(2, -(b*x + a)*d/b) / ((b*x + a)*b)) / b - \frac{1}{2} \cosh(dx + c) / ((b*x + a)^2 * b)$

**Fricas [B]** time = 1.95569, size = 518, normalized size = 4.98

$$\frac{2b^2 \cosh(dx+c) - \left( (b^2 d^2 x^2 + 2abd^2 x + a^2 d^2) \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) + (b^2 d^2 x^2 + 2abd^2 x + a^2 d^2) \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) \right) \cosh\left(-\frac{bc-ad}{b}\right) + 2}{4(b^5 x^2 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")`



[Out] 
$$-1/4*(2*b^2*cosh(d*x + c) - ((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei((b*d*x + a*d)/b) + (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + 2*(b^2*d*x + a*b*d)*sinh(d*x + c) + ((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei((b*d*x + a*d)/b) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x+a)\*\*3,x)

[Out] Timed out

**Giac [B]** time = 1.18275, size = 402, normalized size = 3.87

$$\frac{b^2 d^2 x^2 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + b^2 d^2 x^2 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} + 2abd^2 x \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + 2abd^2 x \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)} + a^2 d^2 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) e^{\left(c-\frac{ad}{b}\right)} + a^2 d^2 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right) e^{\left(-c+\frac{ad}{b}\right)}}{4\left(b^5 x^2 + 2ab^4 x + a^2 b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x+a)^3,x, algorithm="giac")

[Out] 
$$1/4*(b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + b^2*d^2*x^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*a*b*d^2*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a*b*d^2*x*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} + a^2*d^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^2*d^2*Ei(-(b*d*x + a*d)/b)*e^{(-c + a*d/b)} - b^2*d*x*e^{(d*x + c)} + b^2*d*x*e^{(-d*x - c)} - a*b*d*e^{(d*x + c)} + a*b*d*e^{(-d*x - c)} - b^2*e^{(d*x + c)} - b^2*e^{(-d*x - c)})/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)$$

$$3.37 \quad \int \frac{\cosh(c+dx)}{x(a+bx)^3} dx$$

**Optimal.** Leaf size=262

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^2 b} - \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^2 b}$$

[Out] Cosh[c + d\*x]/(2\*a\*(a + b\*x)^2) + Cosh[c + d\*x]/(a^2\*(a + b\*x)) + (Cosh[c]\*CoshIntegral[d\*x])/a^3 - (Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/a^3 - (d^2\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/(2\*a\*b^2) - (d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/(a^2\*b) + (d\*Sinh[c + d\*x])/(2\*a\*b\*(a + b\*x)) + (Sinh[c]\*SinhIntegral[d\*x])/a^3 - (d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/(a^2\*b) - (Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/a^3 - (d^2\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/(2\*a\*b^2)

**Rubi [A]** time = 0.561227, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6742, 3303, 3298, 3301, 3297}

$$\frac{d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^2 b} - \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{\sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(xd + \frac{ad}{b}\right)}{a^2 b}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x\*(a + b\*x)^3), x]

[Out] Cosh[c + d\*x]/(2\*a\*(a + b\*x)^2) + Cosh[c + d\*x]/(a^2\*(a + b\*x)) + (Cosh[c]\*CoshIntegral[d\*x])/a^3 - (Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/a^3 - (d^2\*Cosh[c - (a\*d)/b]\*CoshIntegral[(a\*d)/b + d\*x])/(2\*a\*b^2) - (d\*CoshIntegral[(a\*d)/b + d\*x]\*Sinh[c - (a\*d)/b])/(a^2\*b) + (d\*Sinh[c + d\*x])/(2\*a\*b\*(a + b\*x)) + (Sinh[c]\*SinhIntegral[d\*x])/a^3 - (d\*Cosh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/(a^2\*b) - (Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/a^3 - (d^2\*Sinh[c - (a\*d)/b]\*SinhIntegral[(a\*d)/b + d\*x])/(2\*a\*b^2)

**Rule 6742**

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

**Rule 3303**

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx)^3} dx &= \int \left( \frac{\cosh(c+dx)}{a^3x} - \frac{b \cosh(c+dx)}{a(a+bx)^3} - \frac{b \cosh(c+dx)}{a^2(a+bx)^2} - \frac{b \cosh(c+dx)}{a^3(a+bx)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a^3} - \frac{b \int \frac{\cosh(c+dx)}{a+bx} dx}{a^3} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx)^3} dx}{a} \\
&= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} - \frac{d \int \frac{\sinh(c+dx)}{a+bx} dx}{a^2} - \frac{d \int \frac{\sinh(c+dx)}{(a+bx)^2} dx}{2a} + \frac{\cosh(c) \int \frac{\cosh(dx)}{x} dx}{a^3} - \left( b \cosh(c) \int \frac{\cosh(dx)}{a+bx} dx \right) \\
&= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c) \text{Chi}(dx)}{a^3} - \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} + \frac{d \sinh(c+dx)}{2ab(a+bx)} \\
&= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c) \text{Chi}(dx)}{a^3} - \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} - \frac{d \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^2b} \\
&= \frac{\cosh(c+dx)}{2a(a+bx)^2} + \frac{\cosh(c+dx)}{a^2(a+bx)} + \frac{\cosh(c) \text{Chi}(dx)}{a^3} - \frac{\cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{a^3} - \frac{d^2 \cosh\left(c - \frac{ad}{b}\right)}{2a}
\end{aligned}$$

**Mathematica [B]** time = 4.59218, size = 614, normalized size = 2.34

---


$$a^2b^2d^2x^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{d(a+bx)}{b}\right) + 4a^2b^2dx \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{d(a+bx)}{b}\right) + a^2b^2d^2x^2 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{d(a+bx)}{b}\right) - 2a^2b^2d^2x^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{d(a+bx)}{b}\right)$$


---

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x\*(a + b\*x)^3), x]

[Out]  $-(3a^2b^2 \text{Cosh}[c + d*x] - 2ab^3x \text{Cosh}[c + d*x] - 2b^2(a + b*x)^2 \text{Cosh}[c] * \text{CoshIntegral}[d*x] + 2b^2(a + b*x)^2 \text{Cosh}[c - (a*d)/b] * \text{CoshIntegral}[d*(a/b + x)] + a^4d^2 \text{Cosh}[c - (a*d)/b] * \text{CoshIntegral}[(d*(a + b*x))/b] + 2a^3b^2d^2x \text{Cosh}[c - (a*d)/b] * \text{CoshIntegral}[(d*(a + b*x))/b] + a^2b^2d^2x^2 \text{Cosh}[c - (a*d)/b] * \text{CoshIntegral}[(d*(a + b*x))/b] + 2a^3b^2d \text{CoshIntegral}[(d*(a + b*x))/b] * \text{Sinh}[c - (a*d)/b] + 4a^2b^2d^2x \text{CoshIntegral}[(d*(a + b*x))/b] * \text{Sinh}[c - (a*d)/b] + 2ab^3d^2x^2 \text{CoshIntegral}[(d*(a + b*x))/b] * \text{Sinh}[c - (a*d)/b] - a^3b^2d \text{Sinh}[c + d*x] - a^2b^2d^2x \text{Sinh}[c + d*x] - 2a^2b^2 \text{Sinh}[c] * \text{SinhIntegral}[d*x] - 4ab^3x \text{Sinh}[c] * \text{SinhIntegral}[d*x] - 2b^4x^2 \text{Sinh}[c] * \text{SinhIntegral}[d*x] + 2a^2b^2 \text{Sinh}[c - (a*d)/b] * \text{SinhIntegral}[d*(a/b + x)] + 4ab^3x \text{Sinh}[c - (a*d)/b] * \text{SinhIntegral}[d*(a/b + x)] + 2b^4x^2 \text{Sinh}[c - (a*d)/b] * \text{SinhIntegral}[d*(a/b + x)] + 2a^3b^2d \text{Cosh}[c - (a*d)/b] * \text{SinhIntegral}[(d*(a + b*x))/b] + 4a^2b^2d^2x \text{Cosh}[c - (a*d)/b] * \text{SinhIntegral}[(d*(a + b*x))/b]$

```

gral[(d*(a + b*x))/b] + 2*a*b^3*d*x^2*Cosh[c - (a*d)/b]*SinhIntegral[(d*(a
+ b*x))/b] + a^4*d^2*Sinh[c - (a*d)/b]*SinhIntegral[(d*(a + b*x))/b] + 2*a^
3*b*d^2*x*Sinh[c - (a*d)/b]*SinhIntegral[(d*(a + b*x))/b] + a^2*b^2*d^2*x^2
*Sinh[c - (a*d)/b]*SinhIntegral[(d*(a + b*x))/b])/(2*a^3*b^2*(a + b*x)^2)

```

**Maple [A]** time = 0.055, size = 488, normalized size = 1.9

$$\frac{e^{-dx-c}d\left((dx+c)abd+a^2d^2-cdba-2b^2(dx+c)-3bda+2cb^2\right)}{4a^2b\left(b^2(dx+c)^2+2(dx+c)abd-2(dx+c)b^2c+a^2d^2-2cdba+c^2b^2\right)} - \frac{e^{-c}\text{Ei}(1,dx)}{2a^3} + \frac{d^2}{4ab^2}e^{\frac{da-cb}{b}}\text{Ei}\left(1,dx+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/x/(b\*x+a)^3,x)

[Out] 
$$\begin{aligned} & -1/4*\exp(-d*x-c)*d*((d*x+c)*a*b*d+a^2*d^2-c*d*b*a-2*b^2*(d*x+c)-3*b*d*a+2*c \\ & *b^2)/a^2/b/(b^2*(d*x+c)^2+2*(d*x+c)*a*b*d-2*(d*x+c)*b^2*c+a^2*d^2-2*c*d*b* \\ & a+c^2*b^2)-1/2/a^3*\exp(-c)*\text{Ei}(1,d*x)+1/4/a/b^2*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+ \\ & (a*d-b*c)/b)*d^2-1/2/a^2/b*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)*d+1/2/a \\ & ^3*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)-1/2/a^3*\exp(c)*\text{Ei}(1,-d*x)+1/4*d \\ & ^2/b^2/a*\exp(d*x+c)/(1/b*d*a+d*x)^2+1/4*d^2/b^2/a*\exp(d*x+c)/(1/b*d*a+d*x)+ \\ & 1/4*d^2/b^2/a*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)+1/2*d/b/a^2*\exp(d* \\ & x+c)/(1/b*d*a+d*x)+1/2*d/b/a^2*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)+1 \\ & /2/a^3*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx+a)^3x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x+a)^3,x, algorithm="maxima")

[Out] integrate(cosh(d\*x + c)/((b\*x + a)^3\*x), x)

**Fricas [B]** time = 2.17532, size = 1243, normalized size = 4.74

$$2(2ab^3x + 3a^2b^2) \cosh(dx + c) + 2\left((b^4x^2 + 2ab^3x + a^2b^2) \operatorname{Ei}(dx) + (b^4x^2 + 2ab^3x + a^2b^2) \operatorname{Ei}(-dx)\right) \cosh(c) - \left(a^4d^2\right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x+a)^3,x, algorithm="fricas")

[Out]  $\frac{1}{4} \cdot (2 \cdot (2 \cdot a \cdot b^3 \cdot x + 3 \cdot a^2 \cdot b^2) \cdot \cosh(d \cdot x + c) + 2 \cdot ((b^4 \cdot x^2 + 2 \cdot a \cdot b^3 \cdot x + a^2 \cdot b^2) \cdot \operatorname{Ei}(d \cdot x) + (b^4 \cdot x^2 + 2 \cdot a \cdot b^3 \cdot x + a^2 \cdot b^2) \cdot \operatorname{Ei}(-d \cdot x)) \cdot \cosh(c) - ((a^4 \cdot d^2 + 2 \cdot a^3 \cdot b \cdot d + 2 \cdot a^2 \cdot b^2 + (a^2 \cdot b^2 \cdot d^2 + 2 \cdot a \cdot b^3 \cdot d + 2 \cdot b^4) \cdot x^2 + 2 \cdot (a^3 \cdot b \cdot d^2 + 2 \cdot a^2 \cdot b^2 \cdot d + 2 \cdot a \cdot b^3) \cdot x) \cdot \operatorname{Ei}((b \cdot d \cdot x + a \cdot d) / b) + (a^4 \cdot d^2 - 2 \cdot a^3 \cdot b \cdot d + 2 \cdot a^2 \cdot b^2 + (a^2 \cdot b^2 \cdot d^2 - 2 \cdot a \cdot b^3 \cdot d + 2 \cdot b^4) \cdot x^2 + 2 \cdot (a^3 \cdot b \cdot d^2 - 2 \cdot a^2 \cdot b^2 \cdot d + 2 \cdot a \cdot b^3) \cdot x) \cdot \operatorname{Ei}(-(b \cdot d \cdot x + a \cdot d) / b)) \cdot \cosh(-(b \cdot c - a \cdot d) / b) + 2 \cdot (a^2 \cdot b^2 \cdot d \cdot x + a^3 \cdot b \cdot d) \cdot \sinh(d \cdot x + c) + 2 \cdot ((b^4 \cdot x^2 + 2 \cdot a \cdot b^3 \cdot x + a^2 \cdot b^2) \cdot \operatorname{Ei}(d \cdot x) - (b^4 \cdot x^2 + 2 \cdot a \cdot b^3 \cdot x + a^2 \cdot b^2) \cdot \operatorname{Ei}(-d \cdot x)) \cdot \sinh(c) + ((a^4 \cdot d^2 + 2 \cdot a^3 \cdot b \cdot d + 2 \cdot a^2 \cdot b^2 + (a^2 \cdot b^2 \cdot d^2 + 2 \cdot a \cdot b^3 \cdot d + 2 \cdot b^4) \cdot x^2 + 2 \cdot (a^3 \cdot b \cdot d^2 + 2 \cdot a^2 \cdot b^2 \cdot d + 2 \cdot a \cdot b^3) \cdot x) \cdot \operatorname{Ei}((b \cdot d \cdot x + a \cdot d) / b) - (a^4 \cdot d^2 - 2 \cdot a^3 \cdot b \cdot d + 2 \cdot a^2 \cdot b^2 + (a^2 \cdot b^2 \cdot d^2 - 2 \cdot a \cdot b^3 \cdot d + 2 \cdot b^4) \cdot x^2 + 2 \cdot (a^3 \cdot b \cdot d^2 - 2 \cdot a^2 \cdot b^2 \cdot d + 2 \cdot a \cdot b^3) \cdot x) \cdot \operatorname{Ei}(-(b \cdot d \cdot x + a \cdot d) / b)) \cdot \sinh(-(b \cdot c - a \cdot d) / b)) / (a^3 \cdot b^4 \cdot x^2 + 2 \cdot a^4 \cdot b^3 \cdot x + a^5 \cdot b^2)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx)}{x(a + bx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x+a)\*\*3,x)

[Out] Integral(cosh(c + d\*x)/(x\*(a + b\*x)\*\*3), x)

---

**Giac [B]** time = 1.2106, size = 1130, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x+a)^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/4*(a^2*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + a^2*b^2*d^2*x^2*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*a^3*b*d^2*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*a^3*b*d^2*x*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 2*a*b^3*d*x^2*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 2*b^4*x^2*Ei(-d*x)*e^{(-c)} + a^4*d^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 4*a^2*b^2*d*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 2*b^4*x^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 2*b^4*x^2*Ei(d*x)*e^c + a^4*d^2*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 4*a^2*b^2*d*x*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 2*b^4*x^2*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^2*b^2*d*x*e^{(d*x + c)} + a^2*b^2*d*x*e^{(-d*x - c)} - 4*a*b^3*x*Ei(-d*x)*e^{(-c)} + 2*a^3*b*d*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} + 4*a*b^3*x*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 4*a*b^3*x*Ei(d*x)*e^c - 2*a^3*b*d*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} + 4*a*b^3*x*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} - a^3*b*d*e^{(d*x + c)} - 2*a*b^3*x*e^{(d*x + c)} + a^3*b*d*e^{(-d*x - c)} - 2*a*b^3*x*e^{(-d*x - c)} - 2*a^2*b^2*Ei(-d*x)*e^{(-c)} + 2*a^2*b^2*Ei((b*d*x + a*d)/b)*e^{(c - a*d/b)} - 2*a^2*b^2*Ei(d*x)*e^c + 2*a^2*b^2*Ei(- (b*d*x + a*d)/b)*e^{(-c + a*d/b)} - 3*a^2*b^2*e^{(d*x + c)} - 3*a^2*b^2*e^{(-d*x - c)})/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2) \end{aligned}$$

$$3.38 \quad \int \frac{\cosh(c+dx)}{x^2(a+bx)^3} dx$$

**Optimal.** Leaf size=298

$$\frac{d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2a^2b} + \frac{2d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{3b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^4}$$

[Out]  $-(\text{Cosh}[c + d*x]/(a^3*x)) - (b*\text{Cosh}[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*\text{Cosh}[c + d*x])/(a^3*(a + b*x)) - (3*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/a^4 + (3*b*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/a^4 + (d^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/(2*a^2*b) + (d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a^3 + (2*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/a^3 - (d*\text{Sinh}[c + d*x])/(2*a^2*(a + b*x)) + (d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a^3 - (3*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/a^4 + (2*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^3 + (3*b*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^4 + (d^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/(2*a^2*b)$

**Rubi [A]** time = 0.689996, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{d^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{2a^2b} + \frac{2d \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^3} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{3b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(xd + \frac{ad}{b}\right)}{a^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cosh}[c + d*x]/(x^2*(a + b*x)^3), x]$

[Out]  $-(\text{Cosh}[c + d*x]/(a^3*x)) - (b*\text{Cosh}[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*\text{Cosh}[c + d*x])/(a^3*(a + b*x)) - (3*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/a^4 + (3*b*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/a^4 + (d^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/(2*a^2*b) + (d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a^3 + (2*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/a^3 - (d*\text{Sinh}[c + d*x])/(2*a^2*(a + b*x)) + (d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a^3 - (3*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/a^4 + (2*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^3 + (3*b*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^4 + (d^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/(2*a^2*b)$

Rule 6742



```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx)^3} dx &= \int \left( \frac{\cosh(c+dx)}{a^3x^2} - \frac{3b \cosh(c+dx)}{a^4x} + \frac{b^2 \cosh(c+dx)}{a^2(a+bx)^3} + \frac{2b^2 \cosh(c+dx)}{a^3(a+bx)^2} + \frac{3b^2 \cosh(c+dx)}{a^4(a+bx)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a^3} - \frac{(3b) \int \frac{\cosh(c+dx)}{x} dx}{a^4} + \frac{(3b^2) \int \frac{\cosh(c+dx)}{a+bx} dx}{a^4} + \frac{(2b^2) \int \frac{\cosh(c+dx)}{(a+bx)^2} dx}{a^3} + \frac{b^2 \int \frac{\cosh(c+dx)}{(a+bx)} dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{a^3x} - \frac{b \cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b \cosh(c+dx)}{a^3(a+bx)} + \frac{d \int \frac{\sinh(c+dx)}{x} dx}{a^3} + \frac{(2bd) \int \frac{\sinh(c+dx)}{a+bx} dx}{a^3} + \frac{bd \int \frac{\sinh(c+dx)}{(a+bx)^2} dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{a^3x} - \frac{b \cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b \cosh(c+dx)}{a^3(a+bx)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{3b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{d(a+bx)}{b}\right)}{a^4} \\
&= -\frac{\cosh(c+dx)}{a^3x} - \frac{b \cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b \cosh(c+dx)}{a^3(a+bx)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{3b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{d(a+bx)}{b}\right)}{a^4} \\
&= -\frac{\cosh(c+dx)}{a^3x} - \frac{b \cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b \cosh(c+dx)}{a^3(a+bx)} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{3b \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{d(a+bx)}{b}\right)}{a^4}
\end{aligned}$$

**Mathematica [B]** time = 1.43962, size = 710, normalized size = 2.38

$$a^2b^2d^2x^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{d(a+bx)}{b}\right) + 8a^2b^2dx^2 \sinh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{d(a+bx)}{b}\right) + a^2b^2d^2x^3 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{d(a+bx)}{b}\right) + 4a^2b^2d^2x^2 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{d(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x^2\*(a + b\*x)^3), x]

[Out] (-2\*a^3\*b\*Cosh[c + d\*x] - 9\*a^2\*b^2\*x\*Cosh[c + d\*x] - 6\*a\*b^3\*x^2\*Cosh[c + d\*x] + 6\*b^2\*x\*(a + b\*x)^2\*Cosh[c - (a\*d)/b]\*CoshIntegral[d\*(a/b + x)] + a^4\*d^2\*x\*Cosh[c - (a\*d)/b]\*CoshIntegral[(d\*(a + b\*x))/b] + 2\*a^3\*b\*d^2\*x^2\*Cosh[c - (a\*d)/b]\*CoshIntegral[(d\*(a + b\*x))/b] + a^2\*b^2\*d^2\*x^3\*Cosh[c - (a\*d)/b]\*CoshIntegral[(d\*(a + b\*x))/b] + 2\*b\*x\*(a + b\*x)^2\*CoshIntegral[d\*x] \* (-3\*b\*Cosh[c] + a\*d\*Sinh[c]) + 4\*a^3\*b\*d\*x\*CoshIntegral[(d\*(a + b\*x))/b]\*Sinh[c - (a\*d)/b] + 8\*a^2\*b^2\*d\*x^2\*CoshIntegral[(d\*(a + b\*x))/b]\*Sinh[c - (a\*d)/b] + 4\*a\*b^3\*d\*x^3\*CoshIntegral[(d\*(a + b\*x))/b]\*Sinh[c - (a\*d)/b] - a^3\*b\*d\*x\*Sinh[c + d\*x] - a^2\*b^2\*d\*x^2\*Sinh[c + d\*x] + 2\*a^3\*b\*d\*x\*Cosh[c]\*SinhIntegral[d\*x] + 4\*a^2\*b^2\*d\*x^2\*Cosh[c]\*SinhIntegral[d\*x] + 2\*a\*b^3\*d\*x^3\*Cosh[c]\*SinhIntegral[d\*x] - 6\*a^2\*b^2\*x\*Sinh[c]\*SinhIntegral[d\*x] - 12\*a\*b^3\*x^2\*Sinh[c]\*SinhIntegral[d\*x] - 6\*b^4\*x^3\*Sinh[c]\*SinhIntegral[d\*x] + 6\*a^2\*b^2\*x\*Sinh[c - (a\*d)/b]\*SinhIntegral[d\*(a/b + x)] + 12\*a\*b^3\*x^2\*Sinh

$[c - (a*d)/b]*\text{SinhIntegral}[d*(a/b + x)] + 6*b^4*x^3*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[d*(a/b + x)] + 4*a^3*b*d*x*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(d*(a + b*x))/b] + 8*a^2*b^2*d*x^2*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(d*(a + b*x))/b] + 4*a*b^3*d*x^3*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(d*(a + b*x))/b] + a^4*d^2*x*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(d*(a + b*x))/b] + 2*a^3*b*d^2*x^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(d*(a + b*x))/b] + a^2*b^2*d^2*x^3*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(d*(a + b*x))/b])/(2*a^4*b*x*(a + b*x)^2)$

**Maple [B]** time = 0.073, size = 643, normalized size = 2.2

$$\frac{e^{-dx-c}xd^3b}{4a^2(b^2d^2x^2 + 2abd^2x + a^2d^2)} + \frac{e^{-dx-c}d^3}{4a(b^2d^2x^2 + 2abd^2x + a^2d^2)} - \frac{3e^{-dx-c}xd^2b^2}{2a^3(b^2d^2x^2 + 2abd^2x + a^2d^2)} - \frac{9e^{-dx-c}d^2}{4a^2(b^2d^2x^2 + 2abd^2x + a^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/x^2/(b\*x+a)^3,x)

[Out]  $\frac{1}{4}*\exp(-d*x-c)/a^2*x*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+1/4*\exp(-d*x-c)/a*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-3/2*\exp(-d*x-c)/a^3*x*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b-9/4*\exp(-d*x-c)/a^2*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b-1/2*\exp(-d*x-c)/a/x*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)+1/2*d/a^3*\exp(-c)*\text{Ei}(1,d*x)+3/2/a^4*\exp(-c)*\text{Ei}(1,d*x)*b-1/4/b/a^2*d^2*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)+d/a^3*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)-3/2*b/a^4*\exp((a*d-b*c)/b)*\text{Ei}(1,d*x+c+(a*d-b*c)/b)+3/2/a^4*b*\exp(c)*\text{Ei}(1,-d*x)-1/2/a^3/x*\exp(d*x+c)-1/2*d/a^3*\exp(c)*\text{Ei}(1,-d*x)-1/4*d^2/b/a^2*\exp(d*x+c)/(1/b*d*a+d*x)^2-1/4*d^2/b/a^2*\exp(d*x+c)/(1/b*d*a+d*x)-1/4*d^2/b/a^2*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)-d/a^3*\exp(d*x+c)/(1/b*d*a+d*x)-d/a^3*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)-3/2*b/a^4*\exp(-(a*d-b*c)/b)*\text{Ei}(1,-d*x-c-(a*d-b*c)/b)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx+a)^3x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x+a)^3,x, algorithm="maxima")

```
[Out] integrate(cosh(d*x + c)/((b*x + a)^3*x^2), x)
```

**Fricas [B]** time = 2.2, size = 1563, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(6*a*b^3*x^2 + 9*a^2*b^2*x + 2*a^3*b)*cosh(d*x + c) - 2*(((a*b^3*d
- 3*b^4)*x^3 + 2*(a^2*b^2*d - 3*a*b^3)*x^2 + (a^3*b*d - 3*a^2*b^2)*x)*Ei(d*x
x) - ((a*b^3*d + 3*b^4)*x^3 + 2*(a^2*b^2*d + 3*a*b^3)*x^2 + (a^3*b*d + 3*a^
2*b^2)*x)*Ei(-d*x))*cosh(c) - (((a^2*b^2*d^2 + 4*a*b^3*d + 6*b^4)*x^3 + 2*(
a^3*b*d^2 + 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 + 4*a^3*b*d + 6*a^2*b^2)*
x)*Ei((b*d*x + a*d)/b) + ((a^2*b^2*d^2 - 4*a*b^3*d + 6*b^4)*x^3 + 2*(a^3*b*
d^2 - 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 - 4*a^3*b*d + 6*a^2*b^2)*x)*Ei(
-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + 2*(a^2*b^2*d*x^2 + a^3*b*d*x)*sin
h(d*x + c) - 2*(((a*b^3*d - 3*b^4)*x^3 + 2*(a^2*b^2*d - 3*a*b^3)*x^2 + (a^3
*b*d - 3*a^2*b^2)*x)*Ei(d*x) + ((a*b^3*d + 3*b^4)*x^3 + 2*(a^2*b^2*d + 3*a*
b^3)*x^2 + (a^3*b*d + 3*a^2*b^2)*x)*Ei(-d*x))*sinh(c) + (((a^2*b^2*d^2 + 4*
a*b^3*d + 6*b^4)*x^3 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2
+ 4*a^3*b*d + 6*a^2*b^2)*x)*Ei((b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 4*a*b^3*
d + 6*b^4)*x^3 + 2*(a^3*b*d^2 - 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 - 4*a
^3*b*d + 6*a^2*b^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^4*b^3
*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x**2/(b*x+a)**3,x)
```

```
[Out] Exception raised: ValueError
```

**Giac [B]** time = 1.25263, size = 1358, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x+a)^3,x, algorithm="giac")

[Out] 
$$\frac{1}{4} \cdot (a^2 b^2 d^2 x^3 \operatorname{Ei}((b d x + a d)/b) e^{(c - a d/b)} + a^2 b^2 d^2 x^3 \operatorname{Ei}(- (b d x + a d)/b) e^{(-c + a d/b)} - 2 a^3 b^3 d x^3 \operatorname{Ei}(-d x) e^{(-c)} + 2 a^3 b^3 d^2 x^2 \operatorname{Ei}((b d x + a d)/b) e^{(c - a d/b)} + 4 a^3 b^3 d x^3 \operatorname{Ei}((b d x + a d)/b) e^{(c - a d/b)} + 2 a^3 b^3 d x^3 \operatorname{Ei}(d x) e^c + 2 a^3 b^3 d^2 x^2 \operatorname{Ei}(- (b d x + a d)/b) e^{(-c + a d/b)} - 4 a^3 b^3 d x^3 \operatorname{Ei}(- (b d x + a d)/b) e^{(-c + a d/b)} - 4 a^2 b^2 d x^2 \operatorname{Ei}(-d x) e^{(-c)} - 6 b^4 x^3 \operatorname{Ei}(-d x) e^{(-c)} + a^4 d^2 x^3 \operatorname{Ei}((b d x + a d)/b) e^{(c - a d/b)} + 8 a^2 b^2 d x^2 \operatorname{Ei}((b d x + a d)/b) e^{(c - a d/b)} + 6 b^4 x^3 \operatorname{Ei}((b d x + a d)/b) e^{(c - a d/b)} + 4 a^2 b^2 d x^2 \operatorname{Ei}(d x) e^c - 6 b^4 x^3 \operatorname{Ei}(d x) e^c + a^4 d^2 x^3 \operatorname{Ei}(- (b d x + a d)/b) e^{(-c + a d/b)} - 8 a^2 b^2 d x^2 \operatorname{Ei}(- (b d x + a d)/b) e^{(-c + a d/b)} + 6 b^4 x^3 \operatorname{Ei}(- (b d x + a d)/b) e^{(-c + a d/b)} - a^2 b^2 d x^2 e^{(d x + c)} + a^2 b^2 d x^2 e^{(-d x - c)} - 2 a^3 b^3 d x \operatorname{Ei}(-d x) e^{(-c)} - 12 a^3 b^3 x^2 \operatorname{Ei}(-d x) e^{(-c)} + 4 a^3 b^3 d x \operatorname{Ei}((b d x + a d)/b) e^{(c - a d/b)} + 12 a^3 b^3 x^2 \operatorname{Ei}((b d x + a d)/b) e^{(c - a d/b)} + 2 a^3 b^3 d x \operatorname{Ei}(d x) e^c - 12 a^3 b^3 x^2 \operatorname{Ei}(d x) e^c - 4 a^3 b^3 d x \operatorname{Ei}(- (b d x + a d)/b) e^{(-c + a d/b)} + 12 a^3 b^3 x^2 \operatorname{Ei}(- (b d x + a d)/b) e^{(-c + a d/b)} - a^3 b^3 d x e^{(d x + c)} - 6 a^3 b^3 x^2 e^{(d x + c)} + a^3 b^3 d x e^{(-d x - c)} - 6 a^3 b^3 x^2 e^{(-d x - c)} - 6 a^2 b^2 x \operatorname{Ei}(-d x) e^{(-c)} + 6 a^2 b^2 x \operatorname{Ei}((b d x + a d)/b) e^{(c - a d/b)} - 6 a^2 b^2 x \operatorname{Ei}(d x) e^c + 6 a^2 b^2 x \operatorname{Ei}(- (b d x + a d)/b) e^{(-c + a d/b)} - 9 a^2 b^2 x e^{(d x + c)} - 9 a^2 b^2 x e^{(-d x - c)} - 2 a^3 b^3 e^{(d x + c)} - 2 a^3 b^3 e^{(-d x - c)}) / (a^4 b^3 x^3 + 2 a^5 b^2 x^2 + a^6 b x)$$

### 3.39 $\int \frac{\cosh(c+dx)}{x^3(a+bx)^3} dx$

**Optimal.** Leaf size=377

$$\frac{6b^2 \cosh(c)\text{Chi}(dx)}{a^5} - \frac{6b^2 \cosh\left(c - \frac{ad}{b}\right)\text{Chi}\left(xd + \frac{ad}{b}\right)}{a^5} + \frac{6b^2 \sinh(c)\text{Shi}(dx)}{a^5} - \frac{6b^2 \sinh\left(c - \frac{ad}{b}\right)\text{Shi}\left(xd + \frac{ad}{b}\right)}{a^5} + \frac{3b^2 \cosh(c)\text{Chi}(dx)}{a^4}$$

[Out]  $-\text{Cosh}[c + d*x]/(2*a^3*x^2) + (3*b*\text{Cosh}[c + d*x])/(a^4*x) + (b^2*\text{Cosh}[c + d*x])/(2*a^3*(a + b*x)^2) + (3*b^2*\text{Cosh}[c + d*x])/(a^4*(a + b*x)) + (6*b^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/a^5 + (d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/(2*a^3) - (6*b^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/a^5 - (d^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/(2*a^3) - (3*b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a^4 - (3*b*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/a^4 - (d*\text{Sinh}[c + d*x])/(2*a^3*x) + (b*d*\text{Sinh}[c + d*x])/(2*a^3*(a + b*x)) - (3*b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a^4 + (6*b^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/a^5 + (d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/(2*a^3) - (3*b*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^4 - (6*b^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^5 - (d^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/(2*a^3)$

**Rubi [A]** time = 0.821274, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6742, 3297, 3303, 3298, 3301}

$$\frac{6b^2 \cosh(c)\text{Chi}(dx)}{a^5} - \frac{6b^2 \cosh\left(c - \frac{ad}{b}\right)\text{Chi}\left(xd + \frac{ad}{b}\right)}{a^5} + \frac{6b^2 \sinh(c)\text{Shi}(dx)}{a^5} - \frac{6b^2 \sinh\left(c - \frac{ad}{b}\right)\text{Shi}\left(xd + \frac{ad}{b}\right)}{a^5} + \frac{3b^2 \cosh(c)\text{Chi}(dx)}{a^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cosh}[c + d*x]/(x^3*(a + b*x)^3), x]$

[Out]  $-\text{Cosh}[c + d*x]/(2*a^3*x^2) + (3*b*\text{Cosh}[c + d*x])/(a^4*x) + (b^2*\text{Cosh}[c + d*x])/(2*a^3*(a + b*x)^2) + (3*b^2*\text{Cosh}[c + d*x])/(a^4*(a + b*x)) + (6*b^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/a^5 + (d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/(2*a^3) - (6*b^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/a^5 - (d^2*\text{Cosh}[c - (a*d)/b]*\text{CoshIntegral}[(a*d)/b + d*x])/(2*a^3) - (3*b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a^4 - (3*b*d*\text{CoshIntegral}[(a*d)/b + d*x]*\text{Sinh}[c - (a*d)/b])/a^4 - (d*\text{Sinh}[c + d*x])/(2*a^3*x) + (b*d*\text{Sinh}[c + d*x])/(2*a^3*(a + b*x)) - (3*b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a^4 + (6*b^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/a^5 + (d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/(2*a^3) - (3*b*d*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^4 - (6*b^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/a^5 - (d^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[(a*d)/b + d*x])/(2*a^3)$

$$/a^5 - (d^2 \sinh[c - (a*d)/b] * \text{SinhIntegral}[(a*d)/b + d*x]) / (2*a^3)$$

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^3(a+bx)^3} dx &= \int \left( \frac{\cosh(c+dx)}{a^3x^3} - \frac{3b \cosh(c+dx)}{a^4x^2} + \frac{6b^2 \cosh(c+dx)}{a^5x} - \frac{b^3 \cosh(c+dx)}{a^3(a+bx)^3} - \frac{3b^3 \cosh(c+dx)}{a^4(a+bx)^2} - \frac{3b^3 \cosh(c+dx)}{a^5(a+bx)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{a^3} - \frac{(3b) \int \frac{\cosh(c+dx)}{x^2} dx}{a^4} + \frac{(6b^2) \int \frac{\cosh(c+dx)}{x} dx}{a^5} - \frac{(6b^3) \int \frac{\cosh(c+dx)}{a+bx} dx}{a^5} - \frac{(3b^3) \int \frac{\cosh(c+dx)}{a+bx} dx}{a^4} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b \cosh(c+dx)}{a^4x} + \frac{b^2 \cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2 \cosh(c+dx)}{a^4(a+bx)} + \frac{d \int \frac{\sinh(c+dx)}{x^2} dx}{2a^3} - \frac{(3b^3) \int \frac{\cosh(c+dx)}{a+bx} dx}{a^4} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b \cosh(c+dx)}{a^4x} + \frac{b^2 \cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2 \cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2 \cosh(c) \text{Chi}(dx)}{a^5} - \frac{(3b^3) \int \frac{\cosh(c+dx)}{a+bx} dx}{a^4} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b \cosh(c+dx)}{a^4x} + \frac{b^2 \cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2 \cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2 \cosh(c) \text{Chi}(dx)}{a^5} - \frac{(3b^3) \int \frac{\cosh(c+dx)}{a+bx} dx}{a^4} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} + \frac{3b \cosh(c+dx)}{a^4x} + \frac{b^2 \cosh(c+dx)}{2a^3(a+bx)^2} + \frac{3b^2 \cosh(c+dx)}{a^4(a+bx)} + \frac{6b^2 \cosh(c) \text{Chi}(dx)}{a^5} + \frac{(3b^3) \int \frac{\cosh(c+dx)}{a+bx} dx}{a^4}
\end{aligned}$$

**Mathematica [A]** time = 1.80699, size = 627, normalized size = 1.66

$$\frac{-x^2(a+bx)^2 \text{Chi}(dx) \left( \cosh(c) (a^2d^2 + 12b^2) - 6abd \sinh(c) \right) + x^2(a+bx)^2 \text{Chi} \left( d \left( \frac{a}{b} + x \right) \right) \left( (a^2d^2 + 12b^2) \cosh \left( c - \frac{ad}{b} \right) \right)}{a^5}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x^3\*(a + b\*x)^3), x]

[Out]  $-(a^4 \text{Cosh}[c + d*x] - 4*a^3*b*x*\text{Cosh}[c + d*x] - 18*a^2*b^2*x^2*\text{Cosh}[c + d*x] - 12*a*b^3*x^3*\text{Cosh}[c + d*x] - x^2*(a + b*x)^2*\text{CoshIntegral}[d*x]*((12*b^2 + a^2*d^2)*\text{Cosh}[c] - 6*a*b*d*\text{Sinh}[c]) + x^2*(a + b*x)^2*\text{CoshIntegral}[d*(a/b + x)]*((12*b^2 + a^2*d^2)*\text{Cosh}[c - (a*d)/b] + 6*a*b*d*\text{Sinh}[c - (a*d)/b]) + a^4*d*x*\text{Sinh}[c + d*x] + a^3*b*d*x^2*\text{Sinh}[c + d*x] + 6*a^3*b*d*x^2*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + 12*a^2*b^2*d*x^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + 6*a*b^3*d*x^4*\text{Cosh}[c]*\text{SinhIntegral}[d*x] - 12*a^2*b^2*x^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] - a^4*d^2*x^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] - 24*a*b^3*x^3*\text{Sinh}[c]*\text{SinhIntegral}[d*x] - 2*a^3*b*d^2*x^3*\text{Sinh}[c]*\text{SinhIntegral}[d*x] - 12*b^4*x^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x] - a^2*b^2*d^2*x^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + 6*a^3*b*d*x^2*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[d*(a/b + x)] + 12*a^2*b^2*d*x^3*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[d*(a/b + x)] + 6*a*b^3*d*x^4*\text{Cosh}[c - (a*d)/b]*\text{SinhIntegral}[d*(a/b + x)] + 12*a^2*b^2*x^2*\text{Sinh}[c - (a*d)/b]*\text{SinhIntegral}[d*(a/b + x)]$



$$+ a^4 d^2 x^2 \operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] + 24*a*b^3*x^3*\operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] + 2*a^3*b*d^2*x^3*\operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] + 12*b^4*x^4*\operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] + a^2*b^2*d^2*x^4*\operatorname{Sinh}[c - (a*d)/b] \operatorname{SinhIntegral}[d*(a/b + x)] / (2*a^5*x^2*(a + b*x)^2)$$

**Maple [B]** time = 0.082, size = 760, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/x^3/(b*x+a)^3,x)`

[Out]  $\frac{1}{4}d^3\exp(-d*x-c)/a^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+3*d^2*\exp(-d*x-c)/a^4*x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^3+1/4*d^3*\exp(-d*x-c)/a/x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)+9/2*d^2*\exp(-d*x-c)/a^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^2+d^2*\exp(-d*x-c)/a^2/x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b-1/4*d^2*\exp(-d*x-c)/a/x^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-1/4*d^2/a^3*\exp(-c)*\operatorname{Ei}(1,d*x)-3/2*d/a^4*\exp(-c)*\operatorname{Ei}(1,d*x)*b-3/a^5*\exp(-c)*\operatorname{Ei}(1,d*x)*b^2+1/4*d^2/a^3*\exp((a*d-b*c)/b)*\operatorname{Ei}(1,d*x+c+(a*d-b*c)/b)-3/2*d/a^4*\exp((a*d-b*c)/b)*\operatorname{Ei}(1,d*x+c+(a*d-b*c)/b)*b+3/a^5*\exp((a*d-b*c)/b)*\operatorname{Ei}(1,d*x+c+(a*d-b*c)/b)*b^2-3/a^5*b^2*\exp(c)*\operatorname{Ei}(1,-d*x)-1/4/a^3/x^2*\exp(d*x+c)-1/4*d/a^3/x*\exp(d*x+c)-1/4*d^2/a^3*\exp(c)*\operatorname{Ei}(1,-d*x)+3/2/a^4*b/x*\exp(d*x+c)+3/2*d/a^4*b*\exp(c)*\operatorname{Ei}(1,-d*x)+1/4*d^2/a^3*\exp(d*x+c)/(1/b*d*a+d*x)^2+1/4*d^2/a^3*\exp(d*x+c)/(1/b*d*a+d*x)+1/4*d^2/a^3*\exp(-(a*d-b*c)/b)*\operatorname{Ei}(1,-d*x-c-(a*d-b*c)/b)+3/2*d/a^4*b*\exp(d*x+c)/(1/b*d*a+d*x)+3/2*d/a^4*b*\exp(-(a*d-b*c)/b)*\operatorname{Ei}(1,-d*x-c-(a*d-b*c)/b)+3*b^2/a^5*\exp(-(a*d-b*c)/b)*\operatorname{Ei}(1,-d*x-c-(a*d-b*c)/b)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx+a)^3 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="maxima")`

[Out] `integrate(cosh(d*x + c)/((b*x + a)^3*x^3), x)`

---

**Fricas [B]** time = 2.18189, size = 1845, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^3/(b\*x+a)^3,x, algorithm="fricas")

[Out]  $\frac{1}{4} * (2 * (12 * a * b^3 * x^3 + 18 * a^2 * b^2 * x^2 + 4 * a^3 * b * x - a^4) * \cosh(d * x + c) + ((a^2 * b^2 * d^2 - 6 * a * b^3 * d + 12 * b^4) * x^4 + 2 * (a^3 * b * d^2 - 6 * a^2 * b^2 * d + 12 * a * b^3) * x^3 + (a^4 * d^2 - 6 * a^3 * b * d + 12 * a^2 * b^2) * x^2) * \text{Ei}(d * x) + ((a^2 * b^2 * d^2 + 6 * a * b^3 * d + 12 * b^4) * x^4 + 2 * (a^3 * b * d^2 + 6 * a^2 * b^2 * d + 12 * a * b^3) * x^3 + (a^4 * d^2 + 6 * a^3 * b * d + 12 * a^2 * b^2) * x^2) * \text{Ei}(-d * x)) * \cosh(c) - (((a^2 * b^2 * d^2 + 6 * a * b^3 * d + 12 * b^4) * x^4 + 2 * (a^3 * b * d^2 + 6 * a^2 * b^2 * d + 12 * a * b^3) * x^3 + (a^4 * d^2 + 6 * a^3 * b * d + 12 * a^2 * b^2) * x^2) * \text{Ei}((b * d * x + a * d) / b) + ((a^2 * b^2 * d^2 - 6 * a * b^3 * d + 12 * b^4) * x^4 + 2 * (a^3 * b * d^2 - 6 * a^2 * b^2 * d + 12 * a * b^3) * x^3 + (a^4 * d^2 - 6 * a^3 * b * d + 12 * a^2 * b^2) * x^2) * \text{Ei}(-(b * d * x + a * d) / b)) * \cosh(-(b * c - a * d) / b) - 2 * (a^3 * b * d * x^2 + a^4 * d * x) * \sinh(d * x + c) + (((a^2 * b^2 * d^2 - 6 * a * b^3 * d + 12 * b^4) * x^4 + 2 * (a^3 * b * d^2 - 6 * a^2 * b^2 * d + 12 * a * b^3) * x^3 + (a^4 * d^2 - 6 * a^3 * b * d + 12 * a^2 * b^2) * x^2) * \text{Ei}(d * x) - ((a^2 * b^2 * d^2 + 6 * a * b^3 * d + 12 * b^4) * x^4 + 2 * (a^3 * b * d^2 + 6 * a^2 * b^2 * d + 12 * a * b^3) * x^3 + (a^4 * d^2 + 6 * a^3 * b * d + 12 * a^2 * b^2) * x^2) * \text{Ei}(-d * x)) * \sinh(c) + (((a^2 * b^2 * d^2 + 6 * a * b^3 * d + 12 * b^4) * x^4 + 2 * (a^3 * b * d^2 + 6 * a^2 * b^2 * d + 12 * a * b^3) * x^3 + (a^4 * d^2 + 6 * a^3 * b * d + 12 * a^2 * b^2) * x^2) * \text{Ei}((b * d * x + a * d) / b) - ((a^2 * b^2 * d^2 - 6 * a * b^3 * d + 12 * b^4) * x^4 + 2 * (a^3 * b * d^2 - 6 * a^2 * b^2 * d + 12 * a * b^3) * x^3 + (a^4 * d^2 - 6 * a^3 * b * d + 12 * a^2 * b^2) * x^2) * \text{Ei}(-(b * d * x + a * d) / b)) * \sinh(-(b * c - a * d) / b)) / (a^5 * b^2 * x^4 + 2 * a^6 * b * x^3 + a^7 * x^2)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x\*\*3/(b\*x+a)\*\*3,x)

[Out] Timed out

---

**Giac [B]** time = 1.31946, size = 1578, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^3/(b\*x+a)^3,x, algorithm="giac")

[Out]  $\frac{1}{4}*(a^2*b^2*d^2*x^4*Ei(-d*x)*e^{-c} - a^2*b^2*d^2*x^4*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} + a^2*b^2*d^2*x^4*Ei(d*x)*e^c - a^2*b^2*d^2*x^4*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} + 2*a^3*b*d^2*x^3*Ei(-d*x)*e^{-c} + 6*a*b^3*d*x^4*Ei(-d*x)*e^{-c} - 2*a^3*b*d^2*x^3*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} - 6*a*b^3*d*x^4*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} + 2*a^3*b*d^2*x^3*Ei(d*x)*e^c - 6*a*b^3*d*x^4*Ei(d*x)*e^c - 2*a^3*b*d^2*x^3*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} + 6*a*b^3*d*x^4*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} + a^4*d^2*x^2*Ei(-d*x)*e^{-c} + 12*a^2*b^2*d*x^3*Ei(-d*x)*e^{-c} + 12*b^4*x^4*Ei(-d*x)*e^{-c} - a^4*d^2*x^2*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} - 12*a^2*b^2*d*x^3*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} - 12*b^4*x^4*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} + a^4*d^2*x^2*Ei(d*x)*e^c - 12*a^2*b^2*d*x^3*Ei(d*x)*e^c + 12*b^4*x^4*Ei(d*x)*e^c - a^4*d^2*x^2*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} + 12*a^2*b^2*d*x^3*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} - 12*b^4*x^4*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} + 6*a^3*b*d*x^2*Ei(-d*x)*e^{-c} + 24*a*b^3*x^3*Ei(-d*x)*e^{-c} - 6*a^3*b*d*x^2*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} - 24*a*b^3*x^3*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} - 6*a^3*b*d*x^2*Ei(d*x)*e^c + 24*a*b^3*x^3*Ei(d*x)*e^c + 6*a^3*b*d*x^2*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} - 24*a*b^3*x^3*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} - a^3*b*d*x^2*e^{d*x + c} + 12*a*b^3*x^3*e^{d*x + c} + a^3*b*d*x^2*e^{-d*x - c} + 12*a*b^3*x^3*e^{-d*x - c} + 12*a^2*b^2*x^2*Ei(-d*x)*e^{-c} - 12*a^2*b^2*x^2*Ei((b*d*x + a*d)/b)*e^{c - a*d/b} + 12*a^2*b^2*x^2*Ei(d*x)*e^c - 12*a^2*b^2*x^2*Ei(-(b*d*x + a*d)/b)*e^{-c + a*d/b} - a^4*d*x*e^{d*x + c} + 18*a^2*b^2*x^2*e^{d*x + c} + a^4*d*x*e^{-d*x - c} + 18*a^2*b^2*x^2*e^{-d*x - c} + 4*a^3*b*x*e^{d*x + c} + 4*a^3*b*x*e^{-d*x - c} - a^4*e^{d*x + c} - a^4*e^{-d*x - c})/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)$

### 3.40 $\int x^3 (a + bx^2) \cosh(c + dx) dx$

**Optimal.** Leaf size=139

$$-\frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{6a \cosh(c + dx)}{d^4} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{20bx^3 \sinh(c + dx)}{d^3} - \frac{5bx^4 \cosh(c + dx)}{d^2}$$

[Out]  $(-120*b*Cosh[c + d*x])/d^6 - (6*a*Cosh[c + d*x])/d^4 - (60*b*x^2*Cosh[c + d*x])/d^4 - (3*a*x^2*Cosh[c + d*x])/d^2 - (5*b*x^4*Cosh[c + d*x])/d^2 + (120*b*x*Sinh[c + d*x])/d^5 + (6*a*x*Sinh[c + d*x])/d^3 + (20*b*x^3*Sinh[c + d*x])/d^3 + (a*x^3*Sinh[c + d*x])/d + (b*x^5*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.255416, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {5287, 3296, 2638}

$$-\frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{6a \cosh(c + dx)}{d^4} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{20bx^3 \sinh(c + dx)}{d^3} - \frac{5bx^4 \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(a + b*x^2)*Cosh[c + d*x], x]$

[Out]  $(-120*b*Cosh[c + d*x])/d^6 - (6*a*Cosh[c + d*x])/d^4 - (60*b*x^2*Cosh[c + d*x])/d^4 - (3*a*x^2*Cosh[c + d*x])/d^2 - (5*b*x^4*Cosh[c + d*x])/d^2 + (120*b*x*Sinh[c + d*x])/d^5 + (6*a*x*Sinh[c + d*x])/d^3 + (20*b*x^3*Sinh[c + d*x])/d^3 + (a*x^3*Sinh[c + d*x])/d + (b*x^5*Sinh[c + d*x])/d$

#### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{GtQ}[m, 0]$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int x^3 (a + bx^2) \cosh(c + dx) dx &= \int (ax^3 \cosh(c + dx) + bx^5 \cosh(c + dx)) dx \\
 &= a \int x^3 \cosh(c + dx) dx + b \int x^5 \cosh(c + dx) dx \\
 &= \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^5 \sinh(c + dx)}{d} - \frac{(3a) \int x^2 \sinh(c + dx) dx}{d} - \frac{(5b) \int x^4 \sinh(c + dx) dx}{d} \\
 &= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^5 \sinh(c + dx)}{d} + \frac{(3a) \int x \sinh(c + dx) dx}{d} + \frac{(5b) \int x^3 \sinh(c + dx) dx}{d} \\
 &= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} + \frac{20bx^3 \sinh(c + dx)}{d^3} + \frac{3a \cosh(c + dx)}{d^2} + \frac{5bx^2 \cosh(c + dx)}{d^2} \\
 &= -\frac{6a \cosh(c + dx)}{d^4} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} + \frac{20bx^3 \sinh(c + dx)}{d^3} \\
 &= -\frac{6a \cosh(c + dx)}{d^4} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} + \frac{20bx^3 \sinh(c + dx)}{d^3} \\
 &= -\frac{120b \cosh(c + dx)}{d^6} - \frac{6a \cosh(c + dx)}{d^4} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.149123, size = 92, normalized size = 0.66

$$\frac{dx \left( ad^2 (d^2 x^2 + 6) + b (d^4 x^4 + 20d^2 x^2 + 120) \right) \sinh(c + dx) - \left( 3ad^2 (d^2 x^2 + 2) + 5b (d^4 x^4 + 12d^2 x^2 + 24) \right) \cosh(c + dx)}{d^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*x^2)\*Cosh[c + d\*x], x]

[Out] (-(3\*a\*d^2\*(2 + d^2\*x^2) + 5\*b\*(24 + 12\*d^2\*x^2 + d^4\*x^4))\*Cosh[c + d\*x]) + d\*x\*(a\*d^2\*(6 + d^2\*x^2) + b\*(120 + 20\*d^2\*x^2 + d^4\*x^4))\*Sinh[c + d\*x])/d^6

**Maple [B]** time = 0.009, size = 447, normalized size = 3.2

$$\frac{1}{d^4} \left( \frac{b \left( (dx + c)^5 \sinh(dx + c) - 5(dx + c)^4 \cosh(dx + c) + 20(dx + c)^3 \sinh(dx + c) - 60(dx + c)^2 \cosh(dx + c) + 120(dx + c) \sinh(dx + c) - 120 \cosh(dx + c) \right)}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)*cosh(d*x+c),x)`

[Out]  $\frac{1}{d^4} \left( \frac{1}{d^2} b \left( (d*x+c)^5 \sinh(d*x+c) - 5(d*x+c)^4 \cosh(d*x+c) + 20(d*x+c)^3 \sinh(d*x+c) - 60(d*x+c)^2 \cosh(d*x+c) + 120(d*x+c) \sinh(d*x+c) - 120 \cosh(d*x+c) \right) - 5 \frac{1}{d^2} b c \left( (d*x+c)^4 \sinh(d*x+c) - 4(d*x+c)^3 \cosh(d*x+c) + 12(d*x+c)^2 \sinh(d*x+c) - 24(d*x+c) \cosh(d*x+c) + 24 \sinh(d*x+c) \right) + 10 \frac{1}{d^2} b c^2 \left( (d*x+c)^3 \sinh(d*x+c) - 3(d*x+c)^2 \cosh(d*x+c) + 6(d*x+c) \sinh(d*x+c) - 6 \cosh(d*x+c) \right) - 10 \frac{1}{d^2} b c^3 \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) + 5 \frac{1}{d^2} b c^4 \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) - \frac{1}{d^2} b c^5 \sinh(d*x+c) + a \left( (d*x+c)^3 \sinh(d*x+c) - 3(d*x+c)^2 \cosh(d*x+c) + 6(d*x+c) \sinh(d*x+c) - 6 \cosh(d*x+c) \right) - 3 a c \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) + 3 a c^2 \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) - a c^3 \sinh(d*x+c) \right)$

**Maxima [A]** time = 1.07114, size = 338, normalized size = 2.43

$$-\frac{1}{24} d \left( \frac{3(d^4 x^4 e^c - 4d^3 x^3 e^c + 12d^2 x^2 e^c - 24dx e^c + 24e^c) a e^{(dx)}}{d^5} + \frac{3(d^4 x^4 + 4d^3 x^3 + 12d^2 x^2 + 24dx + 24) a e^{(-dx-c)}}{d^5} + \frac{2}{d^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")`

[Out]  $-\frac{1}{24} d \left( \frac{3(d^4 x^4 e^c - 4d^3 x^3 e^c + 12d^2 x^2 e^c - 24dx e^c + 24e^c) a e^{(dx)}}{d^5} + \frac{3(d^4 x^4 + 4d^3 x^3 + 12d^2 x^2 + 24dx + 24) a e^{(-dx-c)}}{d^5} + \frac{2}{d^5} \right)$

**Fricas [A]** time = 2.01632, size = 212, normalized size = 1.53

$$\frac{(5bd^4x^4 + 6ad^2 + 3(ad^4 + 20bd^2)x^2 + 120b) \cosh(dx + c) - (bd^5x^5 + (ad^5 + 20bd^3)x^3 + 6(ad^3 + 20bd)x) \sinh(dx + c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")`

[Out]  $-\frac{((5*b*d^4*x^4 + 6*a*d^2 + 3*(a*d^4 + 20*b*d^2))*x^2 + 120*b)*\cosh(d*x + c) - (b*d^5*x^5 + (a*d^5 + 20*b*d^3))*x^3 + 6*(a*d^3 + 20*b*d)*x*\sinh(d*x + c)}{d^6}$

**Sympy [A]** time = 5.48512, size = 168, normalized size = 1.21

$$\left\{ \begin{array}{l} \frac{ax^3 \sinh(c+dx)}{d} - \frac{3ax^2 \cosh(c+dx)}{d^2} + \frac{6ax \sinh(c+dx)}{d^3} - \frac{6a \cosh(c+dx)}{d^4} + \frac{bx^5 \sinh(c+dx)}{d} - \frac{5bx^4 \cosh(c+dx)}{d^2} + \frac{20bx^3 \sinh(c+dx)}{d^3} - \frac{60bx^2 \cosh(c+dx)}{d^4} \\ \left( \frac{ax^4}{4} + \frac{bx^6}{6} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)*cosh(d*x+c),x)`

[Out] `Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*sinh(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**5*sinh(c + d*x)/d - 5*b*x**4*cosh(c + d*x)/d**2 + 20*b*x**3*sinh(c + d*x)/d**3 - 60*b*x**2*cosh(c + d*x)/d**4 + 120*b*x*sinh(c + d*x)/d**5 - 120*b*cosh(c + d*x)/d**6, Ne(d, 0)), ((a*x**4/4 + b*x**6/6)*cosh(c), True))`

**Giac [A]** time = 1.20411, size = 235, normalized size = 1.69

$$\frac{(bd^5x^5 + ad^5x^3 - 5bd^4x^4 - 3ad^4x^2 + 20bd^3x^3 + 6ad^3x - 60bd^2x^2 - 6ad^2 + 120bdx - 120b)e^{(dx+c)}}{2d^6} - \frac{(bd^5x^5 + ad^5x^3)}{2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")`

[Out]  $\frac{1}{2}*(b*d^5*x^5 + a*d^5*x^3 - 5*b*d^4*x^4 - 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*a*d^3*x - 60*b*d^2*x^2 - 6*a*d^2 + 120*b*d*x - 120*b)*e^{(d*x + c)}/d^6 - \frac{1}{2}*(b*d^5*x^5 + a*d^5*x^3 + 5*b*d^4*x^4 + 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*a*d^3*x + 60*b*d^2*x^2 + 6*a*d^2 + 120*b*d*x + 120*b)*e^{(-d*x - c)}/d^6$

### 3.41 $\int x^2 (a + bx^2) \cosh(c + dx) dx$

**Optimal.** Leaf size=109

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{24b \sinh(c + dx)}{d^5}$$

[Out]  $(-24*b*x*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (4*b*x^3*Cosh[c + d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (2*a*Sinh[c + d*x])/d^3 + (12*b*x^2*Sinh[c + d*x])/d^3 + (a*x^2*Sinh[c + d*x])/d + (b*x^4*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.187407, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {5287, 3296, 2637}

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{24b \sinh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x^2)\*Cosh[c + d\*x], x]

[Out]  $(-24*b*x*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (4*b*x^3*Cosh[c + d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (2*a*Sinh[c + d*x])/d^3 + (12*b*x^2*Sinh[c + d*x])/d^3 + (a*x^2*Sinh[c + d*x])/d + (b*x^4*Sinh[c + d*x])/d$

#### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]



Rubi steps

$$\begin{aligned}
\int x^2 (a + bx^2) \cosh(c + dx) dx &= \int (ax^2 \cosh(c + dx) + bx^4 \cosh(c + dx)) dx \\
&= a \int x^2 \cosh(c + dx) dx + b \int x^4 \cosh(c + dx) dx \\
&= \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} - \frac{(2a) \int x \sinh(c + dx) dx}{d} - \frac{(4b) \int x^3 \sinh(c + dx) dx}{d} \\
&= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} + \frac{2a}{d^3} \\
&= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{12bx^2 \sinh(c + dx)}{d^3} + \frac{2a}{d^3} \\
&= -\frac{24bx \cosh(c + dx)}{d^4} - \frac{2ax \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{12bx^2 \sinh(c + dx)}{d^3} \\
&= -\frac{24bx \cosh(c + dx)}{d^4} - \frac{2ax \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{24b \sinh(c + dx)}{d^5} + \frac{2a}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.129127, size = 74, normalized size = 0.68

$$\frac{(ad^2(d^2x^2 + 2) + b(d^4x^4 + 12d^2x^2 + 24)) \sinh(c + dx) - 2dx(ad^2 + 2b(d^2x^2 + 6)) \cosh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x^2)\*Cosh[c + d\*x], x]

[Out] (-2\*d\*x\*(a\*d^2 + 2\*b\*(6 + d^2\*x^2))\*Cosh[c + d\*x] + (a\*d^2\*(2 + d^2\*x^2) + b\*(24 + 12\*d^2\*x^2 + d^4\*x^4))\*Sinh[c + d\*x])/d^5

**Maple [B]** time = 0.009, size = 298, normalized size = 2.7

$$\frac{1}{d^3} \left( \frac{b((dx + c)^4 \sinh(dx + c) - 4(dx + c)^3 \cosh(dx + c) + 12(dx + c)^2 \sinh(dx + c) - 24(dx + c) \cosh(dx + c) + 24 \sinh(dx + c))}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)\*cosh(d\*x+c), x)

[Out]  $1/d^3*(1/d^2*b*((d*x+c)^4*\sinh(d*x+c)-4*(d*x+c)^3*\cosh(d*x+c)+12*(d*x+c)^2*\sinh(d*x+c)-24*(d*x+c)*\cosh(d*x+c)+24*\sinh(d*x+c))-4/d^2*b*c*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))+6/d^2*b*c^2*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-4/d^2*b*c^3*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+1/d^2*b*c^4*\sinh(d*x+c)+a*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))-2*a*c*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))+a*c^2*\sinh(d*x+c))$

**Maxima [A]** time = 1.03039, size = 289, normalized size = 2.65

$$-\frac{1}{30}d\left(\frac{5(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)ae^{(dx)}}{d^4} + \frac{5(d^3x^3 + 3d^2x^2 + 6dx + 6)ae^{(-dx-c)}}{d^4} + \frac{3(d^5x^5e^c - 5d^4x^4e^c + 20d^3x^3e^c - 30d^2x^2e^c + 60dxe^c - 60e^c)be^{(dx)}}{d^6} + \frac{3(d^5x^5 + 5d^4x^4 + 20d^3x^3 + 60d^2x^2 + 120dx + 120)be^{(-dx-c)}}{d^6} + \frac{1}{15}(3bx^5 + 5ax^3)*\cosh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)\*cosh(d\*x+c),x, algorithm="maxima")

[Out]  $-1/30*d*(5*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^{(d*x)}/d^4 + 5*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^{(-d*x - c)}/d^4 + 3*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^{(d*x)}/d^6 + 3*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^{(-d*x - c)}/d^6 + 1/15*(3*b*x^5 + 5*a*x^3)*\cosh(d*x + c)$

**Fricas [A]** time = 2.02551, size = 174, normalized size = 1.6

$$\frac{2(2bd^3x^3 + (ad^3 + 12bd)x)\cosh(dx+c) - (bd^4x^4 + 2ad^2 + (ad^4 + 12bd^2)x^2 + 24b)\sinh(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)\*cosh(d\*x+c),x, algorithm="fricas")

[Out]  $-(2*(2*b*d^3*x^3 + (a*d^3 + 12*b*d)*x)*\cosh(d*x + c) - (b*d^4*x^4 + 2*a*d^2 + (a*d^4 + 12*b*d^2)*x^2 + 24*b)*\sinh(d*x + c))/d^5$

**Sympy [A]** time = 2.61537, size = 134, normalized size = 1.23

$$\left\{ \begin{array}{l} \frac{ax^2 \sinh(c+dx)}{d} - \frac{2ax \cosh(c+dx)}{d^2} + \frac{2a \sinh(c+dx)}{d^3} + \frac{bx^4 \sinh(c+dx)}{d} - \frac{4bx^3 \cosh(c+dx)}{d^2} + \frac{12bx^2 \sinh(c+dx)}{d^3} - \frac{24bx \cosh(c+dx)}{d^4} + \frac{24b \sinh(c+dx)}{d^5} \\ \left( \frac{ax^3}{3} + \frac{bx^5}{5} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)\*cosh(d\*x+c),x)

[Out] Piecewise((a\*x\*\*2\*sinh(c + d\*x)/d - 2\*a\*x\*cosh(c + d\*x)/d\*\*2 + 2\*a\*sinh(c + d\*x)/d\*\*3 + b\*x\*\*4\*sinh(c + d\*x)/d - 4\*b\*x\*\*3\*cosh(c + d\*x)/d\*\*2 + 12\*b\*x\*\*2\*sinh(c + d\*x)/d\*\*3 - 24\*b\*x\*cosh(c + d\*x)/d\*\*4 + 24\*b\*sinh(c + d\*x)/d\*\*5, Ne(d, 0)), ((a\*x\*\*3/3 + b\*x\*\*5/5)\*cosh(c), True))

**Giac [A]** time = 1.19823, size = 186, normalized size = 1.71

$$\frac{(bd^4x^4 + ad^4x^2 - 4bd^3x^3 - 2ad^3x + 12bd^2x^2 + 2ad^2 - 24bdx + 24b)e^{(dx+c)}}{2d^5} - \frac{(bd^4x^4 + ad^4x^2 + 4bd^3x^3 + 2ad^3x + 12bd^2x^2 + 2ad^2 + 24bdx + 24b)e^{(-dx-c)}}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)\*cosh(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(b\*d^4\*x^4 + a\*d^4\*x^2 - 4\*b\*d^3\*x^3 - 2\*a\*d^3\*x + 12\*b\*d^2\*x^2 + 2\*a\*d^2 - 24\*b\*d\*x + 24\*b)\*e^(d\*x + c)/d^5 - 1/2\*(b\*d^4\*x^4 + a\*d^4\*x^2 + 4\*b\*d^3\*x^3 + 2\*a\*d^3\*x + 12\*b\*d^2\*x^2 + 2\*a\*d^2 + 24\*b\*d\*x + 24\*b)\*e^(-d\*x - c)/d^5

### 3.42 $\int x(a + bx^2) \cosh(c + dx) dx$

**Optimal.** Leaf size=79

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{6b \cosh(c + dx)}{d^4} + \frac{bx^3 \sinh(c + dx)}{d}$$

[Out]  $(-6*b*Cosh[c + d*x])/d^4 - (a*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*x])/d^2 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x*Sinh[c + d*x])/d + (b*x^3*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.121082, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {5287, 3296, 2638}

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{6b \cosh(c + dx)}{d^4} + \frac{bx^3 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x^2)\*Cosh[c + d\*x], x]

[Out]  $(-6*b*Cosh[c + d*x])/d^4 - (a*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*x])/d^2 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x*Sinh[c + d*x])/d + (b*x^3*Sinh[c + d*x])/d$

#### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] :> -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x(a + bx^2) \cosh(c + dx) dx &= \int (ax \cosh(c + dx) + bx^3 \cosh(c + dx)) dx \\
&= a \int x \cosh(c + dx) dx + b \int x^3 \cosh(c + dx) dx \\
&= \frac{ax \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} - \frac{a \int \sinh(c + dx) dx}{d} - \frac{(3b) \int x^2 \sinh(c + dx) dx}{d} \\
&= -\frac{a \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} + \frac{(6b) \int x \sinh(c + dx) dx}{d} \\
&= -\frac{a \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{ax \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} \\
&= -\frac{6b \cosh(c + dx)}{d^4} - \frac{a \cosh(c + dx)}{d^2} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{ax \sinh(c + dx)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.0997394, size = 57, normalized size = 0.72

$$\frac{dx(ad^2 + b(d^2x^2 + 6)) \sinh(c + dx) - (ad^2 + 3b(d^2x^2 + 2)) \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^2)\*Cosh[c + d\*x], x]

[Out] (-((a\*d^2 + 3\*b\*(2 + d^2\*x^2))\*Cosh[c + d\*x]) + d\*x\*(a\*d^2 + b\*(6 + d^2\*x^2))\*Sinh[c + d\*x])/d^4

**Maple [B]** time = 0.007, size = 183, normalized size = 2.3

$$\frac{1}{d^2} \left( \frac{b((dx + c)^3 \sinh(dx + c) - 3(dx + c)^2 \cosh(dx + c) + 6(dx + c) \sinh(dx + c) - 6 \cosh(dx + c))}{d^2} - 3 \frac{cb((dx + c)^2}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)\*cosh(d\*x+c), x)

[Out] 1/d^2\*(1/d^2\*b\*((d\*x+c)^3\*sinh(d\*x+c)-3\*(d\*x+c)^2\*cosh(d\*x+c)+6\*(d\*x+c)\*sinh(d\*x+c)-6\*cosh(d\*x+c))-3/d^2\*b\*c\*((d\*x+c)^2\*sinh(d\*x+c)-2\*(d\*x+c)\*cosh(d\*x+c)+2\*sinh(d\*x+c))+3/d^2\*b\*c^2\*((d\*x+c)\*sinh(d\*x+c)-cosh(d\*x+c))+a\*((d\*x+c)

$$*\sinh(d*x+c)-\cosh(d*x+c))-1/d^2*b*c^3*\sinh(d*x+c)-c*a*\sinh(d*x+c))$$

**Maxima [B]** time = 1.08556, size = 288, normalized size = 3.65

$$\frac{(bx^2 + a)^2 \cosh(dx + c)}{4b} - \frac{\left( \frac{a^2 e^{(dx+c)}}{d} + \frac{a^2 e^{(-dx-c)}}{d} + \frac{2(d^2 x^2 e^c - 2 dx e^c + 2 e^c) a b e^{(dx)}}{d^3} + \frac{2(d^2 x^2 + 2 dx + 2) a b e^{(-dx-c)}}{d^3} + \frac{(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 24 d x e^c + 24 e^c) b^2 e^{(dx)}}{d^5} + \frac{(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 24 d x e^c + 24 e^c) b^2 e^{(-dx-c)}}{d^5} \right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)\*cosh(d\*x+c),x, algorithm="maxima")

[Out] 1/4\*(b\*x^2 + a)^2\*cosh(d\*x + c)/b - 1/8\*(a^2\*e^(d\*x + c)/d + a^2\*e^(-d\*x - c)/d + 2\*(d^2\*x^2\*e^c - 2\*d\*x\*e^c + 2\*e^c)\*a\*b\*e^(d\*x)/d^3 + 2\*(d^2\*x^2 + 2\*d\*x + 2)\*a\*b\*e^(-d\*x - c)/d^3 + (d^4\*x^4\*e^c - 4\*d^3\*x^3\*e^c + 12\*d^2\*x^2\*e^c - 24\*d\*x\*e^c + 24\*e^c)\*b^2\*e^(d\*x)/d^5 + (d^4\*x^4 + 4\*d^3\*x^3 + 12\*d^2\*x^2 + 24\*d\*x + 24)\*b^2\*e^(-d\*x - c)/d^5)\*d/b

**Fricas [A]** time = 1.98437, size = 132, normalized size = 1.67

$$\frac{(3bd^2x^2 + ad^2 + 6b) \cosh(dx + c) - (bd^3x^3 + (ad^3 + 6bd)x) \sinh(dx + c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)\*cosh(d\*x+c),x, algorithm="fricas")

[Out] -((3\*b\*d^2\*x^2 + a\*d^2 + 6\*b)\*cosh(d\*x + c) - (b\*d^3\*x^3 + (a\*d^3 + 6\*b\*d)\*x)\*sinh(d\*x + c))/d^4

**Sympy [A]** time = 1.31972, size = 99, normalized size = 1.25

$$\begin{cases} \frac{ax \sinh(c+dx)}{d} - \frac{a \cosh(c+dx)}{d^2} + \frac{bx^3 \sinh(c+dx)}{d} - \frac{3bx^2 \cosh(c+dx)}{d^2} + \frac{6bx \sinh(c+dx)}{d^3} - \frac{6b \cosh(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left( \frac{ax^2}{2} + \frac{bx^4}{4} \right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*2+a)\*cosh(d\*x+c),x)

[Out] Piecewise((a\*x\*sinh(c + d\*x)/d - a\*cosh(c + d\*x)/d\*\*2 + b\*x\*\*3\*sinh(c + d\*x)/d - 3\*b\*x\*\*2\*cosh(c + d\*x)/d\*\*2 + 6\*b\*x\*sinh(c + d\*x)/d\*\*3 - 6\*b\*cosh(c + d\*x)/d\*\*4, Ne(d, 0)), ((a\*x\*\*2/2 + b\*x\*\*4/4)\*cosh(c), True))

**Giac [A]** time = 1.17752, size = 136, normalized size = 1.72

$$\frac{(bd^3x^3 + ad^3x - 3bd^2x^2 - ad^2 + 6bdx - 6b)e^{(dx+c)}}{2d^4} - \frac{(bd^3x^3 + ad^3x + 3bd^2x^2 + ad^2 + 6bdx + 6b)e^{(-dx-c)}}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)\*cosh(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(b\*d^3\*x^3 + a\*d^3\*x - 3\*b\*d^2\*x^2 - a\*d^2 + 6\*b\*d\*x - 6\*b)\*e^(d\*x + c)/d^4 - 1/2\*(b\*d^3\*x^3 + a\*d^3\*x + 3\*b\*d^2\*x^2 + a\*d^2 + 6\*b\*d\*x + 6\*b)\*e^(-d\*x - c)/d^4

### 3.43 $\int (a + bx^2) \cosh(c + dx) dx$

**Optimal.** Leaf size=51

$$\frac{a \sinh(c + dx)}{d} + \frac{2b \sinh(c + dx)}{d^3} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{bx^2 \sinh(c + dx)}{d}$$

[Out]  $(-2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/d^3 + (a*Sinh[c + d*x])/d + (b*x^2*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.0667648, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {5277, 2637, 3296}

$$\frac{a \sinh(c + dx)}{d} + \frac{2b \sinh(c + dx)}{d^3} - \frac{2bx \cosh(c + dx)}{d^2} + \frac{bx^2 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)\*Cosh[c + d\*x],x]

[Out]  $(-2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/d^3 + (a*Sinh[c + d*x])/d + (b*x^2*Sinh[c + d*x])/d$

#### Rule 5277

Int[Cosh[(c\_.) + (d\_.)\*(x\_.)]\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[p, 0]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] :> -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rubi steps



$$\begin{aligned}
\int (a + bx^2) \cosh(c + dx) dx &= \int (a \cosh(c + dx) + bx^2 \cosh(c + dx)) dx \\
&= a \int \cosh(c + dx) dx + b \int x^2 \cosh(c + dx) dx \\
&= \frac{a \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d} - \frac{(2b) \int x \sinh(c + dx) dx}{d} \\
&= -\frac{2bx \cosh(c + dx)}{d^2} + \frac{a \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d} + \frac{(2b) \int \cosh(c + dx) dx}{d^2} \\
&= -\frac{2bx \cosh(c + dx)}{d^2} + \frac{2b \sinh(c + dx)}{d^3} + \frac{a \sinh(c + dx)}{d} + \frac{bx^2 \sinh(c + dx)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.0667749, size = 40, normalized size = 0.78

$$\frac{(ad^2 + b(d^2x^2 + 2)) \sinh(c + dx) - 2bdx \cosh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)\*Cosh[c + d\*x],x]

[Out] (-2\*b\*d\*x\*Cosh[c + d\*x] + (a\*d^2 + b\*(2 + d^2\*x^2))\*Sinh[c + d\*x])/d^3

**Maple [A]** time = 0.008, size = 97, normalized size = 1.9

$$\frac{1}{d} \left( \frac{b((dx + c)^2 \sinh(dx + c) - 2(dx + c) \cosh(dx + c) + 2 \sinh(dx + c))}{d^2} - 2 \frac{cb((dx + c) \sinh(dx + c) - \cosh(dx + c))}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)\*cosh(d\*x+c),x)

[Out] 1/d\*(1/d^2\*b\*((d\*x+c)^2\*sinh(d\*x+c)-2\*(d\*x+c)\*cosh(d\*x+c)+2\*sinh(d\*x+c))-2/d^2\*b\*c\*((d\*x+c)\*sinh(d\*x+c)-cosh(d\*x+c))+1/d^2\*b\*c^2\*sinh(d\*x+c)+a\*sinh(d\*x+c))

**Maxima [A]** time = 1.01897, size = 116, normalized size = 2.27

$$\frac{ae^{(dx+c)}}{2d} - \frac{ae^{(-dx-c)}}{2d} + \frac{(d^2x^2e^c - 2dxe^c + 2e^c)be^{(dx)}}{2d^3} - \frac{(d^2x^2 + 2dx + 2)be^{(-dx-c)}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c),x, algorithm="maxima")

[Out] 1/2\*a\*e^(d\*x + c)/d - 1/2\*a\*e^(-d\*x - c)/d + 1/2\*(d^2\*x^2\*e^c - 2\*d\*x\*e^c + 2\*e^c)\*b\*e^(d\*x)/d^3 - 1/2\*(d^2\*x^2 + 2\*d\*x + 2)\*b\*e^(-d\*x - c)/d^3

**Fricas [A]** time = 1.9949, size = 97, normalized size = 1.9

$$\frac{2 b d x \cosh (d x + c) - \left( b d^2 x^2 + a d^2 + 2 b \right) \sinh (d x + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c),x, algorithm="fricas")

[Out] -(2\*b\*d\*x\*cosh(d\*x + c) - (b\*d^2\*x^2 + a\*d^2 + 2\*b)\*sinh(d\*x + c))/d^3

**Sympy [A]** time = 0.674157, size = 65, normalized size = 1.27

$$\begin{cases} \frac{a \sinh (c+d x)}{d} + \frac{b x^2 \sinh (c+d x)}{d} - \frac{2 b x \cosh (c+d x)}{d^2} + \frac{2 b \sinh (c+d x)}{d^3} & \text{for } d \neq 0 \\ \left( a x + \frac{b x^3}{3} \right) \cosh (c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*cosh(d\*x+c),x)

[Out] Piecewise((a\*sinh(c + d\*x)/d + b\*x\*\*2\*sinh(c + d\*x)/d - 2\*b\*x\*cosh(c + d\*x)/d\*\*2 + 2\*b\*sinh(c + d\*x)/d\*\*3, Ne(d, 0)), ((a\*x + b\*x\*\*3/3)\*cosh(c), True))

**Giac [A]** time = 1.15273, size = 95, normalized size = 1.86

$$\frac{(bd^2x^2 + ad^2 - 2bdx + 2b)e^{(dx+c)}}{2d^3} - \frac{(bd^2x^2 + ad^2 + 2bdx + 2b)e^{(-dx-c)}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(b\*d^2\*x^2 + a\*d^2 - 2\*b\*d\*x + 2\*b)\*e^(d\*x + c)/d^3 - 1/2\*(b\*d^2\*x^2 + a\*d^2 + 2\*b\*d\*x + 2\*b)\*e^(-d\*x - c)/d^3

$$3.44 \quad \int \frac{(a+bx^2) \cosh(c+dx)}{x} dx$$

**Optimal.** Leaf size=41

$$a \cosh(c)\text{Chi}(dx) + a \sinh(c)\text{Shi}(dx) - \frac{b \cosh(c+dx)}{d^2} + \frac{bx \sinh(c+dx)}{d}$$

[Out] -((b\*Cosh[c + d\*x])/d^2) + a\*Cosh[c]\*CoshIntegral[d\*x] + (b\*x\*Sinh[c + d\*x])/d + a\*Sinh[c]\*SinhIntegral[d\*x]

**Rubi [A]** time = 0.0980776, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {5287, 3303, 3298, 3301, 3296, 2638}

$$a \cosh(c)\text{Chi}(dx) + a \sinh(c)\text{Shi}(dx) - \frac{b \cosh(c+dx)}{d^2} + \frac{bx \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)\*Cosh[c + d\*x])/x,x]

[Out] -((b\*Cosh[c + d\*x])/d^2) + a\*Cosh[c]\*CoshIntegral[d\*x] + (b\*x\*Sinh[c + d\*x])/d + a\*Sinh[c]\*SinhIntegral[d\*x]

#### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f}

, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \frac{(a + bx^2) \cosh(c + dx)}{x} dx &= \int \left( \frac{a \cosh(c + dx)}{x} + bx \cosh(c + dx) \right) dx \\ &= a \int \frac{\cosh(c + dx)}{x} dx + b \int x \cosh(c + dx) dx \\ &= \frac{bx \sinh(c + dx)}{d} - \frac{b \int \sinh(c + dx) dx}{d} + (a \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (a \sinh(c)) \int \frac{\sinh(dx)}{x} dx \\ &= -\frac{b \cosh(c + dx)}{d^2} + a \cosh(c) \text{Chi}(dx) + \frac{bx \sinh(c + dx)}{d} + a \sinh(c) \text{Shi}(dx) \end{aligned}$$

**Mathematica [A]** time = 0.109761, size = 55, normalized size = 1.34

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) + \frac{b \cosh(dx)(dx \sinh(c) - \cosh(c))}{d^2} + \frac{b \sinh(dx)(dx \cosh(c) - \sinh(c))}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^2)\*Cosh[c + d\*x])/x,x]

[Out] a\*Cosh[c]\*CoshIntegral[d\*x] + (b\*Cosh[d\*x]\*(-Cosh[c] + d\*x\*Sinh[c]))/d^2 + (b\*(d\*x\*Cosh[c] - Sinh[c])\*Sinh[d\*x])/d^2 + a\*Sinh[c]\*SinhIntegral[d\*x]

---

**Maple [A]** time = 0.035, size = 81, normalized size = 2.

$$-\frac{ae^{-c}\text{Ei}(1, dx)}{2} - \frac{be^{-dx-c}x}{2d} - \frac{be^{-dx-c}}{2d^2} - \frac{ae^c\text{Ei}(1, -dx)}{2} + \frac{be^{dx+c}x}{2d} - \frac{be^{dx+c}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)\*cosh(d\*x+c)/x,x)

[Out] -1/2\*a\*exp(-c)\*Ei(1,d\*x)-1/2/d\*b\*exp(-d\*x-c)\*x-1/2/d^2\*b\*exp(-d\*x-c)-1/2\*a\*exp(c)\*Ei(1,-d\*x)+1/2/d\*b\*exp(d\*x+c)\*x-1/2/d^2\*b\*exp(d\*x+c)

---

**Maxima [B]** time = 1.16856, size = 165, normalized size = 4.02

$$-\frac{1}{4} \left( b \left( \frac{(d^2x^2e^c - 2dxe^c + 2e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2dx + 2)e^{(-dx-c)}}{d^3} \right) + \frac{2a \cosh(dx + c) \log(x^2)}{d} - \frac{2(\text{Ei}(-dx)e^{(-c)} + \text{Ei}(dx))}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c)/x,x, algorithm="maxima")

[Out] -1/4\*(b\*((d^2\*x^2\*e^c - 2\*d\*x\*e^c + 2\*e^c)\*e^(d\*x)/d^3 + (d^2\*x^2 + 2\*d\*x + 2)\*e^(-d\*x - c)/d^3) + 2\*a\*cosh(d\*x + c)\*log(x^2)/d - 2\*(Ei(-d\*x)\*e^(-c) + Ei(d\*x)\*e^c)\*a/d\*d + 1/2\*(b\*x^2 + a\*log(x^2))\*cosh(d\*x + c)

---

**Fricas [A]** time = 2.06274, size = 188, normalized size = 4.59

$$\frac{2bdx \sinh(dx + c) - 2b \cosh(dx + c) + (ad^2\text{Ei}(dx) + ad^2\text{Ei}(-dx)) \cosh(c) + (ad^2\text{Ei}(dx) - ad^2\text{Ei}(-dx)) \sinh(c)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c)/x,x, algorithm="fricas")

[Out] 1/2\*(2\*b\*d\*x\*sinh(d\*x + c) - 2\*b\*cosh(d\*x + c) + (a\*d^2\*Ei(d\*x) + a\*d^2\*Ei(-d\*x))\*cosh(c) + (a\*d^2\*Ei(d\*x) - a\*d^2\*Ei(-d\*x))\*sinh(c))/d^2

---

**Sympy [A]** time = 3.56531, size = 49, normalized size = 1.2

$$a \sinh(c) \operatorname{Shi}(dx) + a \cosh(c) \operatorname{Chi}(dx) + b \left( \begin{cases} \frac{x \sinh(c+dx)}{d} - \frac{\cosh(c+dx)}{d^2} & \text{for } d \neq 0 \\ \frac{x^2 \cosh(c)}{2} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*cosh(d\*x+c)/x,x)

[Out] a\*sinh(c)\*Shi(d\*x) + a\*cosh(c)\*Chi(d\*x) + b\*Piecewise((x\*sinh(c + d\*x)/d - cosh(c + d\*x)/d\*\*2, Ne(d, 0)), (x\*\*2\*cosh(c)/2, True))

---

**Giac [A]** time = 1.17627, size = 103, normalized size = 2.51

$$\frac{ad^2 \operatorname{Ei}(-dx) e^{-c} + ad^2 \operatorname{Ei}(dx) e^c + bdx e^{(dx+c)} - bdx e^{(-dx-c)} - be^{(dx+c)} - be^{(-dx-c)}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c)/x,x, algorithm="giac")

[Out] 1/2\*(a\*d^2\*Ei(-d\*x)\*e^(-c) + a\*d^2\*Ei(d\*x)\*e^c + b\*d\*x\*e^(d\*x + c) - b\*d\*x\*e^(-d\*x - c) - b\*e^(d\*x + c) - b\*e^(-d\*x - c))/d^2

$$3.45 \quad \int \frac{(a+bx^2) \cosh(c+dx)}{x^2} dx$$

**Optimal.** Leaf size=42

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{x} + \frac{b \sinh(c+dx)}{d}$$

[Out] -((a\*Cosh[c + d\*x])/x) + a\*d\*CoshIntegral[d\*x]\*Sinh[c] + (b\*Sinh[c + d\*x])/d + a\*d\*Cosh[c]\*SinhIntegral[d\*x]

**Rubi [A]** time = 0.106847, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {5287, 2637, 3297, 3303, 3298, 3301}

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{x} + \frac{b \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)\*Cosh[c + d\*x])/x^2,x]

[Out] -((a\*Cosh[c + d\*x])/x) + a\*d\*CoshIntegral[d\*x]\*Sinh[c] + (b\*Sinh[c + d\*x])/d + a\*d\*Cosh[c]\*SinhIntegral[d\*x]

#### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x]/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]



Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2) \cosh(c + dx)}{x^2} dx &= \int \left( b \cosh(c + dx) + \frac{a \cosh(c + dx)}{x^2} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^2} dx + b \int \cosh(c + dx) dx \\
&= -\frac{a \cosh(c + dx)}{x} + \frac{b \sinh(c + dx)}{d} + (ad) \int \frac{\sinh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{x} + \frac{b \sinh(c + dx)}{d} + (ad \cosh(c)) \int \frac{\sinh(dx)}{x} dx + (ad \sinh(c)) \int \frac{\cosh(dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{x} + ad \operatorname{Chi}(dx) \sinh(c) + \frac{b \sinh(c + dx)}{d} + ad \cosh(c) \operatorname{Shi}(dx)
\end{aligned}$$

**Mathematica [A]** time = 0.0881973, size = 42, normalized size = 1.

$$ad \sinh(c) \operatorname{Chi}(dx) + ad \cosh(c) \operatorname{Shi}(dx) - \frac{a \cosh(c + dx)}{x} + \frac{b \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^2, x]
```

[Out]  $-\left(\frac{a \cosh[c + d x]}{x}\right) + a d \operatorname{CoshIntegral}[d x] \operatorname{Sinh}[c] + (b \operatorname{Sinh}[c + d x]) / d + a d \operatorname{Cosh}[c] \operatorname{SinhIntegral}[d x]$

**Maple [A]** time = 0.046, size = 81, normalized size = 1.9

$$-\frac{a e^{-d x-c}}{2 x} + \frac{d a e^{-c} \operatorname{Ei}(1, d x)}{2} - \frac{b e^{-d x-c}}{2 d} - \frac{a e^{d x+c}}{2 x} - \frac{d a e^c \operatorname{Ei}(1, -d x)}{2} + \frac{b e^{d x+c}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)*cosh(d*x+c)/x^2,x)`

[Out]  $-1/2 * a * \exp(-d * x - c) / x + 1/2 * d * a * \exp(-c) * \operatorname{Ei}(1, d * x) - 1/2 * b / d * \exp(-d * x - c) - 1/2 * a / x * \exp(d * x + c) - 1/2 * d * a * \exp(c) * \operatorname{Ei}(1, -d * x) + 1/2 * b / d * \exp(d * x + c)$

**Maxima [A]** time = 1.18785, size = 108, normalized size = 2.57

$$-\frac{1}{2} \left( a \operatorname{Ei}(-d x) e^{(-c)} - a \operatorname{Ei}(d x) e^c + \frac{(d x e^c - e^c) b e^{(d x)}}{d^2} + \frac{(d x + 1) b e^{(-d x - c)}}{d^2} \right) d + \left( b x - \frac{a}{x} \right) \cosh(d x + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")`

[Out]  $-1/2 * (a * \operatorname{Ei}(-d * x) * e^{(-c)} - a * \operatorname{Ei}(d * x) * e^c + (d * x * e^c - e^c) * b * e^{(d * x)} / d^2 + (d * x + 1) * b * e^{(-d * x - c)} / d^2) * d + (b * x - a / x) * \cosh(d * x + c)$

**Fricas [A]** time = 2.084, size = 203, normalized size = 4.83

$$\frac{2 a d \cosh(d x + c) - 2 b x \sinh(d x + c) - (a d^2 x \operatorname{Ei}(d x) - a d^2 x \operatorname{Ei}(-d x)) \cosh(c) - (a d^2 x \operatorname{Ei}(d x) + a d^2 x \operatorname{Ei}(-d x)) \sinh(c)}{2 d x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")`

[Out]  $-1/2*(2*a*d*cosh(d*x + c) - 2*b*x*sinh(d*x + c) - (a*d^2*x*Ei(d*x) - a*d^2*x*Ei(-d*x))*cosh(c) - (a*d^2*x*Ei(d*x) + a*d^2*x*Ei(-d*x))*sinh(c))/(d*x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2) \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)*cosh(d*x+c)/x**2,x)`

[Out] `Integral((a + b*x**2)*cosh(c + d*x)/x**2, x)`

**Giac [A]** time = 1.22212, size = 108, normalized size = 2.57

$$\frac{ad^2xEi(-dx)e^{(-c)} - ad^2xEi(dx)e^c + ade^{(dx+c)} - bxe^{(dx+c)} + ade^{(-dx-c)} + bxe^{(-dx-c)}}{2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="giac")`

[Out]  $-1/2*(a*d^2*x*Ei(-d*x)*e^{(-c)} - a*d^2*x*Ei(d*x)*e^c + a*d*e^{(d*x + c)} - b*x*e^{(d*x + c)} + a*d*e^{(-d*x - c)} + b*x*e^{(-d*x - c)})/(d*x)$

$$3.46 \quad \int \frac{(a+bx^2) \cosh(c+dx)}{x^3} dx$$

**Optimal.** Leaf size=74

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + b \cosh(c)\text{Chi}(dx) + b \sinh(c)\text{Shi}(dx)$$

[Out]  $-(a*\text{Cosh}[c + d*x])/(2*x^2) + b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (a*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 - (a*d*\text{Sinh}[c + d*x])/(2*x) + b*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + (a*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

**Rubi [A]** time = 0.178131, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + b \cosh(c)\text{Chi}(dx) + b \sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)*\text{Cosh}[c + d*x])/x^3, x]$

[Out]  $-(a*\text{Cosh}[c + d*x])/(2*x^2) + b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (a*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 - (a*d*\text{Sinh}[c + d*x])/(2*x) + b*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + (a*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_)^{(n_)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m, n\}, x\} \ \&\& \ \text{IGtQ}[p, 0]$

### Rule 3297

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_)], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}*\sin[e + f*x]/(d*(m+1)), x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\cos[e + f*x], x], x] /;$   $\text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{LtQ}[m, -1]$

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2) \cosh(c + dx)}{x^3} dx &= \int \left( \frac{a \cosh(c + dx)}{x^3} + \frac{b \cosh(c + dx)}{x} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^3} dx + b \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + \frac{1}{2}(ad) \int \frac{\sinh(c + dx)}{x^2} dx + (b \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (b \sinh(c)) \int \frac{\sinh(dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + b \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{2x} + b \sinh(c) \text{Shi}(dx) + \frac{1}{2} (ad^2) \int \frac{\cosh(dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + b \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{2x} + b \sinh(c) \text{Shi}(dx) + \frac{1}{2} (ad^2 \coth(c) \text{Chi}(dx) + \text{Shi}(dx)) \\
&= -\frac{a \cosh(c + dx)}{2x^2} + b \cosh(c) \text{Chi}(dx) + \frac{1}{2} ad^2 \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{2x} + b \sinh(c) \text{Shi}(dx)
\end{aligned}$$

**Mathematica [A]** time = 0.147441, size = 80, normalized size = 1.08

$$\frac{1}{2} ad^2 (\cosh(c) \text{Chi}(dx) + \sinh(c) \text{Shi}(dx)) - \frac{a \cosh(dx) (dx \sinh(c) + \cosh(c))}{2x^2} - \frac{a \sinh(dx) (dx \cosh(c) + \sinh(c))}{2x^2} + b \coth(c) \text{Chi}(dx) + \text{Shi}(dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*Cosh[c + d*x])/x^3,x]
```

[Out]  $b \cdot \text{Cosh}[c] \cdot \text{CoshIntegral}[d \cdot x] - (a \cdot \text{Cosh}[d \cdot x] \cdot (\text{Cosh}[c] + d \cdot x \cdot \text{Sinh}[c])) / (2 \cdot x^2) - (a \cdot (d \cdot x \cdot \text{Cosh}[c] + \text{Sinh}[c]) \cdot \text{Sinh}[d \cdot x]) / (2 \cdot x^2) + b \cdot \text{Sinh}[c] \cdot \text{SinhIntegral}[d \cdot x] + (a \cdot d^2 \cdot (\text{Cosh}[c] \cdot \text{CoshIntegral}[d \cdot x] + \text{Sinh}[c] \cdot \text{SinhIntegral}[d \cdot x])) / 2$

**Maple [A]** time = 0.044, size = 110, normalized size = 1.5

$$\frac{dae^{-dx-c}}{4x} - \frac{ae^{-dx-c}}{4x^2} - \frac{d^2ae^{-c}\text{Ei}(1, dx)}{4} - \frac{be^{-c}\text{Ei}(1, dx)}{2} - \frac{ae^{dx+c}}{4x^2} - \frac{dae^{dx+c}}{4x} - \frac{d^2ae^c\text{Ei}(1, -dx)}{4} - \frac{be^c\text{Ei}(1, -dx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)*cosh(d*x+c)/x^3,x)`

[Out]  $1/4 \cdot d \cdot a \cdot \exp(-d \cdot x - c) / x - 1/4 \cdot a \cdot \exp(-d \cdot x - c) / x^2 - 1/4 \cdot d^2 \cdot a \cdot \exp(-c) \cdot \text{Ei}(1, d \cdot x) - 1/2 \cdot b \cdot \exp(-c) \cdot \text{Ei}(1, d \cdot x) - 1/4 \cdot a / x^2 \cdot \exp(d \cdot x + c) - 1/4 \cdot d \cdot a / x \cdot \exp(d \cdot x + c) - 1/4 \cdot d^2 \cdot a \cdot \exp(c) \cdot \text{Ei}(1, -d \cdot x) - 1/2 \cdot b \cdot \exp(c) \cdot \text{Ei}(1, -d \cdot x)$

**Maxima [A]** time = 1.19083, size = 122, normalized size = 1.65

$$\frac{1}{4} \left( (de^{(-c)}\Gamma(-1, dx) + de^c\Gamma(-1, -dx))a - \frac{2b \cosh(dx + c) \log(x^2)}{d} + \frac{2(\text{Ei}(-dx)e^{(-c)} + \text{Ei}(dx)e^c)b}{d} \right) d + \frac{1}{2} (b \log(x^2) - \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")`

[Out]  $1/4 \cdot ((d \cdot e^{(-c)} \cdot \text{gamma}(-1, d \cdot x) + d \cdot e^c \cdot \text{gamma}(-1, -d \cdot x)) \cdot a - 2 \cdot b \cdot \cosh(d \cdot x + c) \cdot \log(x^2) / d + 2 \cdot (\text{Ei}(-d \cdot x) \cdot e^{(-c)} + \text{Ei}(d \cdot x) \cdot e^c) \cdot b / d) \cdot d + 1/2 \cdot (b \cdot \log(x^2) - a / x^2) \cdot \cosh(d \cdot x + c)$

**Fricas [A]** time = 2.02335, size = 254, normalized size = 3.43

$$\frac{2 \, adx \sinh(dx + c) + 2 \, a \cosh(dx + c) - ((ad^2 + 2b)x^2 \text{Ei}(dx) + (ad^2 + 2b)x^2 \text{Ei}(-dx)) \cosh(c) - ((ad^2 + 2b)x^2 \text{Ei}(dx) - (ad^2 + 2b)x^2 \text{Ei}(-dx)) \sinh(c)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c)/x^3,x, algorithm="fricas")

[Out] 
$$-1/4*(2*a*d*x*\sinh(d*x + c) + 2*a*\cosh(d*x + c) - ((a*d^2 + 2*b)*x^2*Ei(d*x) + (a*d^2 + 2*b)*x^2*Ei(-d*x))*\cosh(c) - ((a*d^2 + 2*b)*x^2*Ei(d*x) - (a*d^2 + 2*b)*x^2*Ei(-d*x))*\sinh(c))/x^2$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*cosh(d\*x+c)/x\*\*3,x)

[Out] Exception raised: ValueError

**Giac [A]** time = 1.16015, size = 147, normalized size = 1.99

$$\frac{ad^2x^2Ei(-dx)e^{(-c)} + ad^2x^2Ei(dx)e^c + 2bx^2Ei(-dx)e^{(-c)} + 2bx^2Ei(dx)e^c - adxe^{(dx+c)} + adxe^{(-dx-c)} - ae^{(dx+c)} - ae^{(-dx-c)}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c)/x^3,x, algorithm="giac")

[Out] 
$$1/4*(a*d^2*x^2*Ei(-d*x)*e^{(-c)} + a*d^2*x^2*Ei(d*x)*e^c + 2*b*x^2*Ei(-d*x)*e^{(-c)} + 2*b*x^2*Ei(d*x)*e^c - a*d*x*e^{(d*x + c)} + a*d*x*e^{(-d*x - c)} - a*e^{(d*x + c)} - a*e^{(-d*x - c)})/x^2$$

$$3.47 \quad \int \frac{(a+bx^2) \cosh(c+dx)}{x^4} dx$$

**Optimal.** Leaf size=105

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{ad \sinh(c+dx)}{6x^2} - \frac{a \cosh(c+dx)}{3x^3} + bd \sinh(c)\text{Chi}(dx)$$

[Out]  $-(a*\text{Cosh}[c + d*x])/(3*x^3) - (b*\text{Cosh}[c + d*x])/x - (a*d^2*\text{Cosh}[c + d*x])/(6*x) + b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + (a*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 - (a*d*\text{Sinh}[c + d*x])/(6*x^2) + b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + (a*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6$

**Rubi [A]** time = 0.233946, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{ad \sinh(c+dx)}{6x^2} - \frac{a \cosh(c+dx)}{3x^3} + bd \sinh(c)\text{Chi}(dx)$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)\*Cosh[c + d\*x])/x^4, x]

[Out]  $-(a*\text{Cosh}[c + d*x])/(3*x^3) - (b*\text{Cosh}[c + d*x])/x - (a*d^2*\text{Cosh}[c + d*x])/(6*x) + b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + (a*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 - (a*d*\text{Sinh}[c + d*x])/(6*x^2) + b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + (a*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6$

#### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]



Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2) \cosh(c + dx)}{x^4} dx &= \int \left( \frac{a \cosh(c + dx)}{x^4} + \frac{b \cosh(c + dx)}{x^2} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^4} dx + b \int \frac{\cosh(c + dx)}{x^2} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{x} + \frac{1}{3}(ad) \int \frac{\sinh(c + dx)}{x^3} dx + (bd) \int \frac{\sinh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{x} - \frac{ad \sinh(c + dx)}{6x^2} + \frac{1}{6}(ad^2) \int \frac{\cosh(c + dx)}{x^2} dx + (bd) \int \frac{\sinh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{x} - \frac{ad^2 \cosh(c + dx)}{6x} + bd \operatorname{Chi}(dx) \sinh(c) - \frac{ad \sinh(c)}{6x^2} \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{x} - \frac{ad^2 \cosh(c + dx)}{6x} + bd \operatorname{Chi}(dx) \sinh(c) - \frac{ad \sinh(c)}{6x^2} \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{b \cosh(c + dx)}{x} - \frac{ad^2 \cosh(c + dx)}{6x} + bd \operatorname{Chi}(dx) \sinh(c) + \frac{1}{6} ad^3 \operatorname{Chi}(dx)
\end{aligned}$$

**Mathematica [A]** time = 0.22498, size = 95, normalized size = 0.9

$$\frac{-dx^3 \sinh(c) (ad^2 + 6b) \operatorname{Chi}(dx) - dx^3 \cosh(c) (ad^2 + 6b) \operatorname{Shi}(dx) + ad^2 x^2 \cosh(c + dx) + adx \sinh(c + dx) + 2a \cosh(c)}{6x^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^2)\*Cosh[c + d\*x])/x^4,x]

[Out]  $-(2*a*\text{Cosh}[c + d*x] + 6*b*x^2*\text{Cosh}[c + d*x] + a*d^2*x^2*\text{Cosh}[c + d*x] - d*(6*b + a*d^2)*x^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + a*d*x*\text{Sinh}[c + d*x] - d*(6*b + a*d^2)*x^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/(6*x^3)$

**Maple [A]** time = 0.059, size = 172, normalized size = 1.6

$$-\frac{ad^2e^{-dx-c}}{12x} + \frac{dae^{-dx-c}}{12x^2} - \frac{ae^{-dx-c}}{6x^3} + \frac{d^3ae^{-c}\text{Ei}(1,dx)}{12} - \frac{be^{-dx-c}}{2x} + \frac{dbe^{-c}\text{Ei}(1,dx)}{2} - \frac{ae^{dx+c}}{6x^3} - \frac{dae^{dx+c}}{12x^2} - \frac{ad^2e^{dx+c}}{12x} - \frac{d^3ae^c}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)\*cosh(d\*x+c)/x^4,x)

[Out]  $-1/12*d^2*a*\exp(-d*x-c)/x + 1/12*d*a*\exp(-d*x-c)/x^2 - 1/6*a*\exp(-d*x-c)/x^3 + 1/12*d^3*a*\exp(-c)*\text{Ei}(1,d*x) - 1/2*b*\exp(-d*x-c)/x + 1/2*d*b*\exp(-c)*\text{Ei}(1,d*x) - 1/6*a/x^3*\exp(d*x+c) - 1/12*d*a/x^2*\exp(d*x+c) - 1/12*d^2*a/x*\exp(d*x+c) - 1/12*d^3*a*\exp(c)*\text{Ei}(1,-d*x) - 1/2*b/x*\exp(d*x+c) - 1/2*d*b*\exp(c)*\text{Ei}(1,-d*x)$

**Maxima [A]** time = 1.21257, size = 99, normalized size = 0.94

$$\frac{1}{6} \left( ad^2e^{(-c)}\Gamma(-2,dx) - ad^2e^c\Gamma(-2,-dx) - 3b\text{Ei}(-dx)e^{(-c)} + 3b\text{Ei}(dx)e^c \right) d - \frac{(3bx^2 + a)\cosh(dx + c)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c)/x^4,x, algorithm="maxima")

[Out]  $1/6*(a*d^2*e^{(-c)}*\text{gamma}(-2, d*x) - a*d^2*e^c*\text{gamma}(-2, -d*x) - 3*b*\text{Ei}(-d*x)*e^{(-c)} + 3*b*\text{Ei}(d*x)*e^c)*d - 1/3*(3*b*x^2 + a)*\cosh(d*x + c)/x^3$

**Fricas [A]** time = 1.98027, size = 298, normalized size = 2.84

$$\frac{2adx \sinh(dx + c) + 2((ad^2 + 6b)x^2 + 2a)\cosh(dx + c) - ((ad^3 + 6bd)x^3\text{Ei}(dx) - (ad^3 + 6bd)x^3\text{Ei}(-dx))\cosh(c)}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c)/x^4,x, algorithm="fricas")

[Out] 
$$-1/12*(2*a*d*x*\sinh(d*x + c) + 2*((a*d^2 + 6*b)*x^2 + 2*a)*\cosh(d*x + c) - ((a*d^3 + 6*b*d)*x^3*Ei(d*x) - (a*d^3 + 6*b*d)*x^3*Ei(-d*x))*\cosh(c) - ((a*d^3 + 6*b*d)*x^3*Ei(d*x) + (a*d^3 + 6*b*d)*x^3*Ei(-d*x))*\sinh(c))/x^3$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*cosh(d\*x+c)/x\*\*4,x)

[Out] Exception raised: ValueError

**Giac [A]** time = 1.18282, size = 230, normalized size = 2.19

$$\frac{ad^3x^3Ei(-dx)e^{(-c)} - ad^3x^3Ei(dx)e^c + 6bdx^3Ei(-dx)e^{(-c)} - 6bdx^3Ei(dx)e^c + ad^2x^2e^{(dx+c)} + ad^2x^2e^{(-dx-c)} + adxe^{(dx+c)}}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c)/x^4,x, algorithm="giac")

[Out] 
$$-1/12*(a*d^3*x^3*Ei(-d*x)*e^{(-c)} - a*d^3*x^3*Ei(d*x)*e^c + 6*b*d*x^3*Ei(-d*x)*e^{(-c)} - 6*b*d*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^{(d*x + c)} + a*d^2*x^2*e^{(-d*x - c)} + a*d*x*e^{(d*x + c)} + 6*b*x^2*e^{(d*x + c)} - a*d*x*e^{(-d*x - c)} + 6*b*x^2*e^{(-d*x - c)} + 2*a*e^{(d*x + c)} + 2*a*e^{(-d*x - c)})/x^3$$

$$3.48 \quad \int \frac{(a+bx^2) \cosh(c+dx)}{x^5} dx$$

**Optimal.** Leaf size=149

$$\frac{1}{24}ad^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}ad^4 \sinh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{24x^2} - \frac{ad^3 \sinh(c+dx)}{24x} - \frac{ad \sinh(c+dx)}{12x^3} - \frac{a \cosh(c+dx)}{4x^4}$$

[Out]  $-(a*\text{Cosh}[c+d*x])/(4*x^4) - (b*\text{Cosh}[c+d*x])/(2*x^2) - (a*d^2*\text{Cosh}[c+d*x])/(24*x^2) + (b*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (a*d^4*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/24 - (a*d*\text{Sinh}[c+d*x])/(12*x^3) - (b*d*\text{Sinh}[c+d*x])/(2*x) - (a*d^3*\text{Sinh}[c+d*x])/(24*x) + (b*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2 + (a*d^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/24$

**Rubi [A]** time = 0.290435, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{24}ad^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}ad^4 \sinh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{24x^2} - \frac{ad^3 \sinh(c+dx)}{24x} - \frac{ad \sinh(c+dx)}{12x^3} - \frac{a \cosh(c+dx)}{4x^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)*\text{Cosh}[c + d*x])/x^5, x]$

[Out]  $-(a*\text{Cosh}[c+d*x])/(4*x^4) - (b*\text{Cosh}[c+d*x])/(2*x^2) - (a*d^2*\text{Cosh}[c+d*x])/(24*x^2) + (b*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (a*d^4*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/24 - (a*d*\text{Sinh}[c+d*x])/(12*x^3) - (b*d*\text{Sinh}[c+d*x])/(2*x) - (a*d^3*\text{Sinh}[c+d*x])/(24*x) + (b*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2 + (a*d^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/24$

### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))}^{(p_.)}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;$  FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

### Rule 3297

$\text{Int}[(c_.) + (d_.)*(x_.))^{(m_.)*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] :> \text{Simp}[(c + d*x)^{(m+1)}*\text{Sin}[e + f*x]/(d*(m+1)), x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\text{Cos}[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2) \cosh(c + dx)}{x^5} dx &= \int \left( \frac{a \cosh(c + dx)}{x^5} + \frac{b \cosh(c + dx)}{x^3} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^5} dx + b \int \frac{\cosh(c + dx)}{x^3} dx \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} + \frac{1}{4}(ad) \int \frac{\sinh(c + dx)}{x^4} dx + \frac{1}{2}(bd) \int \frac{\sinh(c + dx)}{x^2} dx \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad \sinh(c + dx)}{12x^3} - \frac{bd \sinh(c + dx)}{2x} + \frac{1}{12}(ad^2) \int \frac{\cosh(c + dx)}{x^3} dx \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{24x^2} - \frac{ad \sinh(c + dx)}{12x^3} - \frac{bd \sinh(c + dx)}{2x} \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{24x^2} + \frac{1}{2}bd^2 \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{12x^3} \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{24x^2} + \frac{1}{2}bd^2 \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{12x^3} \\
&= -\frac{a \cosh(c + dx)}{4x^4} - \frac{b \cosh(c + dx)}{2x^2} - \frac{ad^2 \cosh(c + dx)}{24x^2} + \frac{1}{2}bd^2 \cosh(c) \text{Chi}(dx) + \frac{1}{24}ad^3 \int \frac{\cosh(c + dx)}{x} dx
\end{aligned}$$

**Mathematica [A]** time = 0.274468, size = 127, normalized size = 0.85

$$\frac{-d^2x^4 \cosh(c)(ad^2 + 12b) \operatorname{Chi}(dx) - d^2x^4 \sinh(c)(ad^2 + 12b) \operatorname{Shi}(dx) + ad^3x^3 \sinh(c + dx) + ad^2x^2 \cosh(c + dx) + 2ad^2x \sinh(c + dx) + 2ad \cosh(c + dx) + 2a \sinh(c + dx)}{24x^4}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^2)\*Cosh[c + d\*x])/x^5,x]

[Out]  $-(6*a*\operatorname{Cosh}[c + d*x] + 12*b*x^2*\operatorname{Cosh}[c + d*x] + a*d^2*x^2*\operatorname{Cosh}[c + d*x] - d^2*(12*b + a*d^2)*x^4*\operatorname{Cosh}[c]*\operatorname{CoshIntegral}[d*x] + 2*a*d*x*\operatorname{Sinh}[c + d*x] + 12*b*d*x^3*\operatorname{Sinh}[c + d*x] + a*d^3*x^3*\operatorname{Sinh}[c + d*x] - d^2*(12*b + a*d^2)*x^4*\operatorname{SinhIntegral}[d*x])/(24*x^4)$

**Maple [A]** time = 0.066, size = 238, normalized size = 1.6

$$\frac{ad^3e^{-dx-c}}{48x} - \frac{ad^2e^{-dx-c}}{48x^2} + \frac{dae^{-dx-c}}{24x^3} - \frac{ae^{-dx-c}}{8x^4} + \frac{bde^{-dx-c}}{4x} - \frac{be^{-dx-c}}{4x^2} - \frac{d^2be^{-c}\operatorname{Ei}(1, dx)}{4} - \frac{d^4ae^{-c}\operatorname{Ei}(1, dx)}{48} - \frac{d^4ae^c\operatorname{Ei}(1, -dx)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)\*cosh(d\*x+c)/x^5,x)

[Out]  $1/48*d^3*a*\exp(-d*x-c)/x - 1/48*d^2*a*\exp(-d*x-c)/x^2 + 1/24*d*a*\exp(-d*x-c)/x^3 - 1/8*a*\exp(-d*x-c)/x^4 + 1/4*d*b*\exp(-d*x-c)/x - 1/4*b*\exp(-d*x-c)/x^2 - 1/4*d^2*b*\exp(-c)*\operatorname{Ei}(1, d*x) - 1/48*d^4*a*\exp(-c)*\operatorname{Ei}(1, d*x) - 1/48*d^4*a*\exp(c)*\operatorname{Ei}(1, -d*x) - 1/8*a/x^4*\exp(d*x+c) - 1/24*d*a/x^3*\exp(d*x+c) - 1/48*d^2*a/x^2*\exp(d*x+c) - 1/48*d^3*a/x*\exp(d*x+c) - 1/4*b/x^2*\exp(d*x+c) - 1/4*d*b/x*\exp(d*x+c) - 1/4*d^2*b*\exp(c)*\operatorname{Ei}(1, -d*x)$

**Maxima [A]** time = 1.21251, size = 103, normalized size = 0.69

$$\frac{1}{8} \left( ad^3e^{(-c)}\Gamma(-3, dx) + ad^3e^c\Gamma(-3, -dx) + 2bde^{(-c)}\Gamma(-1, dx) + 2bde^c\Gamma(-1, -dx) \right) d - \frac{(2bx^2 + a) \cosh(dx + c)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*cosh(d\*x+c)/x^5,x, algorithm="maxima")

[Out]  $\frac{1}{8}(a*d^3*e^{-c}*\gamma(-3, d*x) + a*d^3*e^c*\gamma(-3, -d*x) + 2*b*d*e^{-c}*\gamma(-1, d*x) + 2*b*d*e^c*\gamma(-1, -d*x))*d - \frac{1}{4}(2*b*x^2 + a)*\cosh(d*x + c)/x^4$

**Fricas [A]** time = 2.00653, size = 352, normalized size = 2.36

$$\frac{2((ad^2 + 12b)x^2 + 6a)\cosh(dx + c) - ((ad^4 + 12bd^2)x^4\text{Ei}(dx) + (ad^4 + 12bd^2)x^4\text{Ei}(-dx))\cosh(c) + 2((ad^3 + 12bd^2)x^3 + 2a*d*x)*\sinh(dx + c) - ((ad^4 + 12bd^2)x^4\text{Ei}(dx) - (ad^4 + 12bd^2)x^4\text{Ei}(-d*x))*\sinh(c)}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="fricas")`

[Out]  $\frac{-1}{48}(2*((a*d^2 + 12*b)*x^2 + 6*a)*\cosh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^4*\text{Ei}(d*x) + (a*d^4 + 12*b*d^2)*x^4*\text{Ei}(-d*x))*\cosh(c) + 2*((a*d^3 + 12*b*d)*x^3 + 2*a*d*x)*\sinh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^4*\text{Ei}(d*x) - (a*d^4 + 12*b*d^2)*x^4*\text{Ei}(-d*x))*\sinh(c))/x^4$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)*cosh(d*x+c)/x**5,x)`

[Out] Timed out

**Giac [A]** time = 1.19852, size = 320, normalized size = 2.15

$$\frac{ad^4x^4\text{Ei}(-dx)e^{(-c)} + ad^4x^4\text{Ei}(dx)e^c + 12bd^2x^4\text{Ei}(-dx)e^{(-c)} + 12bd^2x^4\text{Ei}(dx)e^c - ad^3x^3e^{(dx+c)} + ad^3x^3e^{(-dx-c)} - ad^2x^2e^{(dx+c)} + ad^2x^2e^{(-dx-c)}}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="giac")`

```
[Out] 1/48*(a*d^4*x^4*Ei(-d*x)*e^(-c) + a*d^4*x^4*Ei(d*x)*e^c + 12*b*d^2*x^4*Ei(-
d*x)*e^(-c) + 12*b*d^2*x^4*Ei(d*x)*e^c - a*d^3*x^3*e^(d*x + c) + a*d^3*x^3*
e^(-d*x - c) - a*d^2*x^2*e^(d*x + c) - 12*b*d*x^3*e^(d*x + c) - a*d^2*x^2*e
^(-d*x - c) + 12*b*d*x^3*e^(-d*x - c) - 2*a*d*x*e^(d*x + c) - 12*b*x^2*e^(d
*x + c) + 2*a*d*x*e^(-d*x - c) - 12*b*x^2*e^(-d*x - c) - 6*a*e^(d*x + c) -
6*a*e^(-d*x - c))/x^4
```



### 3.49 $\int x^2 (a + bx^2)^2 \cosh(c + dx) dx$

**Optimal.** Leaf size=234

$$\frac{2a^2 \sinh(c + dx)}{d^3} - \frac{2a^2 x \cosh(c + dx)}{d^2} + \frac{a^2 x^2 \sinh(c + dx)}{d} + \frac{24abx^2 \sinh(c + dx)}{d^3} - \frac{8abx^3 \cosh(c + dx)}{d^2} + \frac{48ab \sinh(c + dx)}{d^5}$$

[Out]  $(-720*b^2*x*Cosh[c + d*x])/d^6 - (48*a*b*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Cos$   
 $sh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (8*a*b*x^3*Cosh[c + d*$   
 $x])/d^2 - (6*b^2*x^5*Cosh[c + d*x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (48$   
 $*a*b*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c +$   
 $d*x])/d^5 + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/d + ($   
 $30*b^2*x^4*Sinh[c + d*x])/d^3 + (2*a*b*x^4*Sinh[c + d*x])/d + (b^2*x^6*Sinh$   
 $[c + d*x])/d$

**Rubi [A]** time = 0.388255, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {5287, 3296, 2637}

$$\frac{2a^2 \sinh(c + dx)}{d^3} - \frac{2a^2 x \cosh(c + dx)}{d^2} + \frac{a^2 x^2 \sinh(c + dx)}{d} + \frac{24abx^2 \sinh(c + dx)}{d^3} - \frac{8abx^3 \cosh(c + dx)}{d^2} + \frac{48ab \sinh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*x^2)^2*\text{Cosh}[c + d*x], x]$

[Out]  $(-720*b^2*x*Cosh[c + d*x])/d^6 - (48*a*b*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Co$   
 $sh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (8*a*b*x^3*Cosh[c + d*$   
 $x])/d^2 - (6*b^2*x^5*Cosh[c + d*x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (48$   
 $*a*b*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c +$   
 $d*x])/d^5 + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/d + ($   
 $30*b^2*x^4*Sinh[c + d*x])/d^3 + (2*a*b*x^4*Sinh[c + d*x])/d + (b^2*x^6*Sinh$   
 $[c + d*x])/d$

#### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;$  FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int x^2 (a + bx^2)^2 \cosh(c + dx) dx &= \int (a^2 x^2 \cosh(c + dx) + 2abx^4 \cosh(c + dx) + b^2 x^6 \cosh(c + dx)) dx \\
&= a^2 \int x^2 \cosh(c + dx) dx + (2ab) \int x^4 \cosh(c + dx) dx + b^2 \int x^6 \cosh(c + dx) dx \\
&= \frac{a^2 x^2 \sinh(c + dx)}{d} + \frac{2abx^4 \sinh(c + dx)}{d} + \frac{b^2 x^6 \sinh(c + dx)}{d} - \frac{(2a^2) \int x \sinh(c + dx)}{d} \\
&= -\frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{6b^2 x^5 \cosh(c + dx)}{d^2} + \frac{a^2 x^2 \sinh(c + dx)}{d} \\
&= -\frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{6b^2 x^5 \cosh(c + dx)}{d^2} + \frac{2a^2 \sinh(c + dx)}{d^3} \\
&= -\frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{120b^2 x^3 \cosh(c + dx)}{d^4} - \frac{8abx^3 \cosh(c + dx)}{d^2} \\
&= -\frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{120b^2 x^3 \cosh(c + dx)}{d^4} - \frac{8abx^3 \cosh(c + dx)}{d^2} \\
&= -\frac{720b^2 x \cosh(c + dx)}{d^6} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{120b^2 x^3 \cosh(c + dx)}{d^4} \\
&= -\frac{720b^2 x \cosh(c + dx)}{d^6} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2a^2 x \cosh(c + dx)}{d^2} - \frac{120b^2 x^3 \cosh(c + dx)}{d^4}
\end{aligned}$$

**Mathematica [A]** time = 0.320592, size = 138, normalized size = 0.59

$$\frac{(a^2 d^4 (d^2 x^2 + 2) + 2abd^2 (d^4 x^4 + 12d^2 x^2 + 24) + b^2 (d^6 x^6 + 30d^4 x^4 + 360d^2 x^2 + 720)) \sinh(c + dx) - 2dx (a^2 d^4 + 4abd^2 x^4)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*Cosh[c + d*x],x]
```

```
[Out] (-2*d*x*(a^2*d^4 + 4*a*b*d^2*(6 + d^2*x^2) + 3*b^2*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c + d*x] + (a^2*d^4*(2 + d^2*x^2) + 2*a*b*d^2*(24 + 12*d^2*x^2 +
```

$$\frac{d^4 x^4 + b^2(720 + 360d^2 x^2 + 30d^4 x^4 + d^6 x^6) \operatorname{Sinh}[c + dx]}{d^7}$$

**Maple [B]** time = 0.009, size = 738, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^2*cosh(d*x+c),x)`

[Out]  $\frac{1}{d^3} \left( \frac{1}{d^4} b^2 \left( (d*x+c)^6 \sinh(d*x+c) - 6(d*x+c)^5 \cosh(d*x+c) + 30(d*x+c)^4 \sinh(d*x+c) - 120(d*x+c)^3 \cosh(d*x+c) + 360(d*x+c)^2 \sinh(d*x+c) - 720(d*x+c) \cosh(d*x+c) + 720 \sinh(d*x+c) \right) + \frac{1}{d^4} b^2 c^6 \sinh(d*x+c) - 2a^2 c \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) + a^2 \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) - \frac{8}{d^2} b^2 c^3 a \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) - \frac{8}{d^2} b^2 a^2 c \left( (d*x+c)^3 \sinh(d*x+c) - 3(d*x+c)^2 \cosh(d*x+c) + 6(d*x+c) \sinh(d*x+c) - 6 \cosh(d*x+c) \right) + \frac{12}{d^2} b^2 c^2 a \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) + \frac{2}{d^2} b^2 c^4 a \sinh(d*x+c) - \frac{6}{d^4} b^2 c^5 \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) - \frac{20}{d^4} b^2 c^3 \left( (d*x+c)^3 \sinh(d*x+c) - 3(d*x+c)^2 \cosh(d*x+c) + 6(d*x+c) \sinh(d*x+c) - 6 \cosh(d*x+c) \right) + \frac{15}{d^4} b^2 c^4 \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) + \frac{2}{d^2} b^2 a \left( (d*x+c)^4 \sinh(d*x+c) - 4(d*x+c)^3 \cosh(d*x+c) + 12(d*x+c)^2 \sinh(d*x+c) - 24(d*x+c) \cosh(d*x+c) + 24 \sinh(d*x+c) \right) - \frac{6}{d^4} b^2 c \left( (d*x+c)^5 \sinh(d*x+c) - 5(d*x+c)^4 \cosh(d*x+c) + 20(d*x+c)^3 \sinh(d*x+c) - 60(d*x+c)^2 \cosh(d*x+c) + 120(d*x+c) \sinh(d*x+c) - 120 \cosh(d*x+c) \right) + \frac{15}{d^4} b^2 c^2 \left( (d*x+c)^4 \sinh(d*x+c) - 4(d*x+c)^3 \cosh(d*x+c) + 12(d*x+c)^2 \sinh(d*x+c) - 24(d*x+c) \cosh(d*x+c) + 24 \sinh(d*x+c) \right) + a^2 c^2 \sinh(d*x+c) \right)$

**Maxima [A]** time = 1.05534, size = 517, normalized size = 2.21

$$-\frac{1}{210} d \left( \frac{35(d^3 x^3 e^c - 3d^2 x^2 e^c + 6dx e^c - 6e^c) a^2 e^{(dx)}}{d^4} + \frac{35(d^3 x^3 + 3d^2 x^2 + 6dx + 6) a^2 e^{(-dx-c)}}{d^4} + \frac{42(d^5 x^5 e^c - 5d^4 x^4 e^c + \dots)}{d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")`

[Out]  $-1/210*d*(35*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a^2*e^{(d*x)}/d^4 + 35*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a^2*e^{(-d*x - c)}/d^4 + 42*(d^5*x$

$$\begin{aligned} &^5e^c - 5d^4x^4e^c + 20d^3x^3e^c - 60d^2x^2e^c + 120dxe^c - 120e^c) * a * b * e^{(dx)} / d^6 + 42 * (d^5x^5 + 5d^4x^4 + 20d^3x^3 + 60d^2x^2 \\ &+ 120dx + 120) * a * b * e^{(-dx - c)} / d^6 + 15 * (d^7x^7e^c - 7d^6x^6e^c + 4 \\ &2d^5x^5e^c - 210d^4x^4e^c + 840d^3x^3e^c - 2520d^2x^2e^c + 5040 \\ &* dx * e^c - 5040e^c) * b^2 * e^{(dx)} / d^8 + 15 * (d^7x^7 + 7d^6x^6 + 42d^5x^5 \\ &+ 210d^4x^4 + 840d^3x^3 + 2520d^2x^2 + 5040dx + 5040) * b^2 * e^{(-dx \\ &- c)} / d^8) + 1/105 * (15 * b^2 * x^7 + 42 * a * b * x^5 + 35 * a^2 * x^3) * \cosh(dx + c) \end{aligned}$$

**Fricas [A]** time = 2.05895, size = 336, normalized size = 1.44

$$\frac{2 \left( 3 b^2 d^5 x^5 + 4 (abd^5 + 15 b^2 d^3) x^3 + (a^2 d^5 + 24 abd^3 + 360 b^2 d) x \right) \cosh(dx + c) - (b^2 d^6 x^6 + 2 a^2 d^4 + 2 (abd^6 + 15 b^2 d^4) x^4 + 48 a b d^2 + (a^2 d^6 + 24 a b d^4 + 360 b^2 d^2) x^2 + 720 b^2) \sinh(dx + c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)^2\*cosh(d\*x+c),x, algorithm="fricas")

[Out]  $-(2 * (3 * b^2 * d^5 * x^5 + 4 * (a * b * d^5 + 15 * b^2 * d^3) * x^3 + (a^2 * d^5 + 24 * a * b * d^3 + 360 * b^2 * d) * x) * \cosh(dx + c) - (b^2 * d^6 * x^6 + 2 * a^2 * d^4 + 2 * (a * b * d^6 + 15 * b^2 * d^4) * x^4 + 48 * a * b * d^2 + (a^2 * d^6 + 24 * a * b * d^4 + 360 * b^2 * d^2) * x^2 + 720 * b^2) * \sinh(dx + c)) / d^7$

**Sympy [A]** time = 10.1603, size = 286, normalized size = 1.22

$$\left\{ \begin{array}{l} \frac{a^2 x^2 \sinh(c+dx)}{d} - \frac{2a^2 x \cosh(c+dx)}{d^2} + \frac{2a^2 \sinh(c+dx)}{d^3} + \frac{2abx^4 \sinh(c+dx)}{d} - \frac{8abx^3 \cosh(c+dx)}{d^2} + \frac{24abx^2 \sinh(c+dx)}{d^3} - \frac{48abx \cosh(c+dx)}{d^4} + \frac{48ab}{d^5} \\ \left( \frac{a^2 x^3}{3} + \frac{2abx^5}{5} + \frac{b^2 x^7}{7} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)\*\*2\*cosh(d\*x+c),x)

[Out] Piecewise((a\*\*2\*x\*\*2\*sinh(c + d\*x)/d - 2\*a\*\*2\*x\*cosh(c + d\*x)/d\*\*2 + 2\*a\*\*2\*sinh(c + d\*x)/d\*\*3 + 2\*a\*b\*x\*\*4\*sinh(c + d\*x)/d - 8\*a\*b\*x\*\*3\*cosh(c + d\*x)/d\*\*2 + 24\*a\*b\*x\*\*2\*sinh(c + d\*x)/d\*\*3 - 48\*a\*b\*x\*cosh(c + d\*x)/d\*\*4 + 48\*a\*b\*sinh(c + d\*x)/d\*\*5 + b\*\*2\*x\*\*6\*sinh(c + d\*x)/d - 6\*b\*\*2\*x\*\*5\*cosh(c + d\*x)/d\*\*2 + 30\*b\*\*2\*x\*\*4\*sinh(c + d\*x)/d\*\*3 - 120\*b\*\*2\*x\*\*3\*cosh(c + d\*x)/d\*\*4 + 360\*b\*\*2\*x\*\*2\*sinh(c + d\*x)/d\*\*5 - 720\*b\*\*2\*x\*cosh(c + d\*x)/d\*\*6 + 720\*b\*\*2\*sinh(c + d\*x)/d\*\*7, Ne(d, 0)), ((a\*\*2\*x\*\*3/3 + 2\*a\*b\*x\*\*5/5 + b\*\*2\*x\*\*7/7) \* cosh(c))

7/7)\*cosh(c), True))

**Giac [A]** time = 1.21762, size = 410, normalized size = 1.75

$$\frac{(b^2 d^6 x^6 + 2 a b d^6 x^4 - 6 b^2 d^5 x^5 + a^2 d^6 x^2 - 8 a b d^5 x^3 + 30 b^2 d^4 x^4 - 2 a^2 d^5 x + 24 a b d^4 x^2 - 120 b^2 d^3 x^3 + 2 a^2 d^4 - 48 a b d^3 x)}{2 d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)^2\*cosh(d\*x+c),x, algorithm="giac")

[Out]  $\frac{1}{2}*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 - 6*b^2*d^5*x^5 + a^2*d^6*x^2 - 8*a*b*d^5*x^3 + 30*b^2*d^4*x^4 - 2*a^2*d^5*x + 24*a*b*d^4*x^2 - 120*b^2*d^3*x^3 + 2*a^2*d^4 - 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b*d^2 - 720*b^2*d*x + 720*b^2)*e^{(d*x + c)}/d^7 - \frac{1}{2}*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 + 6*b^2*d^5*x^5 + a^2*d^6*x^2 + 8*a*b*d^5*x^3 + 30*b^2*d^4*x^4 + 2*a^2*d^5*x + 24*a*b*d^4*x^2 + 120*b^2*d^3*x^3 + 2*a^2*d^4 + 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b*d^2 + 720*b^2*d*x + 720*b^2)*e^{(-d*x - c)}/d^7$

### 3.50 $\int x (a + bx^2)^2 \cosh(c + dx) dx$

**Optimal.** Leaf size=184

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{12ab \cosh(c + dx)}{d^4} + \frac{2abx^3 \sinh(c + dx)}{d}$$

[Out]  $(-120*b^2*Cosh[c + d*x])/d^6 - (12*a*b*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/d^2 - (60*b^2*x^2*Cosh[c + d*x])/d^4 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (5*b^2*x^4*Cosh[c + d*x])/d^2 + (120*b^2*x*Sinh[c + d*x])/d^5 + (12*a*b*x*Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d^3 + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^5*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.27713, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {5287, 3296, 2638}

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{12ab \cosh(c + dx)}{d^4} + \frac{2abx^3 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*x^2)^2*Cosh[c + d*x], x]$

[Out]  $(-120*b^2*Cosh[c + d*x])/d^6 - (12*a*b*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/d^2 - (60*b^2*x^2*Cosh[c + d*x])/d^4 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (5*b^2*x^4*Cosh[c + d*x])/d^2 + (120*b^2*x*Sinh[c + d*x])/d^5 + (12*a*b*x*Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d^3 + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^5*Sinh[c + d*x])/d$

#### Rule 5287

$\text{Int}[Cosh[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m, n\}, x$  &&  $\text{IGtQ}[p, 0]$

#### Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /;$   $\text{FreeQ}\{c, d, e, f\}, x$  &&  $\text{GtQ}[m, 0]$

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned}
 \int x (a + bx^2)^2 \cosh(c + dx) dx &= \int (a^2x \cosh(c + dx) + 2abx^3 \cosh(c + dx) + b^2x^5 \cosh(c + dx)) dx \\
 &= a^2 \int x \cosh(c + dx) dx + (2ab) \int x^3 \cosh(c + dx) dx + b^2 \int x^5 \cosh(c + dx) dx \\
 &= \frac{a^2x \sinh(c + dx)}{d} + \frac{2abx^3 \sinh(c + dx)}{d} + \frac{b^2x^5 \sinh(c + dx)}{d} - \frac{a^2 \int \sinh(c + dx) dx}{d} \\
 &= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{5b^2x^4 \cosh(c + dx)}{d^2} + \frac{a^2x \sinh(c + dx)}{d} \\
 &= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{5b^2x^4 \cosh(c + dx)}{d^2} + \frac{12abx \sinh(c + dx)}{d^3} \\
 &= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{a^2 \cosh(c + dx)}{d^2} - \frac{60b^2x^2 \cosh(c + dx)}{d^4} - \frac{6abx^2 \cosh(c + dx)}{d^2} \\
 &= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{a^2 \cosh(c + dx)}{d^2} - \frac{60b^2x^2 \cosh(c + dx)}{d^4} - \frac{6abx^2 \cosh(c + dx)}{d^2} \\
 &= -\frac{120b^2 \cosh(c + dx)}{d^6} - \frac{12ab \cosh(c + dx)}{d^4} - \frac{a^2 \cosh(c + dx)}{d^2} - \frac{60b^2x^2 \cosh(c + dx)}{d^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.230315, size = 113, normalized size = 0.61

$$\frac{dx (a^2d^4 + 2abd^2(d^2x^2 + 6) + b^2(d^4x^4 + 20d^2x^2 + 120)) \sinh(c + dx) - (a^2d^4 + 6abd^2(d^2x^2 + 2) + 5b^2(d^4x^4 + 12d^2x^2 + 120)) \cosh(c + dx)}{d^6}$$

Antiderivative was successfully verified.

[In] `Integrate[x*(a + b*x^2)^2*Cosh[c + d*x], x]`

[Out] `((-((a^2*d^4 + 6*a*b*d^2*(2 + d^2*x^2) + 5*b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x]) + d*x*(a^2*d^4 + 2*a*b*d^2*(6 + d^2*x^2) + b^2*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6`

**Maple [B]** time = 0.008, size = 513, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)^2*cosh(d*x+c),x)`

[Out]  $\frac{1}{d^2} \left( \frac{1}{d^4} b^2 \left( (d*x+c)^5 \sinh(d*x+c) - 5(d*x+c)^4 \cosh(d*x+c) + 20(d*x+c)^3 \sinh(d*x+c) - 60(d*x+c)^2 \cosh(d*x+c) + 120(d*x+c) \sinh(d*x+c) - 120 \cosh(d*x+c) \right) - \frac{5}{d^4} b^2 c \left( (d*x+c)^4 \sinh(d*x+c) - 4(d*x+c)^3 \cosh(d*x+c) + 12(d*x+c)^2 \sinh(d*x+c) - 24(d*x+c) \cosh(d*x+c) + 24 \sinh(d*x+c) \right) + \frac{10}{d^4} b^2 c^2 \left( (d*x+c)^3 \sinh(d*x+c) - 3(d*x+c)^2 \cosh(d*x+c) + 6(d*x+c) \sinh(d*x+c) - 6 \cosh(d*x+c) \right) + \frac{2}{d^2} b a \left( (d*x+c)^3 \sinh(d*x+c) - 3(d*x+c)^2 \cosh(d*x+c) + 6(d*x+c) \sinh(d*x+c) - 6 \cosh(d*x+c) \right) - \frac{10}{d^4} b^2 c^3 \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) - \frac{6}{d^2} b c a \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) + \frac{5}{d^4} b^2 c^4 \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) + \frac{6}{d^2} b c^2 a \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) + a^2 \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) - \frac{1}{d^4} b^2 c^5 \sinh(d*x+c) - \frac{2}{d^2} b c^3 a \sinh(d*x+c) - c a^2 \sinh(d*x+c) \right)$

**Maxima [A]** time = 1.07846, size = 477, normalized size = 2.59

$$\frac{(bx^2 + a)^3 \cosh(dx + c)}{6b} - \frac{\left( \frac{a^3 e^{dx+c}}{d} + \frac{a^3 e^{-dx-c}}{d} + \frac{3(d^2 x^2 e^c - 2 dx e^c + 2 e^c) a^2 b e^{dx}}{d^3} + \frac{3(d^2 x^2 + 2 dx + 2) a^2 b e^{-dx-c}}{d^3} + \frac{3(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 12 d x e^c + 6 e^c) a^2 b^2 e^{dx}}{d^5} + \frac{3(d^4 x^4 e^{-c} - 4 d^3 x^3 e^{-c} + 12 d^2 x^2 e^{-c} - 12 d x e^{-c} + 6 e^{-c}) a^2 b^2 e^{-dx-c}}{d^5} \right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")`

[Out]  $\frac{1}{6} (bx^2 + a)^3 \cosh(dx + c) / b - \frac{1}{12} (a^3 e^{dx+c}) / d + \frac{a^3 e^{-dx-c}}{d} + \frac{3(d^2 x^2 e^c - 2 dx e^c + 2 e^c) a^2 b e^{dx}}{d^3} + \frac{3(d^2 x^2 + 2 dx + 2) a^2 b e^{-dx-c}}{d^3} + \frac{3(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 24 dx e^c + 24 e^c) a^2 b^2 e^{dx}}{d^5} + \frac{3(d^4 x^4 e^{-c} - 4 d^3 x^3 e^{-c} + 12 d^2 x^2 e^{-c} + 12 dx e^{-c} + 24 e^{-c}) a^2 b^2 e^{-dx-c}}{d^5} + \frac{(d^6 x^6 e^c - 6 d^5 x^5 e^c + 30 d^4 x^4 e^c - 120 d^3 x^3 e^c + 360 d^2 x^2 e^c - 720 dx e^c + 720 e^c) b^3 e^{dx}}{d^7} + \frac{(d^6 x^6 e^{-c} - 6 d^5 x^5 e^{-c} + 30 d^4 x^4 e^{-c} + 120 d^3 x^3 e^{-c} + 360 d^2 x^2 e^{-c} + 720 dx e^{-c} + 720 e^{-c}) b^3 e^{-dx-c}}{d^7} * d / b$

**Fricas [A]** time = 2.08417, size = 273, normalized size = 1.48

$$\frac{(5b^2 d^4 x^4 + a^2 d^4 + 12abd^2 + 6(abd^4 + 10b^2 d^2)x^2 + 120b^2) \cosh(dx + c) - (b^2 d^5 x^5 + 2(abd^5 + 10b^2 d^3)x^3 + (a^2 d^5 + 10abd^3)x) \sinh(dx + c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x\*(b\*x^2+a)^2\*cosh(d\*x+c),x, algorithm="fricas")

[Out]  $-\left(\left(5b^2d^4x^4 + a^2d^4 + 12ab^2d^2 + 6(a^2bd^4 + 10b^2d^2)x^2 + 120b^2\right)cosh(dx + c) - \left(b^2d^5x^5 + 2(a^2bd^5 + 10b^2d^3)x^3 + (a^2d^5 + 12ab^2d^3 + 120b^2d)x\right)sinh(dx + c)\right)/d^6$

**Sympy [A]** time = 5.25899, size = 226, normalized size = 1.23

$$\left\{ \begin{array}{l} \frac{a^2x \sinh(c+dx)}{d} - \frac{a^2 \cosh(c+dx)}{d^2} + \frac{2abx^3 \sinh(c+dx)}{d} - \frac{6abx^2 \cosh(c+dx)}{d^2} + \frac{12abx \sinh(c+dx)}{d^3} - \frac{12ab \cosh(c+dx)}{d^4} + \frac{b^2x^5 \sinh(c+dx)}{d} - \frac{5b^2x^4 \cosh(c+dx)}{d^2} \\ \left( \frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*2+a)\*\*2\*cosh(d\*x+c),x)

[Out] Piecewise((a\*\*2\*x\*sinh(c + d\*x)/d - a\*\*2\*cosh(c + d\*x)/d\*\*2 + 2\*a\*b\*x\*\*3\*sinh(c + d\*x)/d - 6\*a\*b\*x\*\*2\*cosh(c + d\*x)/d\*\*2 + 12\*a\*b\*x\*sinh(c + d\*x)/d\*\*3 - 12\*a\*b\*cosh(c + d\*x)/d\*\*4 + b\*\*2\*x\*\*5\*sinh(c + d\*x)/d - 5\*b\*\*2\*x\*\*4\*cosh(c + d\*x)/d\*\*2 + 20\*b\*\*2\*x\*\*3\*sinh(c + d\*x)/d\*\*3 - 60\*b\*\*2\*x\*\*2\*cosh(c + d\*x)/d\*\*4 + 120\*b\*\*2\*x\*sinh(c + d\*x)/d\*\*5 - 120\*b\*\*2\*cosh(c + d\*x)/d\*\*6, Ne(d, 0)), ((a\*\*2\*x\*\*2/2 + a\*b\*x\*\*4/2 + b\*\*2\*x\*\*6/6)\*cosh(c), True))

**Giac [A]** time = 1.17159, size = 323, normalized size = 1.76

$$\frac{(b^2d^5x^5 + 2abd^5x^3 - 5b^2d^4x^4 + a^2d^5x - 6abd^4x^2 + 20b^2d^3x^3 - a^2d^4 + 12abd^3x - 60b^2d^2x^2 - 12abd^2 + 120b^2dx - 120b^2)cosh(dx + c) - (b^2d^5x^5 + 2abd^5x^3 - 5b^2d^4x^4 + a^2d^5x - 6abd^4x^2 + 20b^2d^3x^3 - a^2d^4 + 12abd^3x - 60b^2d^2x^2 - 12abd^2 + 120b^2dx - 120b^2)sinh(dx + c)}{2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)^2\*cosh(d\*x+c),x, algorithm="giac")

[Out]  $\frac{1}{2}(b^2d^5x^5 + 2a^2bd^5x^3 - 5b^2d^4x^4 + a^2d^5x - 6a^2bd^4x^2 + 20b^2d^3x^3 - a^2d^4 + 12a^2bd^3x - 60b^2d^2x^2 - 12a^2bd^2 + 120b^2d^2x - 120b^2)e^{(dx + c)}/d^6 - \frac{1}{2}(b^2d^5x^5 + 2a^2bd^5x^3 + 5b^2d^4x^4 + a^2d^5x + 6a^2bd^4x^2 + 20b^2d^3x^3 + a^2d^4 + 12a^2bd^3x + 60b^2d^2x^2 + 12a^2bd^2 + 120b^2d^2x + 120b^2)e^{-(dx + c)}/d^6$

### 3.51 $\int (a + bx^2)^2 \cosh(c + dx) dx$

**Optimal.** Leaf size=136

$$\frac{a^2 \sinh(c + dx)}{d} + \frac{4ab \sinh(c + dx)}{d^3} - \frac{4abx \cosh(c + dx)}{d^2} + \frac{2abx^2 \sinh(c + dx)}{d} + \frac{12b^2x^2 \sinh(c + dx)}{d^3} - \frac{4b^2x^3 \cosh(c + dx)}{d^2}$$

[Out]  $(-24*b^2*x*Cosh[c + d*x])/d^4 - (4*a*b*x*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Cos$   
 $sh[c + d*x])/d^2 + (24*b^2*Sinh[c + d*x])/d^5 + (4*a*b*Sinh[c + d*x])/d^3 +$   
 $(a^2*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (2*a*b*x^2*Sinh[c$   
 $+ d*x])/d + (b^2*x^4*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.18139, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {5277, 2637, 3296}

$$\frac{a^2 \sinh(c + dx)}{d} + \frac{4ab \sinh(c + dx)}{d^3} - \frac{4abx \cosh(c + dx)}{d^2} + \frac{2abx^2 \sinh(c + dx)}{d} + \frac{12b^2x^2 \sinh(c + dx)}{d^3} - \frac{4b^2x^3 \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^2)^2*Cosh[c + d*x], x]$

[Out]  $(-24*b^2*x*Cosh[c + d*x])/d^4 - (4*a*b*x*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Cos$   
 $sh[c + d*x])/d^2 + (24*b^2*Sinh[c + d*x])/d^5 + (4*a*b*Sinh[c + d*x])/d^3 +$   
 $(a^2*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (2*a*b*x^2*Sinh[c$   
 $+ d*x])/d + (b^2*x^4*Sinh[c + d*x])/d$

#### Rule 5277

$\text{Int}[Cosh[(c_.) + (d_.)*(x_.)]*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Cosh[c + d*x], (a + b*x^n)^p, x], x] /;$   $\text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d*x]/d, x] /;$   $\text{FreeQ}[\{c, d\}, x]$

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int (a + bx^2)^2 \cosh(c + dx) dx &= \int (a^2 \cosh(c + dx) + 2abx^2 \cosh(c + dx) + b^2x^4 \cosh(c + dx)) dx \\
&= a^2 \int \cosh(c + dx) dx + (2ab) \int x^2 \cosh(c + dx) dx + b^2 \int x^4 \cosh(c + dx) dx \\
&= \frac{a^2 \sinh(c + dx)}{d} + \frac{2abx^2 \sinh(c + dx)}{d} + \frac{b^2x^4 \sinh(c + dx)}{d} - \frac{(4ab) \int x \sinh(c + dx) dx}{d} \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{4b^2x^3 \cosh(c + dx)}{d^2} + \frac{a^2 \sinh(c + dx)}{d} + \frac{2abx^2 \sinh(c + dx)}{d} + \frac{b^2x^4 \sinh(c + dx)}{d} \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{4b^2x^3 \cosh(c + dx)}{d^2} + \frac{4ab \sinh(c + dx)}{d^3} + \frac{a^2 \sinh(c + dx)}{d} + \frac{12b^2x \sinh(c + dx)}{d^3} \\
&= -\frac{24b^2x \cosh(c + dx)}{d^4} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{4b^2x^3 \cosh(c + dx)}{d^2} + \frac{4ab \sinh(c + dx)}{d^3} + \frac{a^2 \sinh(c + dx)}{d} \\
&= -\frac{24b^2x \cosh(c + dx)}{d^4} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{4b^2x^3 \cosh(c + dx)}{d^2} + \frac{24b^2 \sinh(c + dx)}{d^5}
\end{aligned}$$

**Mathematica [A]** time = 0.17899, size = 85, normalized size = 0.62

$$\frac{(a^2d^4 + 2abd^2(d^2x^2 + 2) + b^2(d^4x^4 + 12d^2x^2 + 24)) \sinh(c + dx) - 4bdx(a^2d^2 + b(d^2x^2 + 6)) \cosh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)^2\*Cosh[c + d\*x], x]

[Out] (-4\*b\*d\*x\*(a\*d^2 + b\*(6 + d^2\*x^2))\*Cosh[c + d\*x] + (a^2\*d^4 + 2\*a\*b\*d^2\*(2 + d^2\*x^2) + b^2\*(24 + 12\*d^2\*x^2 + d^4\*x^4))\*Sinh[c + d\*x])/d^5

**Maple [B]** time = 0.008, size = 332, normalized size = 2.4

$$\frac{1}{d} \left( \frac{b^2 ((dx + c)^4 \sinh(dx + c) - 4(dx + c)^3 \cosh(dx + c) + 12(dx + c)^2 \sinh(dx + c) - 24(dx + c) \cosh(dx + c) + 24 \sinh(dx + c))}{d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^2*cosh(d*x+c),x)`

[Out]  $\frac{1}{d} \left( \frac{1}{d^4 b^2} \left( (d*x+c)^4 \sinh(d*x+c) - 4(d*x+c)^3 \cosh(d*x+c) + 12(d*x+c)^2 \sinh(d*x+c) - 24(d*x+c) \cosh(d*x+c) + 24 \sinh(d*x+c) \right) - \frac{4}{d^4 b^2 c} \left( (d*x+c)^3 \sinh(d*x+c) - 3(d*x+c)^2 \cosh(d*x+c) + 6(d*x+c) \sinh(d*x+c) - 6 \cosh(d*x+c) \right) + \frac{6}{d^4 b^2 c^2} \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) + \frac{2}{d^2 b a} \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) - \frac{4}{d^4 b^2 c^3} \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) - \frac{4}{d^2 b c a} \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) + \frac{1}{d^4 b^2 c^4} \sinh(d*x+c) + \frac{2}{d^2 b c^2 a} \sinh(d*x+c) + a^2 \sinh(d*x+c) \right)$

**Maxima [A]** time = 1.03875, size = 255, normalized size = 1.88

$$\frac{a^2 e^{(dx+c)}}{2d} - \frac{a^2 e^{(-dx-c)}}{2d} + \frac{(d^2 x^2 e^c - 2 dx e^c + 2 e^c) a b e^{(dx)}}{d^3} - \frac{(d^2 x^2 + 2 dx + 2) a b e^{(-dx-c)}}{d^3} + \frac{(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 24 d x e^c + 24 e^c) b^2 e^{(dx)}}{2 d^5} - \frac{(d^4 x^4 + 4 d^3 x^3 + 12 d^2 x^2 + 24 d x + 24) b^2 e^{(-dx-c)}}{2 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")`

[Out]  $\frac{1}{2} a^2 e^{(dx+c)} / d - \frac{1}{2} a^2 e^{(-dx-c)} / d + \frac{(d^2 x^2 e^c - 2 d x e^c + 2 e^c) a b e^{(dx)}}{d^3} - \frac{(d^2 x^2 + 2 d x + 2) a b e^{(-dx-c)}}{d^3} + \frac{1}{2} \frac{(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 24 d x e^c + 24 e^c) b^2 e^{(dx)}}{d^5} - \frac{1}{2} \frac{(d^4 x^4 + 4 d^3 x^3 + 12 d^2 x^2 + 24 d x + 24) b^2 e^{(-dx-c)}}{d^5}$

**Fricas [A]** time = 2.18069, size = 207, normalized size = 1.52

$$\frac{4 \left( b^2 d^3 x^3 + (abd^3 + 6 b^2 d) x \right) \cosh(dx+c) - \left( b^2 d^4 x^4 + a^2 d^4 + 4 abd^2 + 2 (abd^4 + 6 b^2 d^2) x^2 + 24 b^2 \right) \sinh(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")`

[Out]  $-(4 * (b^2 * d^3 * x^3 + (a * b * d^3 + 6 * b^2 * d) * x) * \cosh(d * x + c) - (b^2 * d^4 * x^4 + a^2 * d^4 + 4 * a * b * d^2 + 2 * (a * b * d^4 + 6 * b^2 * d^2) * x^2 + 24 * b^2) * \sinh(d * x + c)) / d^5$

---

**Sympy [A]** time = 2.95396, size = 172, normalized size = 1.26

$$\left\{ \begin{array}{l} \frac{a^2 \sinh(c+dx)}{d} + \frac{2abx^2 \sinh(c+dx)}{d} - \frac{4abx \cosh(c+dx)}{d^2} + \frac{4ab \sinh(c+dx)}{d^3} + \frac{b^2 x^4 \sinh(c+dx)}{d} - \frac{4b^2 x^3 \cosh(c+dx)}{d^2} + \frac{12b^2 x^2 \sinh(c+dx)}{d^3} - \frac{24b^2 x}{d^4} \\ \left( a^2 x + \frac{2abx^3}{3} + \frac{b^2 x^5}{5} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2\*cosh(d\*x+c),x)

[Out] Piecewise((a\*\*2\*sinh(c + d\*x)/d + 2\*a\*b\*x\*\*2\*sinh(c + d\*x)/d - 4\*a\*b\*x\*cosh(c + d\*x)/d\*\*2 + 4\*a\*b\*sinh(c + d\*x)/d\*\*3 + b\*\*2\*x\*\*4\*sinh(c + d\*x)/d - 4\*b\*\*2\*x\*\*3\*cosh(c + d\*x)/d\*\*2 + 12\*b\*\*2\*x\*\*2\*sinh(c + d\*x)/d\*\*3 - 24\*b\*\*2\*x\*cosh(c + d\*x)/d\*\*4 + 24\*b\*\*2\*sinh(c + d\*x)/d\*\*5, Ne(d, 0)), ((a\*\*2\*x + 2\*a\*b\*x\*\*3/3 + b\*\*2\*x\*\*5/5)\*cosh(c), True))

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**Giac [A]** time = 1.17539, size = 243, normalized size = 1.79

$$\frac{(b^2 d^4 x^4 + 2 abd^4 x^2 - 4 b^2 d^3 x^3 + a^2 d^4 - 4 abd^3 x + 12 b^2 d^2 x^2 + 4 abd^2 - 24 b^2 dx + 24 b^2) e^{(dx+c)}}{2 d^5} - \frac{(b^2 d^4 x^4 + 2 abd^4 x^2 + a^2 d^4 - 4 abd^3 x + 12 b^2 d^2 x^2 + 4 abd^2 - 24 b^2 dx + 24 b^2) e^{-(dx+c)}}{2 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(b^2\*d^4\*x^4 + 2\*a\*b\*d^4\*x^2 - 4\*b^2\*d^3\*x^3 + a^2\*d^4 - 4\*a\*b\*d^3\*x + 12\*b^2\*d^2\*x^2 + 4\*a\*b\*d^2 - 24\*b^2\*d\*x + 24\*b^2)\*e^(d\*x + c)/d^5 - 1/2\*(b^2\*d^4\*x^4 + 2\*a\*b\*d^4\*x^2 + 4\*b^2\*d^3\*x^3 + a^2\*d^4 + 4\*a\*b\*d^3\*x + 12\*b^2\*d^2\*x^2 + 4\*a\*b\*d^2 + 24\*b^2\*d\*x + 24\*b^2)\*e^(-d\*x - c)/d^5

$$3.52 \quad \int \frac{(a+bx^2)^2 \cosh(c+dx)}{x} dx$$

**Optimal.** Leaf size=110

$$a^2 \cosh(c)\text{Chi}(dx) + a^2 \sinh(c)\text{Shi}(dx) - \frac{2ab \cosh(c+dx)}{d^2} + \frac{2abx \sinh(c+dx)}{d} - \frac{3b^2x^2 \cosh(c+dx)}{d^2} + \frac{6b^2x \sinh(c+dx)}{d^3}$$

[Out]  $(-6*b^2*Cosh[c + d*x])/d^4 - (2*a*b*Cosh[c + d*x])/d^2 - (3*b^2*x^2*Cosh[c + d*x])/d^2 + a^2*Cosh[c]*CoshIntegral[d*x] + (6*b^2*x*Sinh[c + d*x])/d^3 + (2*a*b*x*Sinh[c + d*x])/d + (b^2*x^3*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]$

**Rubi [A]** time = 0.188808, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5287, 3303, 3298, 3301, 3296, 2638}

$$a^2 \cosh(c)\text{Chi}(dx) + a^2 \sinh(c)\text{Shi}(dx) - \frac{2ab \cosh(c+dx)}{d^2} + \frac{2abx \sinh(c+dx)}{d} - \frac{3b^2x^2 \cosh(c+dx)}{d^2} + \frac{6b^2x \sinh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)^2\*Cosh[c + d\*x])/x,x]

[Out]  $(-6*b^2*Cosh[c + d*x])/d^4 - (2*a*b*Cosh[c + d*x])/d^2 - (3*b^2*x^2*Cosh[c + d*x])/d^2 + a^2*Cosh[c]*CoshIntegral[d*x] + (6*b^2*x*Sinh[c + d*x])/d^3 + (2*a*b*x*Sinh[c + d*x])/d + (b^2*x^3*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]$

### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
  := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
  && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
  := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
  && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
  ((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
  e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[
  {c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^2)^2 \cosh(c + dx)}{x} dx &= \int \left( \frac{a^2 \cosh(c + dx)}{x} + 2abx \cosh(c + dx) + b^2x^3 \cosh(c + dx) \right) dx \\
 &= a^2 \int \frac{\cosh(c + dx)}{x} dx + (2ab) \int x \cosh(c + dx) dx + b^2 \int x^3 \cosh(c + dx) dx \\
 &= \frac{2abx \sinh(c + dx)}{d} + \frac{b^2x^3 \sinh(c + dx)}{d} - \frac{(2ab) \int \sinh(c + dx) dx}{d} - \frac{(3b^2) \int x^2 \sinh(c + dx) dx}{d} \\
 &= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{3b^2x^2 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) + \frac{2abx \sinh(c + dx)}{d} + \frac{6b^2x^3 \sinh(c + dx)}{d^3} \\
 &= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{3b^2x^2 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) + \frac{6b^2x \sinh(c + dx)}{d^3} + \frac{6b^2x^3 \sinh(c + dx)}{d^3} \\
 &= -\frac{6b^2 \cosh(c + dx)}{d^4} - \frac{2ab \cosh(c + dx)}{d^2} - \frac{3b^2x^2 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) + \frac{6b^2x \sinh(c + dx)}{d^3} + \frac{6b^2x^3 \sinh(c + dx)}{d^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.360715, size = 82, normalized size = 0.75

$$a^2 \cosh(c) \text{Chi}(dx) + a^2 \sinh(c) \text{Shi}(dx) + \frac{bx(2ad^2 + b(d^2x^2 + 6)) \sinh(c + dx)}{d^3} - \frac{b(2ad^2 + 3b(d^2x^2 + 2)) \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^2)^2\*Cosh[c + d\*x])/x,x]

[Out] -((b\*(2\*a\*d^2 + 3\*b\*(2 + d^2\*x^2))\*Cosh[c + d\*x])/d^4) + a^2\*Cosh[c]\*CoshIntegral[d\*x] + (b\*x\*(2\*a\*d^2 + b\*(6 + d^2\*x^2))\*Sinh[c + d\*x])/d^3 + a^2\*SinhIntegral[c]\*SinhIntegral[d\*x]

**Maple [B]** time = 0.05, size = 226, normalized size = 2.1

$$-\frac{a^2 e^{-c} \text{Ei}(1, dx)}{2} - \frac{a b e^{-dx-c}}{d^2} - 3 \frac{e^{dx+c} b^2}{d^4} - \frac{a^2 e^c \text{Ei}(1, -dx)}{2} - 3 \frac{b^2 e^{-dx-c}}{d^4} + \frac{a b e^{dx+c} x}{d} - \frac{a b e^{dx+c}}{d^2} + \frac{e^{dx+c} b^2 x^3}{2d} - \frac{3 e^{dx+c} b^2 x^2}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2\*cosh(d\*x+c)/x,x)

[Out] -1/2\*a^2\*exp(-c)\*Ei(1,d\*x)-a/d^2\*b\*exp(-d\*x-c)-3/d^4\*b^2\*exp(d\*x+c)-1/2\*a^2\*exp(c)\*Ei(1,-d\*x)-3/d^4\*b^2\*exp(-d\*x-c)+a/d\*b\*exp(d\*x+c)\*x-a/d^2\*b\*exp(d\*x+c)+1/2/d\*b^2\*exp(d\*x+c)\*x^3-3/2/d^2\*b^2\*exp(d\*x+c)\*x^2+3/d^3\*b^2\*exp(d\*x+c)\*x-a/d\*b\*exp(-d\*x-c)\*x-1/2/d\*b^2\*exp(-d\*x-c)\*x^3-3/2/d^2\*b^2\*exp(-d\*x-c)\*x^2-3/d^3\*b^2\*exp(-d\*x-c)\*x

**Maxima [B]** time = 1.18744, size = 317, normalized size = 2.88

$$-\frac{1}{8} \left( 4ab \left( \frac{(d^2 x^2 e^c - 2 dx e^c + 2 e^c) e^{(dx)}}{d^3} + \frac{(d^2 x^2 + 2 dx + 2) e^{(-dx-c)}}{d^3} \right) + b^2 \left( \frac{(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 24 dx e^c + 24 e^c)}{d^5} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x,x, algorithm="maxima")

[Out] -1/8\*(4\*a\*b\*((d^2\*x^2\*e^c - 2\*d\*x\*e^c + 2\*e^c)\*e^(d\*x))/d^3 + (d^2\*x^2 + 2\*d\*x + 2)\*e^(-d\*x - c)/d^3) + b^2\*((d^4\*x^4\*e^c - 4\*d^3\*x^3\*e^c + 12\*d^2\*x^2\*e^c - 24\*d\*x\*e^c + 24\*e^c)\*e^(d\*x))/d^5 + (d^4\*x^4 + 4\*d^3\*x^3 + 12\*d^2\*x^2 + 24\*d\*x + 24)\*e^(-d\*x - c)/d^5) + 4\*a^2\*cosh(d\*x + c)\*log(x^2)/d - 4\*(Ei(-d\*x)\*e^(-c) + Ei(d\*x)\*e^c)\*a^2/d\*d + 1/4\*(b^2\*x^4 + 4\*a\*b\*x^2 + 2\*a^2\*log(x^2))\*cosh(d\*x + c)



---

**Fricas [A]** time = 2.47665, size = 292, normalized size = 2.65

$$\frac{2(3b^2d^2x^2 + 2abd^2 + 6b^2)\cosh(dx + c) - (a^2d^4\text{Ei}(dx) + a^2d^4\text{Ei}(-dx))\cosh(c) - 2(b^2d^3x^3 + 2(abd^3 + 3b^2d)x)\sinh(dx + c) - (a^2d^4\text{Ei}(d*x) - a^2d^4\text{Ei}(-d*x))*\sinh(c)}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x,x, algorithm="fricas")

[Out] -1/2\*(2\*(3\*b^2\*d^2\*x^2 + 2\*a\*b\*d^2 + 6\*b^2)\*cosh(d\*x + c) - (a^2\*d^4\*Ei(d\*x) + a^2\*d^4\*Ei(-d\*x))\*cosh(c) - 2\*(b^2\*d^3\*x^3 + 2\*(a\*b\*d^3 + 3\*b^2\*d)\*x)\*sinh(d\*x + c) - (a^2\*d^4\*Ei(d\*x) - a^2\*d^4\*Ei(-d\*x))\*sinh(c))/d^4

---

**Sympy [A]** time = 5.73347, size = 121, normalized size = 1.1

$$a^2 \sinh(c) \text{Shi}(dx) + a^2 \cosh(c) \text{Chi}(dx) + 2ab \left( \begin{cases} \frac{x \sinh(c+dx)}{x^2 \cosh(c)} - \frac{\cosh(c+dx)}{d^2} & \text{for } d \neq 0 \\ \frac{d}{2} & \text{otherwise} \end{cases} \right) + b^2 \left( \begin{cases} \frac{x^3 \sinh(c+dx)}{x^4 \cosh(c)} - \frac{3x^2 \cosh(c)}{d^2} & \text{for } d \neq 0 \\ \frac{d}{4} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2\*cosh(d\*x+c)/x,x)

[Out] a\*\*2\*sinh(c)\*Shi(d\*x) + a\*\*2\*cosh(c)\*Chi(d\*x) + 2\*a\*b\*Piecewise((x\*sinh(c + d\*x)/d - cosh(c + d\*x)/d\*\*2, Ne(d, 0)), (x\*\*2\*cosh(c)/2, True)) + b\*\*2\*Piecewise((x\*\*3\*sinh(c + d\*x)/d - 3\*x\*\*2\*cosh(c + d\*x)/d\*\*2 + 6\*x\*sinh(c + d\*x)/d\*\*3 - 6\*cosh(c + d\*x)/d\*\*4, Ne(d, 0)), (x\*\*4\*cosh(c)/4, True))

---

**Giac [B]** time = 1.20143, size = 300, normalized size = 2.73

$$\frac{b^2d^3x^3e^{(dx+c)} - b^2d^3x^3e^{(-dx-c)} + a^2d^4\text{Ei}(-dx)e^{(-c)} + a^2d^4\text{Ei}(dx)e^c + 2abd^3xe^{(dx+c)} - 3b^2d^2x^2e^{(dx+c)} - 2abd^3xe^{(-dx-c)} - 2abd^3xe^{(-dx-c)}}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x,x, algorithm="giac")

[Out] 1/2\*(b^2\*d^3\*x^3\*e^(d\*x + c) - b^2\*d^3\*x^3\*e^(-d\*x - c) + a^2\*d^4\*Ei(-d\*x)\*e^(-c) + a^2\*d^4\*Ei(d\*x)\*e^c + 2\*a\*b\*d^3\*x\*e^(d\*x + c) - 3\*b^2\*d^2\*x^2\*e^(d\*x + c) - 2\*b^2\*d^2\*x^2\*e^(-d\*x - c) - 2\*b^2\*d^2\*x^2\*e^(-d\*x - c))/d^4

$$\begin{aligned} & *x + c) - 2*a*b*d^3*x*e^{(-d*x - c)} - 3*b^2*d^2*x^2*e^{(-d*x - c)} - 2*a*b*d^2 \\ & *e^{(d*x + c)} + 6*b^2*d*x*e^{(d*x + c)} - 2*a*b*d^2*e^{(-d*x - c)} - 6*b^2*d*x*e \\ & ^{(-d*x - c)} - 6*b^2*e^{(d*x + c)} - 6*b^2*e^{(-d*x - c)})/d^4 \end{aligned}$$

$$3.53 \quad \int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2} dx$$

**Optimal.** Leaf size=95

$$a^2 d \sinh(c) \text{Chi}(dx) + a^2 d \cosh(c) \text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + \frac{2ab \sinh(c+dx)}{d} + \frac{2b^2 \sinh(c+dx)}{d^3} - \frac{2b^2 x \cosh(c+dx)}{d^2}$$

[Out] -((a^2\*Cosh[c + d\*x])/x) - (2\*b^2\*x\*Cosh[c + d\*x])/d^2 + a^2\*d\*CoshIntegral[d\*x]\*Sinh[c] + (2\*b^2\*Sinh[c + d\*x])/d^3 + (2\*a\*b\*Sinh[c + d\*x])/d + (b^2\*x^2\*Sinh[c + d\*x])/d + a^2\*d\*Cosh[c]\*SinhIntegral[d\*x]

**Rubi [A]** time = 0.176575, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {5287, 2637, 3297, 3303, 3298, 3301, 3296}

$$a^2 d \sinh(c) \text{Chi}(dx) + a^2 d \cosh(c) \text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + \frac{2ab \sinh(c+dx)}{d} + \frac{2b^2 \sinh(c+dx)}{d^3} - \frac{2b^2 x \cosh(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)^2\*Cosh[c + d\*x])/x^2,x]

[Out] -((a^2\*Cosh[c + d\*x])/x) - (2\*b^2\*x\*Cosh[c + d\*x])/d^2 + a^2\*d\*CoshIntegral[d\*x]\*Sinh[c] + (2\*b^2\*Sinh[c + d\*x])/d^3 + (2\*a\*b\*Sinh[c + d\*x])/d + (b^2\*x^2\*Sinh[c + d\*x])/d + a^2\*d\*Cosh[c]\*SinhIntegral[d\*x]

#### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c

+ d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^2} dx &= \int \left( 2ab \cosh(c + dx) + \frac{a^2 \cosh(c + dx)}{x^2} + b^2 x^2 \cosh(c + dx) \right) dx \\
 &= a^2 \int \frac{\cosh(c + dx)}{x^2} dx + (2ab) \int \cosh(c + dx) dx + b^2 \int x^2 \cosh(c + dx) dx \\
 &= -\frac{a^2 \cosh(c + dx)}{x} + \frac{2ab \sinh(c + dx)}{d} + \frac{b^2 x^2 \sinh(c + dx)}{d} - \frac{(2b^2) \int x \sinh(c + dx) dx}{d} \\
 &= -\frac{a^2 \cosh(c + dx)}{x} - \frac{2b^2 x \cosh(c + dx)}{d^2} + \frac{2ab \sinh(c + dx)}{d} + \frac{b^2 x^2 \sinh(c + dx)}{d} + \frac{(2b^2)}{d} \\
 &= -\frac{a^2 \cosh(c + dx)}{x} - \frac{2b^2 x \cosh(c + dx)}{d^2} + a^2 d \operatorname{Chi}(dx) \sinh(c) + \frac{2b^2 \sinh(c + dx)}{d^3} + \frac{2ab}{d}
 \end{aligned}$$

**Mathematica [A]** time = 0.262884, size = 95, normalized size = 1.

$$a^2 d \sinh(c) \operatorname{Chi}(dx) + a^2 d \cosh(c) \operatorname{Shi}(dx) - \frac{a^2 \cosh(c + dx)}{x} + \frac{2ab \sinh(c + dx)}{d} + \frac{2b^2 \sinh(c + dx)}{d^3} - \frac{2b^2 x \cosh(c + dx)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^2)^2\*Cosh[c + d\*x])/x^2,x]

[Out] -((a^2\*Cosh[c + d\*x])/x) - (2\*b^2\*x\*Cosh[c + d\*x])/d^2 + a^2\*d\*CoshIntegral[d\*x]\*Sinh[c] + (2\*b^2\*Sinh[c + d\*x])/d^3 + (2\*a\*b\*Sinh[c + d\*x])/d + (b^2\*x^2\*Sinh[c + d\*x])/d + a^2\*d\*Cosh[c]\*SinhIntegral[d\*x]

**Maple [A]** time = 0.069, size = 190, normalized size = 2.

$$\frac{da^2e^{-c}\operatorname{Ei}(1,dx)}{2} - \frac{abe^{-dx-c}}{d} - \frac{a^2e^{-dx-c}}{2x} - \frac{b^2e^{-dx-c}}{d^3} - \frac{b^2e^{-dx-c}x^2}{2d} - \frac{b^2e^{-dx-c}x}{d^2} + \frac{e^{dx+c}b^2}{d^3} - \frac{da^2e^c\operatorname{Ei}(1,-dx)}{2} + \frac{abe^{dx+c}}{d} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2\*cosh(d\*x+c)/x^2,x)

[Out] 1/2\*d\*a^2\*exp(-c)\*Ei(1,d\*x)-a\*b/d\*exp(-d\*x-c)-1/2\*a^2\*exp(-d\*x-c)/x-1/d^3\*b^2\*exp(-d\*x-c)-1/2/d\*b^2\*exp(-d\*x-c)\*x^2-1/d^2\*b^2\*exp(-d\*x-c)\*x+1/d^3\*b^2\*exp(d\*x+c)-1/2\*d\*a^2\*exp(c)\*Ei(1,-d\*x)+a\*b/d\*exp(d\*x+c)+1/2/d\*b^2\*exp(d\*x+c)\*x^2-1/d^2\*b^2\*exp(d\*x+c)\*x-1/2\*a^2/x\*exp(d\*x+c)

**Maxima [A]** time = 1.20407, size = 242, normalized size = 2.55

$$-\frac{1}{6} \left( 3a^2 \operatorname{Ei}(-dx) e^{(-c)} - 3a^2 \operatorname{Ei}(dx) e^c + \frac{6(dx e^c - e^c) a b e^{(dx)}}{d^2} + \frac{6(dx + 1) a b e^{(-dx-c)}}{d^2} + \frac{(d^3 x^3 e^c - 3d^2 x^2 e^c + 6dx e^c - 6e^c) b^2}{d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x^2,x, algorithm="maxima")

[Out] -1/6\*(3\*a^2\*Ei(-d\*x)\*e^(-c) - 3\*a^2\*Ei(d\*x)\*e^c + 6\*(d\*x\*e^c - e^c)\*a\*b\*e^(d\*x)/d^2 + 6\*(d\*x + 1)\*a\*b\*e^(-d\*x - c)/d^2 + (d^3\*x^3\*e^c - 3\*d^2\*x^2\*e^c + 6\*d\*x\*e^c - 6\*e^c)\*b^2\*e^(d\*x)/d^4 + (d^3\*x^3 + 3\*d^2\*x^2 + 6\*d\*x + 6)\*b^2

$$2e^{-(dx - c)/d^4} * d + 1/3 * (b^2 * x^3 + 6 * a * b * x - 3 * a^2 / x) * \cosh(dx + c)$$

**Fricas [A]** time = 2.0144, size = 286, normalized size = 3.01

$$\frac{2(a^2 d^3 + 2 b^2 dx^2) \cosh(dx + c) - (a^2 d^4 x \operatorname{Ei}(dx) - a^2 d^4 x \operatorname{Ei}(-dx)) \cosh(c) - 2(b^2 d^2 x^3 + 2(abd^2 + b^2)x) \sinh(dx + c)}{2 d^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x^2,x, algorithm="fricas")

[Out]  $-1/2 * (2 * (a^2 * d^3 + 2 * b^2 * d * x^2) * \cosh(d * x + c) - (a^2 * d^4 * x * \operatorname{Ei}(d * x) - a^2 * d^4 * x * \operatorname{Ei}(-d * x)) * \cosh(c) - 2 * (b^2 * d^2 * x^3 + 2 * (a * b * d^2 + b^2) * x) * \sinh(d * x + c) - (a^2 * d^4 * x * \operatorname{Ei}(d * x) + a^2 * d^4 * x * \operatorname{Ei}(-d * x)) * \sinh(c)) / (d^3 * x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2\*cosh(d\*x+c)/x\*\*2,x)

[Out] Integral((a + b\*x\*\*2)\*\*2\*cosh(c + d\*x)/x\*\*2, x)

**Giac [B]** time = 1.19945, size = 266, normalized size = 2.8

$$\frac{a^2 d^4 x \operatorname{Ei}(-dx) e^{(-c)} - a^2 d^4 x \operatorname{Ei}(dx) e^c - b^2 d^2 x^3 e^{(dx+c)} + b^2 d^2 x^3 e^{(-dx-c)} + a^2 d^3 e^{(dx+c)} - 2abd^2 x e^{(dx+c)} + 2b^2 dx^2 e^{(dx+c)} + a^2 d^3 e^{(-dx-c)}}{2 d^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x^2,x, algorithm="giac")

[Out]  $-1/2 * (a^2 * d^4 * x * \operatorname{Ei}(-d * x) * e^{(-c)} - a^2 * d^4 * x * \operatorname{Ei}(d * x) * e^c - b^2 * d^2 * x^3 * e^{(d * x + c)} + b^2 * d^2 * x^3 * e^{(-d * x - c)} + a^2 * d^3 * e^{(d * x + c)} - 2 * a * b * d^2 * x * e^{(d * x + c)} + 2 * b^2 * d * x^2 * e^{(d * x + c)} + a^2 * d^3 * e^{(-d * x - c)})$

$$\frac{x + c + 2*b^2*d*x^2*e^{(d*x + c)} + a^2*d^3*e^{(-d*x - c)} + 2*a*b*d^2*x*e^{(-d*x - c)} + 2*b^2*d*x^2*e^{(-d*x - c)} - 2*b^2*x*e^{(d*x + c)} + 2*b^2*x*e^{(-d*x - c)}}{(d^3*x)}$$

$$3.54 \quad \int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} dx$$

**Optimal.** Leaf size=114

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + 2ab \cosh(c)\text{Chi}(dx) + 2ab \sinh(c)\text{Shi}(dx)$$

[Out]  $-\left(\frac{b^2 \text{Cosh}[c + d*x]}{d^2}\right) - \frac{a^2 \text{Cosh}[c + d*x]}{(2*x^2)} + 2*a*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + \frac{a^2*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x]}{2} - \frac{a^2*d*\text{Sinh}[c + d*x]}{(2*x)} + \frac{b^2*x*\text{Sinh}[c + d*x]}{d} + 2*a*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + \frac{a^2*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x]}{2}$

**Rubi [A]** time = 0.225079, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638}

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + 2ab \cosh(c)\text{Chi}(dx) + 2ab \sinh(c)\text{Shi}(dx)$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)^2\*Cosh[c + d\*x])/x^3,x]

[Out]  $-\left(\frac{b^2 \text{Cosh}[c + d*x]}{d^2}\right) - \frac{a^2 \text{Cosh}[c + d*x]}{(2*x^2)} + 2*a*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + \frac{a^2*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x]}{2} - \frac{a^2*d*\text{Sinh}[c + d*x]}{(2*x)} + \frac{b^2*x*\text{Sinh}[c + d*x]}{d} + 2*a*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + \frac{a^2*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x]}{2}$

### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]



Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^3} dx &= \int \left( \frac{a^2 \cosh(c + dx)}{x^3} + \frac{2ab \cosh(c + dx)}{x} + b^2 x \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^3} dx + (2ab) \int \frac{\cosh(c + dx)}{x} dx + b^2 \int x \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} + \frac{b^2 x \sinh(c + dx)}{d} - \frac{b^2 \int \sinh(c + dx) dx}{d} + \frac{1}{2} (a^2 d) \int \frac{\sinh(c + dx)}{x^2} \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{2x^2} + 2ab \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{2x} + \frac{b^2 x \sinh(c + dx)}{2} \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{2x^2} + 2ab \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{2x} + \frac{b^2 x \sinh(c + dx)}{2} \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{2x^2} + 2ab \cosh(c) \text{Chi}(dx) + \frac{1}{2} a^2 d^2 \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{2x} + \frac{b^2 x \sinh(c + dx)}{2}
\end{aligned}$$

**Mathematica [A]** time = 0.348361, size = 97, normalized size = 0.85

$$\frac{1}{2} \left( -\frac{a^2 \cosh(c + dx)}{x^2} - \frac{a^2 d \sinh(c + dx)}{x} + a \cosh(c) (ad^2 + 4b) \text{Chi}(dx) + a \sinh(c) (ad^2 + 4b) \text{Shi}(dx) - \frac{2b^2 \cosh(c + dx)}{d^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^2)^2\*Cosh[c + d\*x])/x^3,x]

[Out] ((-2\*b^2\*Cosh[c + d\*x])/d^2 - (a^2\*Cosh[c + d\*x])/x^2 + a\*(4\*b + a\*d^2)\*Cosh[c]\*CoshIntegral[d\*x] - (a^2\*d\*Sinh[c + d\*x])/x + (2\*b^2\*x\*Sinh[c + d\*x])/d + a\*(4\*b + a\*d^2)\*Sinh[c]\*SinhIntegral[d\*x])/2

**Maple [A]** time = 0.084, size = 188, normalized size = 1.7

$$-\frac{d^2 a^2 e^{-c} \text{Ei}(1, dx)}{4} + \frac{d a^2 e^{-dx-c}}{4x} - \frac{a^2 e^{-dx-c}}{4x^2} - \frac{b^2 e^{-dx-c}}{2d^2} - a b e^{-c} \text{Ei}(1, dx) - \frac{b^2 e^{-dx-c} x}{2d} - \frac{e^{dx+c} b^2}{2d^2} - \frac{d^2 a^2 e^c \text{Ei}(1, -dx)}{4} + \frac{e^{dx+c} a^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2\*cosh(d\*x+c)/x^3,x)

[Out] -1/4\*d^2\*a^2\*exp(-c)\*Ei(1,d\*x)+1/4\*d\*a^2\*exp(-d\*x-c)/x-1/4\*a^2\*exp(-d\*x-c)/x^2-1/2\*b^2/d^2\*exp(-d\*x-c)-a\*b\*exp(-c)\*Ei(1,d\*x)-1/2\*b^2/d\*exp(-d\*x-c)\*x-1/2\*b^2/d^2\*exp(d\*x+c)-1/4\*d^2\*a^2\*exp(c)\*Ei(1,-d\*x)+1/2\*b^2/d\*exp(d\*x+c)\*x-

$$1/4*a^2/x^2*\exp(d*x+c)-1/4*d*a^2/x*\exp(d*x+c)-a*b*\exp(c)*\text{Ei}(1,-d*x)$$

**Maxima [A]** time = 1.3475, size = 223, normalized size = 1.96

$$\frac{1}{4} \left( (de^{(-c)}\Gamma(-1, dx) + de^c\Gamma(-1, -dx))a^2 - b^2 \left( \frac{(d^2x^2e^c - 2dxe^c + 2e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2dx + 2)e^{(-dx-c)}}{d^3} \right) - \frac{4ab \cosh(dx + c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x^3,x, algorithm="maxima")

[Out] 1/4\*((d\*e^(-c)\*gamma(-1, d\*x) + d\*e^c\*gamma(-1, -d\*x))\*a^2 - b^2\*((d^2\*x^2\*e^c - 2\*d\*x\*e^c + 2\*e^c)\*e^(d\*x)/d^3 + (d^2\*x^2 + 2\*d\*x + 2)\*e^(-d\*x - c)/d^3) - 4\*a\*b\*cosh(d\*x + c)\*log(x^2)/d + 4\*(Ei(-d\*x)\*e^(-c) + Ei(d\*x)\*e^c)\*a\*b/d\*d + 1/2\*(b^2\*x^2 + 2\*a\*b\*log(x^2) - a^2/x^2)\*cosh(d\*x + c)

**Fricas [A]** time = 1.99347, size = 359, normalized size = 3.15

$$\frac{2(a^2d^2 + 2b^2x^2) \cosh(dx + c) - ((a^2d^4 + 4abd^2)x^2\text{Ei}(dx) + (a^2d^4 + 4abd^2)x^2\text{Ei}(-dx)) \cosh(c) + 2(a^2d^3x - 2b^2dx) \sinh(c)}{4d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x^3,x, algorithm="fricas")

[Out] -1/4\*(2\*(a^2\*d^2 + 2\*b^2\*x^2)\*cosh(d\*x + c) - ((a^2\*d^4 + 4\*a\*b\*d^2)\*x^2\*Ei(d\*x) + (a^2\*d^4 + 4\*a\*b\*d^2)\*x^2\*Ei(-d\*x))\*cosh(c) + 2\*(a^2\*d^3\*x - 2\*b^2\*d\*x^3)\*sinh(d\*x + c) - ((a^2\*d^4 + 4\*a\*b\*d^2)\*x^2\*Ei(d\*x) - (a^2\*d^4 + 4\*a\*b\*d^2)\*x^2\*Ei(-d\*x))\*sinh(c))/(d^2\*x^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2\*cosh(d\*x+c)/x\*\*3,x)

[Out] Integral((a + b\*x\*\*2)\*\*2\*cosh(c + d\*x)/x\*\*3, x)

**Giac [A]** time = 1.21957, size = 278, normalized size = 2.44

$$\frac{a^2 d^4 x^2 \operatorname{Ei}(-dx) e^{(-c)} + a^2 d^4 x^2 \operatorname{Ei}(dx) e^c + 4 a b d^2 x^2 \operatorname{Ei}(-dx) e^{(-c)} + 4 a b d^2 x^2 \operatorname{Ei}(dx) e^c - a^2 d^3 x e^{(dx+c)} + 2 b^2 d x^3 e^{(dx+c)} + a^2 d^3 x e^{(-dx-c)}}{4 d^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x^3,x, algorithm="giac")

[Out]  $\frac{1}{4} * (a^2 * d^4 * x^2 * \operatorname{Ei}(-d*x) * e^{(-c)} + a^2 * d^4 * x^2 * \operatorname{Ei}(d*x) * e^c + 4 * a * b * d^2 * x^2 * \operatorname{Ei}(-d*x) * e^{(-c)} + 4 * a * b * d^2 * x^2 * \operatorname{Ei}(d*x) * e^c - a^2 * d^3 * x * e^{(d*x + c)} + 2 * b^2 * d * x^3 * e^{(d*x + c)} + a^2 * d^3 * x * e^{(-d*x - c)} - 2 * b^2 * d * x^3 * e^{(-d*x - c)} - a^2 * d^2 * e^{(d*x + c)} - 2 * b^2 * x^2 * e^{(d*x + c)} - a^2 * d^2 * e^{(-d*x - c)} - 2 * b^2 * x^2 * e^{(-d*x - c)}) / (d^2 * x^2)$

$$3.55 \quad \int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4} dx$$

**Optimal.** Leaf size=133

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2d \sinh(c+dx)}{6x^2} - \frac{a^2 \cosh(c+dx)}{3x^3} + 2abd \sinh$$

[Out]  $-(a^2 \text{Cosh}[c + d*x])/(3*x^3) - (2*a*b*\text{Cosh}[c + d*x])/x - (a^2*d^2*\text{Cosh}[c + d*x])/(6*x) + 2*a*b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + (a^2*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 + (b^2*\text{Sinh}[c + d*x])/d - (a^2*d*\text{Sinh}[c + d*x])/(6*x^2) + 2*a*b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + (a^2*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6$

**Rubi [A]** time = 0.258916, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5287, 2637, 3297, 3303, 3298, 3301}

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2d \sinh(c+dx)}{6x^2} - \frac{a^2 \cosh(c+dx)}{3x^3} + 2abd \sinh$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)^2\*Cosh[c + d\*x])/x^4,x]

[Out]  $-(a^2 \text{Cosh}[c + d*x])/(3*x^3) - (2*a*b*\text{Cosh}[c + d*x])/x - (a^2*d^2*\text{Cosh}[c + d*x])/(6*x) + 2*a*b*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + (a^2*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 + (b^2*\text{Sinh}[c + d*x])/d - (a^2*d*\text{Sinh}[c + d*x])/(6*x^2) + 2*a*b*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + (a^2*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6$

#### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^4} dx &= \int \left( b^2 \cosh(c + dx) + \frac{a^2 \cosh(c + dx)}{x^4} + \frac{2ab \cosh(c + dx)}{x^2} \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^4} dx + (2ab) \int \frac{\cosh(c + dx)}{x^2} dx + b^2 \int \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{2ab \cosh(c + dx)}{x} + \frac{b^2 \sinh(c + dx)}{d} + \frac{1}{3} (a^2 d) \int \frac{\sinh(c + dx)}{x^3} dx + \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{2ab \cosh(c + dx)}{x} + \frac{b^2 \sinh(c + dx)}{d} - \frac{a^2 d \sinh(c + dx)}{6x^2} + \frac{1}{6} (a^2 d^2) \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{2ab \cosh(c + dx)}{x} - \frac{a^2 d^2 \cosh(c + dx)}{6x} + 2abd \operatorname{Chi}(dx) \sinh(c) + \frac{b^2}{6} \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{2ab \cosh(c + dx)}{x} - \frac{a^2 d^2 \cosh(c + dx)}{6x} + 2abd \operatorname{Chi}(dx) \sinh(c) + \frac{b^2}{6} \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{2ab \cosh(c + dx)}{x} - \frac{a^2 d^2 \cosh(c + dx)}{6x} + 2abd \operatorname{Chi}(dx) \sinh(c) + \frac{1}{6} a
\end{aligned}$$

**Mathematica [A]** time = 0.401937, size = 114, normalized size = 0.86

$$\frac{1}{6} \left( -\frac{a^2 d^2 \cosh(c + dx)}{x} - \frac{a^2 d \sinh(c + dx)}{x^2} - \frac{2a^2 \cosh(c + dx)}{x^3} + ad \sinh(c) (ad^2 + 12b) \text{Chi}(dx) + ad \cosh(c) (ad^2 + 12b) \text{Chi}(dx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^2)^2\*Cosh[c + d\*x])/x^4,x]

[Out] ((-2\*a^2\*Cosh[c + d\*x])/x^3 - (12\*a\*b\*Cosh[c + d\*x])/x - (a^2\*d^2\*Cosh[c + d\*x])/x + a\*d\*(12\*b + a\*d^2)\*CoshIntegral[d\*x]\*Sinh[c] + (6\*b^2\*Sinh[c + d\*x])/d - (a^2\*d\*Sinh[c + d\*x])/x^2 + a\*d\*(12\*b + a\*d^2)\*Cosh[c]\*SinhIntegral[d\*x])/6

**Maple [A]** time = 0.092, size = 222, normalized size = 1.7

$$\frac{d^3 a^2 e^{-c} \text{Ei}(1, dx)}{12} - \frac{b^2 e^{-dx-c}}{2d} - \frac{abe^{-dx-c}}{x} + dabe^{-c} \text{Ei}(1, dx) - \frac{a^2 d^2 e^{-dx-c}}{12x} + \frac{da^2 e^{-dx-c}}{12x^2} - \frac{a^2 e^{-dx-c}}{6x^3} - \frac{e^{dx+c} a^2}{6x^3} - \frac{da^2 e^{dx+c}}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2\*cosh(d\*x+c)/x^4,x)

[Out] 1/12\*d^3\*a^2\*exp(-c)\*Ei(1,d\*x)-1/2\*b^2/d\*exp(-d\*x-c)-a\*b\*exp(-d\*x-c)/x+d\*a\*b\*exp(-c)\*Ei(1,d\*x)-1/12\*d^2\*a^2\*exp(-d\*x-c)/x+1/12\*d\*a^2\*exp(-d\*x-c)/x^2-1/6\*a^2\*exp(-d\*x-c)/x^3-1/6\*a^2/x^3\*exp(d\*x+c)-1/12\*d\*a^2/x^2\*exp(d\*x+c)-1/12\*d^2\*a^2/x\*exp(d\*x+c)+1/2\*b^2/d\*exp(d\*x+c)-1/12\*d^3\*a^2\*exp(c)\*Ei(1,-d\*x)-a\*b/x\*exp(d\*x+c)-d\*a\*b\*exp(c)\*Ei(1,-d\*x)

**Maxima [A]** time = 1.23669, size = 182, normalized size = 1.37

$$\frac{1}{6} \left( a^2 d^2 e^{(-c)} \Gamma(-2, dx) - a^2 d^2 e^c \Gamma(-2, -dx) - 6 ab \text{Ei}(-dx) e^{(-c)} + 6 ab \text{Ei}(dx) e^c - \frac{3(dx e^c - e^c) b^2 e^{(dx)}}{d^2} - \frac{3(dx + 1) b^2 e^{(-dx-c)}}{d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x^4,x, algorithm="maxima")

[Out] 1/6\*(a^2\*d^2\*e^(-c)\*gamma(-2, d\*x) - a^2\*d^2\*e^c\*gamma(-2, -d\*x) - 6\*a\*b\*Ei(-d\*x)\*e^(-c) + 6\*a\*b\*Ei(d\*x)\*e^c - 3\*(d\*x\*e^c - e^c)\*b^2\*e^(d\*x)/d^2 - 3\*(

$d*x + 1)*b^2*e^{(-d*x - c)/d^2}*d + 1/3*(3*b^2*x - (6*a*b*x^2 + a^2)/x^3)*\cosh(d*x + c)$

**Fricas [A]** time = 2.04377, size = 381, normalized size = 2.86

$$\frac{2 \left( 2 a^2 d + (a^2 d^3 + 12 a b d) x^2 \right) \cosh(dx + c) - \left( (a^2 d^4 + 12 a b d^2) x^3 \operatorname{Ei}(dx) - (a^2 d^4 + 12 a b d^2) x^3 \operatorname{Ei}(-dx) \right) \cosh(c) + 2 \left( a^2 d^2 x - 6 b^2 x^3 \right) \sinh(dx + c) - \left( (a^2 d^4 + 12 a b d^2) x^3 \operatorname{Ei}(dx) + (a^2 d^4 + 12 a b d^2) x^3 \operatorname{Ei}(-dx) \right) \sinh(c)}{12 dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x^4,x, algorithm="fricas")

[Out]  $-1/12*(2*(2*a^2*d + (a^2*d^3 + 12*a*b*d)*x^2)*\cosh(d*x + c) - ((a^2*d^4 + 12*a*b*d^2)*x^3*\operatorname{Ei}(d*x) - (a^2*d^4 + 12*a*b*d^2)*x^3*\operatorname{Ei}(-d*x))*\cosh(c) + 2*(a^2*d^2*x - 6*b^2*x^3)*\sinh(d*x + c) - ((a^2*d^4 + 12*a*b*d^2)*x^3*\operatorname{Ei}(d*x) + (a^2*d^4 + 12*a*b*d^2)*x^3*\operatorname{Ei}(-d*x))*\sinh(c))/(d*x^3)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*2\*cosh(d\*x+c)/x\*\*4,x)

[Out] Integral((a + b\*x\*\*2)\*\*2\*cosh(c + d\*x)/x\*\*4, x)

**Giac [A]** time = 1.1575, size = 319, normalized size = 2.4

$$\frac{a^2 d^4 x^3 \operatorname{Ei}(-dx) e^{(-c)} - a^2 d^4 x^3 \operatorname{Ei}(dx) e^c + 12 a b d^2 x^3 \operatorname{Ei}(-dx) e^{(-c)} - 12 a b d^2 x^3 \operatorname{Ei}(dx) e^c + a^2 d^3 x^2 e^{(dx+c)} + a^2 d^3 x^2 e^{(-dx-c)}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x^4,x, algorithm="giac")



```
[Out] -1/12*(a^2*d^4*x^3*Ei(-d*x)*e^(-c) - a^2*d^4*x^3*Ei(d*x)*e^c + 12*a*b*d^2*x^3*Ei(-d*x)*e^(-c) - 12*a*b*d^2*x^3*Ei(d*x)*e^c + a^2*d^3*x^2*e^(d*x + c) + a^2*d^3*x^2*e^(-d*x - c) + a^2*d^2*x*e^(d*x + c) + 12*a*b*d*x^2*e^(d*x + c) - 6*b^2*x^3*e^(d*x + c) - a^2*d^2*x*e^(-d*x - c) + 12*a*b*d*x^2*e^(-d*x - c) + 6*b^2*x^3*e^(-d*x - c) + 2*a^2*d*e^(d*x + c) + 2*a^2*d*e^(-d*x - c))/ (d*x^3)
```

$$3.56 \quad \int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} dx$$

**Optimal.** Leaf size=175

$$\frac{1}{24}a^2d^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4 \sinh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{24x^2} - \frac{a^2d^3 \sinh(c+dx)}{24x} - \frac{a^2d \sinh(c+dx)}{12x^3} - \frac{a^2 \cosh(c+dx)}{4x^4}$$

[Out]  $-(a^2 \text{Cosh}[c + d*x])/(4*x^4) - (a*b*\text{Cosh}[c + d*x])/x^2 - (a^2*d^2*\text{Cosh}[c + d*x])/(24*x^2) + b^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + a*b*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (a^2*d^4*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/24 - (a^2*d*\text{Sinh}[c + d*x])/(12*x^3) - (a*b*d*\text{Sinh}[c + d*x])/x - (a^2*d^3*\text{Sinh}[c + d*x])/(24*x) + b^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + a*b*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + (a^2*d^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/24$

**Rubi [A]** time = 0.35956, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{24}a^2d^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4 \sinh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{24x^2} - \frac{a^2d^3 \sinh(c+dx)}{24x} - \frac{a^2d \sinh(c+dx)}{12x^3} - \frac{a^2 \cosh(c+dx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^2)^2\*Cosh[c + d\*x])/x^5,x]

[Out]  $-(a^2 \text{Cosh}[c + d*x])/(4*x^4) - (a*b*\text{Cosh}[c + d*x])/x^2 - (a^2*d^2*\text{Cosh}[c + d*x])/(24*x^2) + b^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + a*b*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (a^2*d^4*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/24 - (a^2*d*\text{Sinh}[c + d*x])/(12*x^3) - (a*b*d*\text{Sinh}[c + d*x])/x - (a^2*d^3*\text{Sinh}[c + d*x])/(24*x) + b^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + a*b*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + (a^2*d^4*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/24$

#### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c

+ d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^5} dx &= \int \left( \frac{a^2 \cosh(c + dx)}{x^5} + \frac{2ab \cosh(c + dx)}{x^3} + \frac{b^2 \cosh(c + dx)}{x} \right) dx \\
 &= a^2 \int \frac{\cosh(c + dx)}{x^5} dx + (2ab) \int \frac{\cosh(c + dx)}{x^3} dx + b^2 \int \frac{\cosh(c + dx)}{x} dx \\
 &= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} + \frac{1}{4} (a^2 d) \int \frac{\sinh(c + dx)}{x^4} dx + (abd) \int \frac{\sinh(c + dx)}{x^2} dx \\
 &= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} + b^2 \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{12x^3} - \frac{abd \sinh(c + dx)}{2x} \\
 &= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} + b^2 \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{12x^3} \\
 &= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} + b^2 \cosh(c) \text{Chi}(dx) + \frac{abd^2 \sinh(c + dx)}{2x} \\
 &= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} + b^2 \cosh(c) \text{Chi}(dx) + \frac{abd^2 \sinh(c + dx)}{2x} \\
 &= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{ab \cosh(c + dx)}{x^2} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} + b^2 \cosh(c) \text{Chi}(dx) + \frac{abd^2 \sinh(c + dx)}{2x}
 \end{aligned}$$

**Mathematica [A]** time = 0.438824, size = 124, normalized size = 0.71

$$\frac{x^4 \cosh(c) (a^2 d^4 + 24abd^2 + 24b^2) \operatorname{Chi}(dx) + x^4 \sinh(c) (a^2 d^4 + 24abd^2 + 24b^2) \operatorname{Shi}(dx) - a (dx (ad^2 x^2 + 2a + 24bx^2) \operatorname{Si}(dx) - a^2 dx^2)}{24x^4}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^2)^2\*Cosh[c + d\*x])/x^5,x]

[Out] ((24\*b^2 + 24\*a\*b\*d^2 + a^2\*d^4)\*x^4\*Cosh[c]\*CoshIntegral[d\*x] - a\*((6\*a + 24\*b\*x^2 + a\*d^2\*x^2)\*Cosh[c + d\*x] + d\*x\*(2\*a + 24\*b\*x^2 + a\*d^2\*x^2)\*Sinh[c + d\*x]) + (24\*b^2 + 24\*a\*b\*d^2 + a^2\*d^4)\*x^4\*Sinh[c]\*SinhIntegral[d\*x]) / (24\*x^4)

**Maple [A]** time = 0.116, size = 291, normalized size = 1.7

$$-\frac{d^4 a^2 e^{-c} \operatorname{Ei}(1, dx)}{48} + \frac{bd a e^{-dx-c}}{2x} - \frac{ab e^{-dx-c}}{2x^2} - \frac{d^2 ab e^{-c} \operatorname{Ei}(1, dx)}{2} - \frac{b^2 e^{-c} \operatorname{Ei}(1, dx)}{2} + \frac{a^2 d^3 e^{-dx-c}}{48x} - \frac{a^2 d^2 e^{-dx-c}}{48x^2} + \frac{da^2 e^{-dx-c}}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^2\*cosh(d\*x+c)/x^5,x)

[Out] -1/48\*d^4\*a^2\*exp(-c)\*Ei(1,d\*x)+1/2\*d\*a\*b\*exp(-d\*x-c)/x-1/2\*a\*b\*exp(-d\*x-c)/x^2-1/2\*d^2\*a\*b\*exp(-c)\*Ei(1,d\*x)-1/2\*b^2\*exp(-c)\*Ei(1,d\*x)+1/48\*d^3\*a^2\*exp(-d\*x-c)/x-1/48\*d^2\*a^2\*exp(-d\*x-c)/x^2+1/24\*d\*a^2\*exp(-d\*x-c)/x^3-1/8\*a^2\*exp(-d\*x-c)/x^4-1/8\*a^2/x^4\*exp(d\*x+c)-1/24\*d\*a^2/x^3\*exp(d\*x+c)-1/48\*d^2\*a^2/x^2\*exp(d\*x+c)-1/48\*d^3\*a^2/x\*exp(d\*x+c)-1/48\*d^4\*a^2\*exp(c)\*Ei(1,-d\*x)-1/2\*a\*b/x^2\*exp(d\*x+c)-1/2\*d\*a\*b/x\*exp(d\*x+c)-1/2\*d^2\*a\*b\*exp(c)\*Ei(1,-d\*x)-1/2\*b^2\*exp(c)\*Ei(1,-d\*x)

**Maxima [A]** time = 1.27655, size = 188, normalized size = 1.07

$$\frac{1}{8} \left( (d^3 e^{(-c)} \Gamma(-3, dx) + d^3 e^c \Gamma(-3, -dx)) a^2 + 4 (d e^{(-c)} \Gamma(-1, dx) + d e^c \Gamma(-1, -dx)) ab - \frac{4 b^2 \cosh(dx + c) \log(x^2)}{d} + \frac{4 (\operatorname{Ei}(1, dx) + \operatorname{Ei}(1, -dx))}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x^5,x, algorithm="maxima")

```
[Out] 1/8*((d^3*e^(-c)*gamma(-3, d*x) + d^3*e^c*gamma(-3, -d*x))*a^2 + 4*(d*e^(-c)
)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a*b - 4*b^2*cosh(d*x + c)*log(x^2
)/d + 4*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b^2/d*d + 1/4*(2*b^2*log(x^2) - (4
*a*b*x^2 + a^2)/x^4)*cosh(d*x + c)
```

**Fricas [A]** time = 1.9739, size = 439, normalized size = 2.51

$$\frac{2\left((a^2d^2 + 24ab)x^2 + 6a^2\right)\cosh(dx + c) - \left((a^2d^4 + 24abd^2 + 24b^2)x^4\text{Ei}(dx) + (a^2d^4 + 24abd^2 + 24b^2)x^4\text{Ei}(-dx)\right)}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

```
[Out] -1/48*(2*((a^2*d^2 + 24*a*b)*x^2 + 6*a^2)*cosh(d*x + c) - ((a^2*d^4 + 24*a*
b*d^2 + 24*b^2)*x^4*Ei(d*x) + (a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(-d*x))
*cosh(c) + 2*(2*a^2*d*x + (a^2*d^3 + 24*a*b*d)*x^3)*sinh(d*x + c) - ((a^2*d
^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(d*x) - (a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4
*Ei(-d*x))*sinh(c))/x^4
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^2 \cosh(c + dx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*cosh(d*x+c)/x**5,x)
```

```
[Out] Integral((a + b*x**2)**2*cosh(c + d*x)/x**5, x)
```

**Giac [A]** time = 1.17539, size = 397, normalized size = 2.27

$$a^2d^4x^4\text{Ei}(-dx)e^{(-c)} + a^2d^4x^4\text{Ei}(dx)e^c + 24abd^2x^4\text{Ei}(-dx)e^{(-c)} + 24abd^2x^4\text{Ei}(dx)e^c - a^2d^3x^3e^{(dx+c)} + a^2d^3x^3e^{(-dx-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)^2\*cosh(d\*x+c)/x^5,x, algorithm="giac")

[Out]  $\frac{1}{48}(a^2d^4x^4\text{Ei}(-dx)e^{-c} + a^2d^4x^4\text{Ei}(dx)e^c + 24ab^2d^2x^4\text{Ei}(-dx)e^{-c} + 24ab^2d^2x^4\text{Ei}(dx)e^c - a^2d^3x^3e^{(dx+c)} + a^2d^3x^3e^{-(dx-c)} + 24b^2x^4\text{Ei}(-dx)e^{-c} + 24b^2x^4\text{Ei}(dx)e^c - a^2d^2x^2e^{(dx+c)} - 24abd^2x^3e^{(dx+c)} - a^2d^2x^2e^{-(dx-c)} + 24abd^2x^3e^{-(dx-c)} - 2a^2d^2x^2e^{(dx+c)} - 24ab^2x^2e^{(dx+c)} + 2a^2d^2x^2e^{-(dx-c)} - 24ab^2x^2e^{-(dx-c)} - 6a^2e^{(dx+c)} - 6a^2e^{-(dx-c)})/x^4$

$$3.57 \quad \int \frac{x^4 \cosh(c+dx)}{a+bx^2} dx$$

**Optimal.** Leaf size=273

$$\frac{(-a)^{3/2} \cosh\left(\frac{\sqrt{-a}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}}{\sqrt{b}} - dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2} \cosh\left(c - \frac{\sqrt{-a}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}}{\sqrt{b}}\right)}{2b^{5/2}} - \frac{(-a)^{3/2} \sinh\left(\frac{\sqrt{-a}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right)}{2b^{5/2}}$$

[Out]  $(-2*x*\text{Cosh}[c + d*x])/(b*d^2) + ((-a)^{(3/2)}*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^{(5/2)}) - ((-a)^{(3/2)}*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^{(5/2)}) + (2*\text{Sinh}[c + d*x])/(b*d^3) - (a*\text{Sinh}[c + d*x])/(b^2*d) + (x^2*\text{Sinh}[c + d*x])/(b*d) - ((-a)^{(3/2)}*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^{(5/2)}) - ((-a)^{(3/2)}*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^{(5/2)})$

**Rubi [A]** time = 0.728719, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {5293, 2637, 3296, 5281, 3303, 3298, 3301}

$$\frac{(-a)^{3/2} \cosh\left(\frac{\sqrt{-a}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-a}}{\sqrt{b}} - dx\right)}{2b^{5/2}} - \frac{(-a)^{3/2} \cosh\left(c - \frac{\sqrt{-a}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-a}}{\sqrt{b}}\right)}{2b^{5/2}} - \frac{(-a)^{3/2} \sinh\left(\frac{\sqrt{-a}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^4*\text{Cosh}[c + d*x])/(a + b*x^2), x]$

[Out]  $(-2*x*\text{Cosh}[c + d*x])/(b*d^2) + ((-a)^{(3/2)}*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^{(5/2)}) - ((-a)^{(3/2)}*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^{(5/2)}) + (2*\text{Sinh}[c + d*x])/(b*d^3) - (a*\text{Sinh}[c + d*x])/(b^2*d) + (x^2*\text{Sinh}[c + d*x])/(b*d) - ((-a)^{(3/2)}*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*b^{(5/2)}) - ((-a)^{(3/2)}*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*b^{(5/2)})$

### Rule 5293

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], x^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{IntegerQ}[p, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ (\text{EqQ}[n, 2] \ || \ \text{EqQ}[p, -1])$

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps



$$\begin{aligned}
\int \frac{x^4 \cosh(c + dx)}{a + bx^2} dx &= \int \left( -\frac{a \cosh(c + dx)}{b^2} + \frac{x^2 \cosh(c + dx)}{b} + \frac{a^2 \cosh(c + dx)}{b^2 (a + bx^2)} \right) dx \\
&= -\frac{a \int \cosh(c + dx) dx}{b^2} + \frac{a^2 \int \frac{\cosh(c+dx)}{a+bx^2} dx}{b^2} + \frac{\int x^2 \cosh(c + dx) dx}{b} \\
&= -\frac{a \sinh(c + dx)}{b^2 d} + \frac{x^2 \sinh(c + dx)}{bd} + \frac{a^2 \int \left( \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx}{b^2} - \frac{2 \int x \sinh(c + dx) dx}{bd} \\
&= -\frac{2x \cosh(c + dx)}{bd^2} - \frac{a \sinh(c + dx)}{b^2 d} + \frac{x^2 \sinh(c + dx)}{bd} - \frac{(-a)^{3/2} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2b^2} - \frac{(-a)^{3/2} \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2b^2} \\
&= -\frac{2x \cosh(c + dx)}{bd^2} + \frac{2 \sinh(c + dx)}{bd^3} - \frac{a \sinh(c + dx)}{b^2 d} + \frac{x^2 \sinh(c + dx)}{bd} - \frac{((-a)^{3/2} \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) + (-a)^{3/2} \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right))}{2b^{5/2}} \\
&= -\frac{2x \cosh(c + dx)}{bd^2} + \frac{(-a)^{3/2} \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) - (-a)^{3/2} \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{2b^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.418781, size = 274, normalized size = 1.

$$ia^{3/2}d^3 \cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) - ia^{3/2}d^3 \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) - a^{3/2}d^3 \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) + a^{3/2}d^3 \sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4\*Cosh[c + d\*x])/(a + b\*x^2), x]

[Out]  $(-4*b^{(3/2)}*d*x*Cosh[c + d*x] + I*a^{(3/2)}*d^3*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]])*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] - I*a^{(3/2)}*d^3*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 4*b^{(3/2)}*Sinh[c + d*x] - 2*a*Sqrt[b]*d^2*Sinh[c + d*x] + 2*b^{(3/2)}*d^2*x^2*Sinh[c + d*x] - a^{(3/2)}*d^3*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - a^{(3/2)}*d^3*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(2*b^{(5/2)}*d^3)$

**Maple [A]** time = 0.189, size = 369, normalized size = 1.4

$$-\frac{e^{-dx-c}x^2}{2bd} + \frac{e^{-dx-c}a}{2db^2} - \frac{e^{-dx-c}x}{bd^2} - \frac{e^{-dx-c}}{d^3b} - \frac{a^2}{4b^2}e^{-\frac{1}{b}(d\sqrt{-ab}+cb)}\text{Ei}\left(1, -\frac{1}{b}\left(d\sqrt{-ab} - (dx+c)b + cb\right)\right)\frac{1}{\sqrt{-ab}} + \frac{a^2}{4b^2}e^{\frac{1}{b}(d\sqrt{-ab}-cb)}\text{Ei}\left(1, \frac{1}{b}\left(d\sqrt{-ab} + (dx+c)b - cb\right)\right)\frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^4 \cosh(dx+c)/(b*x^2+a), x)$

[Out] 
$$-1/2/d*\exp(-d*x-c)/b*x^2+1/2/d*\exp(-d*x-c)/b^2*a-1/d^2*\exp(-d*x-c)/b*x-1/d^3*\exp(-d*x-c)/b-1/4/b^2/(-a*b)^{(1/2)}*\exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)*a^2+1/4/b^2/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)*a^2-1/4/b^2/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)*a^2+1/4/b^2/(-a*b)^{(1/2)}*\exp(-(d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)*a^2+1/2/d/b*\exp(d*x+c)*x^2-1/2/d/b^2*a*\exp(d*x+c)-1/d^2/b*\exp(d*x+c)*x+1/d^3/b*\exp(d*x+c)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^4 \cosh(dx+c)/(b*x^2+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.19521, size = 1296, normalized size = 4.75

$$8bdx \cosh(dx+c) + \left( (ad^2 \cosh(dx+c)^2 - ad^2 \sinh(dx+c)^2) \sqrt{-\frac{ad^2}{b}} \text{Ei} \left( dx - \sqrt{-\frac{ad^2}{b}} \right) + (ad^2 \cosh(dx+c)^2 - ad^2 \sinh(dx+c)^2) \sqrt{-\frac{ad^2}{b}} \text{Ei} \left( dx + \sqrt{-\frac{ad^2}{b}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^4 \cosh(dx+c)/(b*x^2+a), x, \text{algorithm}="fricas")$

[Out] 
$$-1/4*(8*b*d*x*\cosh(d*x+c) + ((a*d^2*\cosh(d*x+c)^2 - a*d^2*\sinh(d*x+c)^2)*\sqrt{-a*d^2/b}*\text{Ei}(d*x - \sqrt{-a*d^2/b}) + (a*d^2*\cosh(d*x+c)^2 - a*d^2*\sinh(d*x+c)^2)*\sqrt{-a*d^2/b}*\text{Ei}(-d*x + \sqrt{-a*d^2/b}))*\cosh(c + \sqrt{-a*d^2/b}) - ((a*d^2*\cosh(d*x+c)^2 - a*d^2*\sinh(d*x+c)^2)*\sqrt{-a*d^2/b}*\text{Ei}(d*x + \sqrt{-a*d^2/b}) + (a*d^2*\cosh(d*x+c)^2 - a*d^2*\sinh(d*x+c)^2)*\sqrt{-a*d^2/b}*\text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\cosh(-c + \sqrt{-a*d^2/b}) - 4*($$

$$\frac{b*d^2*x^2 - a*d^2 + 2*b)*\sinh(d*x + c) + ((a*d^2*\cosh(d*x + c)^2 - a*d^2*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b}*Ei(d*x - \sqrt{-a*d^2/b}) - (a*d^2*\cosh(d*x + c)^2 - a*d^2*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b}*Ei(-d*x + \sqrt{-a*d^2/b}))*\sinh(c + \sqrt{-a*d^2/b}) + ((a*d^2*\cosh(d*x + c)^2 - a*d^2*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b}*Ei(d*x + \sqrt{-a*d^2/b}) - (a*d^2*\cosh(d*x + c)^2 - a*d^2*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b}*Ei(-d*x - \sqrt{-a*d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))}{(b^2*d^3*\cosh(d*x + c)^2 - b^2*d^3*\sinh(d*x + c)^2)}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(c + dx)}{a + bx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*cosh(d\*x+c)/(b\*x\*\*2+a), x)

[Out] Integral(x\*\*4\*cosh(c + d\*x)/(a + b\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(dx + c)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*cosh(d\*x+c)/(b\*x^2+a), x, algorithm="giac")

[Out] integrate(x^4\*cosh(d\*x + c)/(b\*x^2 + a), x)

$$3.58 \quad \int \frac{x^3 \cosh(c+dx)}{a+bx^2} dx$$

**Optimal.** Leaf size=209

$$\frac{a \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2} - \frac{a \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b^2} + \frac{a \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2} - \frac{a \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b^2}$$

[Out]  $-(\operatorname{Cosh}[c + d*x]/(b*d^2)) - (a*\operatorname{Cosh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(2*b^2) - (a*\operatorname{Cosh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(2*b^2) + (x*\operatorname{Sinh}[c + d*x])/(b*d) + (a*\operatorname{Sinh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(2*b^2) - (a*\operatorname{Sinh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(2*b^2)$

**Rubi [A]** time = 0.35826, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5293, 3296, 2638, 3303, 3298, 3301}

$$\frac{a \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2} - \frac{a \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b^2} + \frac{a \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \operatorname{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2} - \frac{a \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^3*\operatorname{Cosh}[c + d*x])/(a + b*x^2), x]$

[Out]  $-(\operatorname{Cosh}[c + d*x]/(b*d^2)) - (a*\operatorname{Cosh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(2*b^2) - (a*\operatorname{Cosh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{CoshIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(2*b^2) + (x*\operatorname{Sinh}[c + d*x])/(b*d) + (a*\operatorname{Sinh}[c + (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] - d*x])/(2*b^2) - (a*\operatorname{Sinh}[c - (\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b]]*\operatorname{SinhIntegral}[(\operatorname{Sqrt}[-a]*d)/\operatorname{Sqrt}[b] + d*x])/(2*b^2)$

### Rule 5293

$\operatorname{Int}[\operatorname{Cosh}[(c_.) + (d_.)*(x_.)]*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[\operatorname{Cosh}[c + d*x], x^m*(a + b*x^n)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{ILtQ}[p, 0] \ \&\& \operatorname{IntegerQ}[m] \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& (\operatorname{EqQ}[n, 2] \ || \ \operatorname{EqQ}[p, -1])$

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c + dx)}{a + bx^2} dx &= \int \left( \frac{x \cosh(c + dx)}{b} - \frac{ax \cosh(c + dx)}{b(a + bx^2)} \right) dx \\
&= \frac{\int x \cosh(c + dx) dx}{b} - \frac{a \int \frac{x \cosh(c + dx)}{a + bx^2} dx}{b} \\
&= \frac{x \sinh(c + dx)}{bd} - \frac{a \int \left( -\frac{\cosh(c + dx)}{2\sqrt{b}(\sqrt{-a} - \sqrt{bx})} + \frac{\cosh(c + dx)}{2\sqrt{b}(\sqrt{-a} + \sqrt{bx})} \right) dx}{b} - \frac{\int \sinh(c + dx) dx}{bd} \\
&= -\frac{\cosh(c + dx)}{bd^2} + \frac{x \sinh(c + dx)}{bd} + \frac{a \int \frac{\cosh(c + dx)}{\sqrt{-a} - \sqrt{bx}} dx}{2b^{3/2}} - \frac{a \int \frac{\cosh(c + dx)}{\sqrt{-a} + \sqrt{bx}} dx}{2b^{3/2}} \\
&= -\frac{\cosh(c + dx)}{bd^2} + \frac{x \sinh(c + dx)}{bd} - \frac{\left( a \cosh \left( c - \frac{\sqrt{-ad}}{\sqrt{b}} \right) \right) \int \frac{\cosh \left( \frac{\sqrt{-ad}}{\sqrt{b}} + dx \right)}{\sqrt{-a} + \sqrt{bx}} dx}{2b^{3/2}} + \frac{\left( a \cosh \left( c + \frac{\sqrt{-ad}}{\sqrt{b}} \right) \right) \int \frac{\cosh \left( \frac{\sqrt{-ad}}{\sqrt{b}} + dx \right)}{\sqrt{-a} - \sqrt{bx}} dx}{2b^{3/2}} \\
&= -\frac{\cosh(c + dx)}{bd^2} - \frac{a \cosh \left( c + \frac{\sqrt{-ad}}{\sqrt{b}} \right) \text{Chi} \left( \frac{\sqrt{-ad}}{\sqrt{b}} - dx \right)}{2b^2} - \frac{a \cosh \left( c - \frac{\sqrt{-ad}}{\sqrt{b}} \right) \text{Chi} \left( \frac{\sqrt{-ad}}{\sqrt{b}} + dx \right)}{2b^2} + \dots
\end{aligned}$$

**Mathematica [C]** time = 0.348637, size = 210, normalized size = 1.

$$\frac{ad^2 \cosh \left( c - \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{CosIntegral} \left( -\frac{\sqrt{ad}}{\sqrt{b}} + idx \right) + ad^2 \cosh \left( c + \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{CosIntegral} \left( \frac{\sqrt{ad}}{\sqrt{b}} + idx \right) + iad^2 \sinh \left( c - \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{Si} \left( \frac{\sqrt{ad}}{\sqrt{b}} + idx \right) + iad^2 \sinh \left( c + \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{Si} \left( -\frac{\sqrt{ad}}{\sqrt{b}} + idx \right)}{2b^2 d^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3\*Cosh[c + d\*x])/(a + b\*x^2),x]

[Out]  $-(2*b*Cosh[c + d*x] + a*d^2*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x] + a*d^2*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] - 2*b*d*x*Sinh[c + d*x] + I*a*d^2*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*a*d^2*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x])/(2*b^2*d^2)$

**Maple [A]** time = 0.06, size = 268, normalized size = 1.3

$$-\frac{e^{-dx-c}x}{2bd} - \frac{e^{-dx-c}}{2bd^2} + \frac{a}{4b^2} e^{\frac{1}{b}(d\sqrt{-ab}-cb)} \text{Ei} \left( 1, \frac{1}{b} (d\sqrt{-ab} + (dx+c)b - cb) \right) + \frac{a}{4b^2} e^{-\frac{1}{b}(d\sqrt{-ab}+cb)} \text{Ei} \left( 1, -\frac{1}{b} (d\sqrt{-ab} - (dx+c)b + cb) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cosh(d*x+c)/(b*x^2+a),x)`

[Out] 
$$-1/2/d*\exp(-d*x-c)/b*x-1/2/d^2*\exp(-d*x-c)/b+1/4/b^2*\exp((d*(-a*b)^(1/2)-c*b)/b)*\text{Ei}(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a+1/4/b^2*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*\text{Ei}(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a+1/2/d/b*\exp(d*x+c)*x-1/2/d^2/b*\exp(d*x+c)+1/4/b^2*\exp((d*(-a*b)^(1/2)+c*b)/b)*\text{Ei}(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a+1/4/b^2*\exp(-(d*(-a*b)^(1/2)-c*b)/b)*\text{Ei}(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(dx^3e^{(2c)} - x^2e^{(2c)})e^{(dx)} - (dx^3 + x^2)e^{(-dx)}}{2(bd^2x^2e^c + ad^2e^c)} - \frac{1}{2} \int \frac{2(adx^2e^c - axe^c)e^{(dx)}}{b^2d^2x^4 + 2abd^2x^2 + a^2d^2} dx + \frac{1}{2} \int \frac{2(adx^2 + ax)e^{(-dx)}}{b^2d^2x^4e^c + 2abd^2x^2e^c + a^2d^2e^c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")`

[Out] 
$$1/2*((d*x^3*e^{(2*c)} - x^2*e^{(2*c)})*e^{(d*x)} - (d*x^3 + x^2)*e^{(-d*x)})/(b*d^2*x^2*e^c + a*d^2*e^c) - 1/2*\text{integrate}(2*(a*d*x^2*e^c - a*x*e^c)*e^{(d*x)}/(b^2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2), x) + 1/2*\text{integrate}(2*(a*d*x^2 + a*x)*e^{(-d*x)}/(b^2*d^2*x^4*e^c + 2*a*b*d^2*x^2*e^c + a^2*d^2*e^c), x)$$

**Fricas [B]** time = 2.10293, size = 1100, normalized size = 5.26

$$4 b d x \sinh (d x + c) - 4 b \cosh (d x + c) - \left( (a d^2 \cosh (d x + c))^2 - a d^2 \sinh (d x + c)^2 \right) \text{Ei} \left( d x - \sqrt{-\frac{a d^2}{b}} \right) + (a d^2 \cosh (d x + c) + a d^2 \sinh (d x + c)) \sqrt{-\frac{a d^2}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")`

[Out] 
$$1/4*(4*b*d*x*\sinh(d*x + c) - 4*b*\cosh(d*x + c) - ((a*d^2*\cosh(d*x + c))^2 - a*d^2*\sinh(d*x + c)^2)*\text{Ei}(d*x - \text{sqrt}(-a*d^2/b)) + (a*d^2*\cosh(d*x + c)^2 - a*d^2*\sinh(d*x + c)^2)*\text{Ei}(-d*x + \text{sqrt}(-a*d^2/b)))*\cosh(c + \text{sqrt}(-a*d^2/b))$$

$$\begin{aligned}
& - ((a*d^2*\cosh(d*x + c)^2 - a*d^2*\sinh(d*x + c)^2)*\text{Ei}(d*x + \sqrt{-a*d^2/b}) \\
& + (a*d^2*\cosh(d*x + c)^2 - a*d^2*\sinh(d*x + c)^2)*\text{Ei}(-d*x - \sqrt{-a*d^2/b}) \\
& ))*\cosh(-c + \sqrt{-a*d^2/b}) - ((a*d^2*\cosh(d*x + c)^2 - a*d^2*\sinh(d*x + c \\
& )^2)*\text{Ei}(d*x - \sqrt{-a*d^2/b}) - (a*d^2*\cosh(d*x + c)^2 - a*d^2*\sinh(d*x + c \\
& )^2)*\text{Ei}(-d*x + \sqrt{-a*d^2/b}))*\sinh(c + \sqrt{-a*d^2/b}) + ((a*d^2*\cosh(d*x \\
& + c)^2 - a*d^2*\sinh(d*x + c)^2)*\text{Ei}(d*x + \sqrt{-a*d^2/b}) - (a*d^2*\cosh(d*x \\
& + c)^2 - a*d^2*\sinh(d*x + c)^2)*\text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))/ \\
& (b^2*d^2*\cosh(d*x + c)^2 - b^2*d^2*\sinh(d*x + c)^2)
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(c + dx)}{a + bx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*cosh(d\*x+c)/(b\*x\*\*2+a),x)

[Out] Integral(x\*\*3\*cosh(c + d\*x)/(a + b\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(dx + c)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x^2+a),x, algorithm="giac")

[Out] integrate(x^3\*cosh(d\*x + c)/(b\*x^2 + a), x)



$$3.59 \quad \int \frac{x^2 \cosh(c+dx)}{a+bx^2} dx$$

**Optimal.** Leaf size=226

$$\frac{\sqrt{-a} \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^{3/2}} - \frac{\sqrt{-a} \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{\sqrt{-a} \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^{3/2}}$$

[Out] (Sqrt[-a]\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b^(3/2)) - (Sqrt[-a]\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b^(3/2)) + Sinh[c + d\*x]/(b\*d) - (Sqrt[-a]\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b^(3/2)) - (Sqrt[-a]\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b^(3/2))

**Rubi [A]** time = 0.375852, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5293, 2637, 5281, 3303, 3298, 3301}

$$\frac{\sqrt{-a} \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^{3/2}} - \frac{\sqrt{-a} \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{\sqrt{-a} \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*Cosh[c + d\*x])/(a + b\*x^2),x]

[Out] (Sqrt[-a]\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b^(3/2)) - (Sqrt[-a]\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b^(3/2)) + Sinh[c + d\*x]/(b\*d) - (Sqrt[-a]\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b^(3/2)) - (Sqrt[-a]\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b^(3/2))

### Rule 5293

Int[Cosh[(c\_.) + (d\_.)\*(x\_.)]\*(x\_.)^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], x^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^2 \cosh(c + dx)}{a + bx^2} dx &= \int \left( \frac{\cosh(c + dx)}{b} - \frac{a \cosh(c + dx)}{b(a + bx^2)} \right) dx \\
&= \frac{\int \cosh(c + dx) dx}{b} - \frac{a \int \frac{\cosh(c+dx)}{a+bx^2} dx}{b} \\
&= \frac{\sinh(c + dx)}{bd} - \frac{a \int \left( \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx}{b} \\
&= \frac{\sinh(c + dx)}{bd} - \frac{\sqrt{-a} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2b} - \frac{\sqrt{-a} \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2b} \\
&= \frac{\sinh(c + dx)}{bd} - \frac{\left( \sqrt{-a} \cosh \left( c - \frac{\sqrt{-ad}}{\sqrt{b}} \right) \right) \int \frac{\cosh \left( \frac{\sqrt{-ad}}{\sqrt{b}} + dx \right)}{\sqrt{-a}+\sqrt{bx}} dx}{2b} - \frac{\left( \sqrt{-a} \cosh \left( c + \frac{\sqrt{-ad}}{\sqrt{b}} \right) \right) \int \frac{\cosh \left( \frac{\sqrt{-ad}}{\sqrt{b}} - dx \right)}{\sqrt{-a}-\sqrt{bx}} dx}{2b} \\
&= \frac{\sqrt{-a} \cosh \left( c + \frac{\sqrt{-ad}}{\sqrt{b}} \right) \text{Chi} \left( \frac{\sqrt{-ad}}{\sqrt{b}} - dx \right)}{2b^{3/2}} - \frac{\sqrt{-a} \cosh \left( c - \frac{\sqrt{-ad}}{\sqrt{b}} \right) \text{Chi} \left( \frac{\sqrt{-ad}}{\sqrt{b}} + dx \right)}{2b^{3/2}} + \frac{\sinh(c + dx)}{bd}
\end{aligned}$$

**Mathematica [C]** time = 0.251724, size = 213, normalized size = 0.94

$$\frac{-i\sqrt{ad} \cosh \left( c - \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{CosIntegral} \left( -\frac{\sqrt{ad}}{\sqrt{b}} + idx \right) + i\sqrt{ad} \cosh \left( c + \frac{i\sqrt{ad}}{\sqrt{b}} \right) \text{CosIntegral} \left( \frac{\sqrt{ad}}{\sqrt{b}} + idx \right) + \sqrt{ad} \sinh \left( c - \frac{i\sqrt{ad}}{\sqrt{b}} \right)}{2b^{3/2}d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*Cosh[c + d\*x])/(a + b\*x^2),x]

[Out] ((-I)\*Sqrt[a]\*d\*Cosh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]\*CosIntegral[-((Sqrt[a]\*d)/Sqrt[b]) + I\*d\*x] + I\*Sqrt[a]\*d\*Cosh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]\*CosIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x] + 2\*Sqrt[b]\*Sinh[c + d\*x] + Sqrt[a]\*d\*Sinh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] + Sqrt[a]\*d\*Sinh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x])/(2\*b^(3/2)\*d)

**Maple [A]** time = 0.045, size = 259, normalized size = 1.2

$$-\frac{e^{-dx-c}}{2bd} - \frac{a}{4b} e^{\frac{1}{b}(d\sqrt{-ab}-cb)} \text{Ei} \left( 1, \frac{1}{b} \left( d\sqrt{-ab} + (dx+c)b - cb \right) \right) \frac{1}{\sqrt{-ab}} + \frac{a}{4b} e^{-\frac{1}{b}(d\sqrt{-ab}+cb)} \text{Ei} \left( 1, -\frac{1}{b} \left( d\sqrt{-ab} - (dx+c)b + cb \right) \right) \frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^2+a),x)
```

```
[Out] -1/2/d*exp(-d*x-c)/b-1/4/b/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d
*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*a+1/4/b/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c
*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*a+1/2/d/b*exp(d*x+c)-1/4/b/(
-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*
b)/b)*a+1/4/b/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)
-(d*x+c)*b+c*b)/b)*a
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.16906, size = 1087, normalized size = 4.81

$$\left(\sqrt{-\frac{ad^2}{b}}(\cosh(dx+c)^2 - \sinh(dx+c)^2)Ei\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + \sqrt{-\frac{ad^2}{b}}(\cosh(dx+c)^2 - \sinh(dx+c)^2)Ei\left(-dx + \sqrt{-\frac{ad^2}{b}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d
^2/b)) + sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + sqrt(
-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - s
inh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*(cosh(d*x + c)^2
- sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (
sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b))
- sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2
/b)))*sinh(c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*
```

$$(x + c)^2 \operatorname{Ei}(d x + \sqrt{-a d^2/b}) - \sqrt{-a d^2/b} (\cosh(d x + c)^2 - \sinh(d x + c)^2) \operatorname{Ei}(-d x - \sqrt{-a d^2/b}) + 4 \sinh(d x + c) / (b d \cosh(d x + c)^2 - b d \sinh(d x + c)^2)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(c + dx)}{a + bx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cosh(d\*x+c)/(b\*x\*\*2+a),x)

[Out] Integral(x\*\*2\*cosh(c + d\*x)/(a + b\*x\*\*2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(dx + c)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x^2+a),x, algorithm="giac")

[Out] integrate(x^2\*cosh(d\*x + c)/(b\*x^2 + a), x)

$$3.60 \quad \int \frac{x \cosh(c+dx)}{a+bx^2} dx$$

**Optimal.** Leaf size=177

$$\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b} + \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b} - \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b} + \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b}$$

[Out] (Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b) + (Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b) - (Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b) + (Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b)

**Rubi [A]** time = 0.254037, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {5293, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b} + \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b} - \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b} + \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(x\*Cosh[c + d\*x])/(a + b\*x^2), x]

[Out] (Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b) + (Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b) - (Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b) + (Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b)

### Rule 5293

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[Cosh[c + d\*x], x^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x]

)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned} \int \frac{x \cosh(c + dx)}{a + bx^2} dx &= \int \left( -\frac{\cosh(c + dx)}{2\sqrt{b}(\sqrt{-a} - \sqrt{bx})} + \frac{\cosh(c + dx)}{2\sqrt{b}(\sqrt{-a} + \sqrt{bx})} \right) dx \\ &= -\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2\sqrt{b}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2\sqrt{b}} \\ &= \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{bx}} dx}{2\sqrt{b}} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{bx}} dx}{2\sqrt{b}} + \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \int \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{bx}} dx}{2\sqrt{b}} \\ &= \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b} + \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{2b} - \frac{\sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b} \end{aligned}$$

**Mathematica [C]** time = 0.174738, size = 171, normalized size = 0.97

$$\frac{\cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) + \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) + i\left(\sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} - idx\right) - \sinh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right)\right)}{2b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x\*Cosh[c + d\*x])/(a + b\*x^2), x]

[Out] (Cosh[c - (I\*sqrt[a]\*d)/sqrt[b]]\*CosIntegral[-((sqrt[a]\*d)/sqrt[b]) + I\*d\*x] + Cosh[c + (I\*sqrt[a]\*d)/sqrt[b]]\*CosIntegral[(sqrt[a]\*d)/sqrt[b] + I\*d\*x])

$$\left] + I \left( \frac{\sinh\left[c - \frac{I\sqrt{a}d}{\sqrt{b}}\right] \operatorname{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}} - I d x\right) - \sinh\left[c + \frac{I\sqrt{a}d}{\sqrt{b}}\right] \operatorname{Si}\left(\frac{\sqrt{a}d}{\sqrt{b}} + I d x\right)}{2b} \right)$$

**Maple [A]** time = 0.032, size = 200, normalized size = 1.1

$$-\frac{1}{4b} e^{\frac{1}{b}(d\sqrt{-ab}-cb)} \operatorname{Ei}\left(1, \frac{1}{b}\left(d\sqrt{-ab} + (dx+c)b - cb\right)\right) - \frac{1}{4b} e^{-\frac{1}{b}(d\sqrt{-ab}+cb)} \operatorname{Ei}\left(1, -\frac{1}{b}\left(d\sqrt{-ab} - (dx+c)b + cb\right)\right) - \frac{1}{4b} e^{\frac{1}{b}(d\sqrt{-ab}-cb)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cosh(d*x+c)/(b*x^2+a), x)`

[Out] 
$$-1/4/b*\exp((d*(-a*b)^{(1/2)}-c*b)/b)*\operatorname{Ei}\left(1, (d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b\right) - 1/4/b*\exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*\operatorname{Ei}\left(1, -(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b\right) - 1/4/b*\exp((d*(-a*b)^{(1/2)}+c*b)/b)*\operatorname{Ei}\left(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b\right) - 1/4/b*\exp(-(d*(-a*b)^{(1/2)}-c*b)/b)*\operatorname{Ei}\left(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b\right)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{x e^{(dx+2c)} - x e^{-dx}}{2(bdx^2e^c + ade^c)} + \frac{1}{2} \int \frac{(bx^2e^c - ae^c)e^{(dx)}}{b^2dx^4 + 2abdx^2 + a^2d} dx - \frac{1}{2} \int \frac{(bx^2 - a)e^{-dx}}{b^2dx^4e^c + 2abdx^2e^c + a^2de^c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cosh(d*x+c)/(b*x^2+a), x, algorithm="maxima")`

[Out] 
$$1/2*(x*e^{(d*x + 2*c)} - x*e^{(-d*x)})/(b*d*x^2*e^c + a*d*e^c) + 1/2*integrate((b*x^2*e^c - a*e^c)*e^{(d*x)})/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) - 1/2*integrate((b*x^2 - a)*e^{(-d*x)})/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d*e^c), x)$$

**Fricas [A]** time = 2.09171, size = 455, normalized size = 2.57

$$\left( \operatorname{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + \operatorname{Ei}\left(-dx + \sqrt{-\frac{ad^2}{b}}\right) \right) \cosh\left(c + \sqrt{-\frac{ad^2}{b}}\right) + \left( \operatorname{Ei}\left(dx + \sqrt{-\frac{ad^2}{b}}\right) + \operatorname{Ei}\left(-dx - \sqrt{-\frac{ad^2}{b}}\right) \right) \cosh\left(-c + \sqrt{-\frac{ad^2}{b}}\right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*((Ei(d*x - sqrt(-a*d^2/b)) + Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) + (Ei(d*x + sqrt(-a*d^2/b)) + Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (Ei(d*x - sqrt(-a*d^2/b)) - Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - (Ei(d*x + sqrt(-a*d^2/b)) - Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/b
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(c + dx)}{a + bx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(x*cosh(c + d*x)/(a + b*x**2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(dx + c)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(x*cosh(d*x + c)/(b*x^2 + a), x)
```

### 3.61 $\int \frac{\cosh(c+dx)}{a+bx^2} dx$

**Optimal.** Leaf size=213

$$\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}$$

```
[Out] (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*Sqrt[-a]*Sqrt[b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]
]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]
*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c -
(Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a
]*Sqrt[b])
```

**Rubi [A]** time = 0.258696, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5281, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x^2), x]
```

```
[Out] (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(
2*Sqrt[-a]*Sqrt[b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]
]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]
*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c -
(Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a
]*Sqrt[b])
```

#### Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

#### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\cosh(c+dx)}{a+bx^2} dx &= \int \left( \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx \\ &= -\frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2\sqrt{-a}} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2\sqrt{-a}} \\ &= -\frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{bx}} dx}{2\sqrt{-a}} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{bx}} dx}{2\sqrt{-a}} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \int \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{bx}} dx}{2\sqrt{-a}} \\ &= \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}} \end{aligned}$$

**Mathematica [C]** time = 0.200531, size = 180, normalized size = 0.85

$$\frac{i \left( \cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) - \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) + i \left( \sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} - dx\right) - \sinh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} + dx\right) \right) \right)}{2\sqrt{a}\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[c + d*x]/(a + b*x^2), x]
```

```
[Out] ((I/2)*(Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[-((Sqrt[a]*d)/Sqrt[b])
+ I*d*x] - Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*CosIntegral[(Sqrt[a]*d)/Sqrt[b]
+ I*d*x] + I*(Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[
b] - I*d*x] + Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[
b] + I*d*x])))/(Sqrt[a]*Sqrt[b])
```

**Maple [A]** time = 0.021, size = 212, normalized size = 1.

$$\frac{1}{4}e^{\frac{1}{b}(d\sqrt{-ab}-cb)}\text{Ei}\left(1, \frac{1}{b}\left(d\sqrt{-ab} + (dx+c)b - cb\right)\right)\frac{1}{\sqrt{-ab}} - \frac{1}{4}e^{-\frac{1}{b}(d\sqrt{-ab}+cb)}\text{Ei}\left(1, -\frac{1}{b}\left(d\sqrt{-ab} - (dx+c)b + cb\right)\right)\frac{1}{\sqrt{-ab}} + \frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(b*x^2+a), x)
```

```
[Out] 1/4/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1, (d*(-a*b)^(1/2)+(d*x+c)*b
-c*b)/b)-1/4/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1, -(d*(-a*b)^(1/2)
)-(d*x+c)*b+c*b)/b)+1/4/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1, -(d*
(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/4/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)
*Ei(1, (d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a), x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.15703, size = 624, normalized size = 2.93

$$\left(\sqrt{-\frac{ad^2}{b}}\text{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + \sqrt{-\frac{ad^2}{b}}\text{Ei}\left(-dx + \sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c + \sqrt{-\frac{ad^2}{b}}\right) - \left(\sqrt{-\frac{ad^2}{b}}\text{Ei}\left(dx + \sqrt{-\frac{ad^2}{b}}\right) + \sqrt{-\frac{ad^2}{b}}\text{Ei}\left(-dx\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x + s
qrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a
*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/
b)) + (sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x + s
qrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a
*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/
b)))/(a*d)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx)}{a + bx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x**2+a),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*x**2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^2 + a), x)
```

$$3.62 \quad \int \frac{\cosh(c+dx)}{x(a+bx^2)} dx$$

**Optimal.** Leaf size=197

$$\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a} + \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a}$$

[Out] (Cosh[c]\*CoshIntegral[d\*x])/a - (Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*a) - (Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*a) + (Sinh[c]\*SinhIntegral[d\*x])/a + (Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*a) - (Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*a)

**Rubi [A]** time = 0.372347, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {5293, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a} + \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x\*(a + b\*x^2)),x]

[Out] (Cosh[c]\*CoshIntegral[d\*x])/a - (Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*a) - (Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*a) + (Sinh[c]\*SinhIntegral[d\*x])/a + (Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*a) - (Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*a)

### Rule 5293

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[Cosh[c + d\*x], x^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx^2)} dx &= \int \left( \frac{\cosh(c+dx)}{ax} - \frac{bx \cosh(c+dx)}{a(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a} - \frac{b \int \frac{x \cosh(c+dx)}{a+bx^2} dx}{a} \\
&= -\frac{b \int \left( -\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \right) dx}{a} + \frac{\cosh(c) \int \frac{\cosh(dx)}{x} dx}{a} + \frac{\sinh(c) \int \frac{\sinh(dx)}{x} dx}{a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} + \frac{\sqrt{b} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2a} - \frac{\sqrt{b} \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\left(\sqrt{b} \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right)\right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{bx}} dx}{2a} + \frac{\left(\sqrt{b} \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right)\right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{bx}} dx}{2a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{2a} + \frac{\sinh(c)\text{Shi}(dx)}{a}
\end{aligned}$$

**Mathematica [C]** time = 0.310288, size = 187, normalized size = 0.95

$$\frac{\cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) + \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) + i \sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} - idx\right) + i \sinh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right)}{2a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d\*x]/(x\*(a + b\*x^2)),x]

[Out]  $-(2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + \text{Cosh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x] + \text{Cosh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] - 2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + I*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] - I*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])/(2*a)$

**Maple [A]** time = 0.043, size = 227, normalized size = 1.2

$$\frac{1}{4a}e^{-\frac{1}{b}(d\sqrt{-ab}+cb)}\text{Ei}\left(1, -\frac{1}{b}\left(d\sqrt{-ab} - (dx + c)b + cb\right)\right) + \frac{1}{4a}e^{\frac{1}{b}(d\sqrt{-ab}-cb)}\text{Ei}\left(1, \frac{1}{b}\left(d\sqrt{-ab} + (dx + c)b - cb\right)\right) - \frac{e^{-c}\text{Ei}(1, dx)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/x/(b\*x^2+a),x)

[Out]  $1/4/a*\exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b) + 1/4/a*\exp((d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b) - 1/2/a*\exp(-c)*\text{Ei}(1, d*x) + 1/4/a*\exp(-(d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b) + 1/4/a*\exp((d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b) - 1/2/a*\exp(c)*\text{Ei}(1, -d*x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x^2+a),x, algorithm="maxima")

[Out] integrate(cosh(d\*x + c)/((b\*x^2 + a)\*x), x)



**Fricas [A]** time = 2.20263, size = 545, normalized size = 2.77

$$\frac{\left(\operatorname{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + \operatorname{Ei}\left(-dx + \sqrt{-\frac{ad^2}{b}}\right)\right) \cosh\left(c + \sqrt{-\frac{ad^2}{b}}\right) - 2(\operatorname{Ei}(dx) + \operatorname{Ei}(-dx)) \cosh(c) + \left(\operatorname{Ei}\left(dx + \sqrt{-\frac{ad^2}{b}}\right) + \operatorname{Ei}\left(-dx - \sqrt{-\frac{ad^2}{b}}\right)\right) \cosh\left(-c + \sqrt{-\frac{ad^2}{b}}\right) + (\operatorname{Ei}(dx) + \operatorname{Ei}(-dx)) \sinh(c) - (\operatorname{Ei}(dx) - \operatorname{Ei}(-dx)) \sinh(-c) - (\operatorname{Ei}(dx + \sqrt{-\frac{ad^2}{b}}) - \operatorname{Ei}(-dx - \sqrt{-\frac{ad^2}{b}})) \sinh(-c + \sqrt{-\frac{ad^2}{b}})}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x^2+a),x, algorithm="fricas")

[Out] -1/4\*((Ei(d\*x - sqrt(-a\*d^2/b)) + Ei(-d\*x + sqrt(-a\*d^2/b)))\*cosh(c + sqrt(-a\*d^2/b)) - 2\*(Ei(d\*x) + Ei(-d\*x))\*cosh(c) + (Ei(d\*x + sqrt(-a\*d^2/b)) + Ei(-d\*x - sqrt(-a\*d^2/b)))\*cosh(-c + sqrt(-a\*d^2/b)) + (Ei(d\*x - sqrt(-a\*d^2/b)) + Ei(-d\*x + sqrt(-a\*d^2/b)))\*sinh(c + sqrt(-a\*d^2/b)) - 2\*(Ei(d\*x) - Ei(-d\*x))\*sinh(c) - (Ei(d\*x + sqrt(-a\*d^2/b)) - Ei(-d\*x - sqrt(-a\*d^2/b)))\*sinh(-c + sqrt(-a\*d^2/b)))/a

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx)}{x(a + bx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x\*\*2+a),x)

[Out] Integral(cosh(c + d\*x)/(x\*(a + b\*x\*\*2)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x^2+a),x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((b\*x^2 + a)\*x), x)

### 3.63 $\int \frac{\cosh(c+dx)}{x^2(a+bx^2)} dx$

**Optimal.** Leaf size=249

$$\frac{\sqrt{b} \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b} \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2(-a)^{3/2}} - \frac{\sqrt{b} \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2(-a)^{3/2}} - \dots$$

[Out]  $-(\text{Cosh}[c + d*x]/(a*x)) + (\text{Sqrt}[b]*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*(-a)^{(3/2)}) - (\text{Sqrt}[b]*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*(-a)^{(3/2)}) + (d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a + (d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a - (\text{Sqrt}[b]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*(-a)^{(3/2)}) - (\text{Sqrt}[b]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*(-a)^{(3/2)})$

**Rubi [A]** time = 0.495823, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{\sqrt{b} \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b} \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2(-a)^{3/2}} - \frac{\sqrt{b} \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2(-a)^{3/2}} - \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cosh}[c + d*x]/(x^2*(a + b*x^2)), x]$

[Out]  $-(\text{Cosh}[c + d*x]/(a*x)) + (\text{Sqrt}[b]*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*(-a)^{(3/2)}) - (\text{Sqrt}[b]*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*(-a)^{(3/2)}) + (d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a + (d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a - (\text{Sqrt}[b]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(2*(-a)^{(3/2)}) - (\text{Sqrt}[b]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(2*(-a)^{(3/2)})$

#### Rule 5293

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))^{(p_.)}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], x^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{ILtQ}[p, 0] \&\& \text{IntegerQ}[m] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n,$

2] || EqQ[p, -1])

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx^2)} dx &= \int \left( \frac{\cosh(c+dx)}{ax^2} - \frac{b \cosh(c+dx)}{a(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{a+bx^2} dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} - \frac{b \int \left( \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx}{a} + \frac{d \int \frac{\sinh(c+dx)}{x} dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} - \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2(-a)^{3/2}} - \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2(-a)^{3/2}} + \frac{(d \cosh(c)) \int \frac{\sinh(dx)}{x} dx}{a} + \frac{(d \sinh(c)) \int \frac{\cosh(dx)}{x} dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} + \frac{d \operatorname{Chi}(dx) \sinh(c)}{a} + \frac{d \cosh(c) \operatorname{Shi}(dx)}{a} - \frac{\left( b \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{\sqrt{-a} + \sqrt{bx}} dx}{2(-a)^{3/2}} \\
&= -\frac{\cosh(c+dx)}{ax} + \frac{\sqrt{b} \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b} \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{2(-a)^{3/2}} + \frac{d \cosh(c) \operatorname{Shi}(dx)}{a} + \frac{d \sinh(c) \operatorname{Chi}(dx)}{a}
\end{aligned}$$

**Mathematica [C]** time = 0.357135, size = 243, normalized size = 0.98

$$\frac{-i\sqrt{bx} \cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) + i\sqrt{bx} \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) + \sqrt{bx} \sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{ad}}{\sqrt{b}} - dx\right) + \sqrt{bx} \sinh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{ad}}{\sqrt{b}} + dx\right)}{2(-a)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d\*x]/(x^2\*(a + b\*x^2)),x]

[Out]  $(-2\sqrt{a} \operatorname{Cosh}[c + d*x] - I\sqrt{b} * x * \operatorname{Cosh}[c - (I\sqrt{a} * d) / \sqrt{b}]) * \operatorname{CosIntegral}[-((\sqrt{a} * d) / \sqrt{b}) + I * d * x] + I\sqrt{b} * x * \operatorname{Cosh}[c + (I\sqrt{a} * d) / \sqrt{b}] * \operatorname{CosIntegral}[(\sqrt{a} * d) / \sqrt{b} + I * d * x] + 2\sqrt{a} * d * x * \operatorname{CoshIntegral}[d * x] * \operatorname{Sinh}[c] + 2\sqrt{a} * d * x * \operatorname{Cosh}[c] * \operatorname{SinhIntegral}[d * x] + \sqrt{b} * x * \operatorname{Sinh}[c - (I\sqrt{a} * d) / \sqrt{b}] * \operatorname{SinIntegral}[(\sqrt{a} * d) / \sqrt{b} - I * d * x] + \sqrt{b} * x * \operatorname{Sinh}[c + (I\sqrt{a} * d) / \sqrt{b}] * \operatorname{SinIntegral}[(\sqrt{a} * d) / \sqrt{b} + I * d * x]) / (2 * a^{(3/2)} * x)$

**Maple [A]** time = 0.055, size = 288, normalized size = 1.2

$$-\frac{e^{-dx-c}}{2ax} + \frac{b}{4a} e^{-\frac{1}{b}(d\sqrt{-ab}+cb)} \operatorname{Ei}\left(1, -\frac{1}{b}\left(d\sqrt{-ab} - (dx+c)b + cb\right)\right) \frac{1}{\sqrt{-ab}} - \frac{b}{4a} e^{\frac{1}{b}(d\sqrt{-ab}-cb)} \operatorname{Ei}\left(1, \frac{1}{b}\left(d\sqrt{-ab} + (dx+c)b - cb\right)\right) \frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/x^2/(b*x^2+a),x)`

[Out] 
$$-1/2*\exp(-d*x-c)/a/x+1/4*b/a/(-a*b)^{(1/2)}*\exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)-1/4*b/a/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)+1/2*d/a*\exp(-c)*\text{Ei}(1, d*x)-1/2/a/x*\exp(d*x+c)+1/4*b/a/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)-1/4*b/a/(-a*b)^{(1/2)}*\exp(-(d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)-1/2*d/a*\exp(c)*\text{Ei}(1, -d*x)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.49935, size = 1323, normalized size = 5.31

$$4ad \cosh(dx+c) - \left( (bx \cosh(dx+c)^2 - bx \sinh(dx+c)^2) \sqrt{-\frac{ad^2}{b}} \text{Ei} \left( dx - \sqrt{-\frac{ad^2}{b}} \right) + (bx \cosh(dx+c)^2 - bx \sinh(dx+c)^2) \sqrt{-\frac{ad^2}{b}} \text{Ei} \left( dx + \sqrt{-\frac{ad^2}{b}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="fricas")`

[Out] 
$$-1/4*(4*a*d*\cosh(d*x+c) - ((b*x*\cosh(d*x+c)^2 - b*x*\sinh(d*x+c)^2)*\text{sqrt}(-a*d^2/b)*\text{Ei}(d*x - \text{sqrt}(-a*d^2/b)) + (b*x*\cosh(d*x+c)^2 - b*x*\sinh(d*x+c)^2)*\text{sqrt}(-a*d^2/b)*\text{Ei}(-d*x + \text{sqrt}(-a*d^2/b)))*\cosh(c + \text{sqrt}(-a*d^2/b)) - 2*(a*d^2*x*\text{Ei}(d*x) - a*d^2*x*\text{Ei}(-d*x))*\cosh(c) + ((b*x*\cosh(d*x+c)^2 - b*x*\sinh(d*x+c)^2)*\text{sqrt}(-a*d^2/b)*\text{Ei}(d*x + \text{sqrt}(-a*d^2/b)) + (b*x*\cosh(d*x+c)^2 - b*x*\sinh(d*x+c)^2)*\text{sqrt}(-a*d^2/b)*\text{Ei}(-d*x - \text{sqrt}(-a*d^2/b)))*\cosh(-c + \text{sqrt}(-a*d^2/b)) - ((b*x*\cosh(d*x+c)^2 - b*x*\sinh(d*x+c)^2)*\text{sqrt}(-a*d^2/b)*\text{Ei}(d*x - \text{sqrt}(-a*d^2/b)) - (b*x*\cosh(d*x+c)^2 - b*x*\sinh(d*x+c)^2)*\text{sqrt}(-a*d^2/b)*\text{Ei}(-d*x + \text{sqrt}(-a*d^2/b)))$$

+ c)^2)\*sqrt(-a\*d^2/b)\*Ei(-d\*x + sqrt(-a\*d^2/b))\*sinh(c + sqrt(-a\*d^2/b)) - 2\*(a\*d^2\*x\*Ei(d\*x) + a\*d^2\*x\*Ei(-d\*x))\*sinh(c) - ((b\*x\*cosh(d\*x + c)^2 - b\*x\*sinh(d\*x + c)^2)\*sqrt(-a\*d^2/b)\*Ei(d\*x + sqrt(-a\*d^2/b)) - (b\*x\*cosh(d\*x + c)^2 - b\*x\*sinh(d\*x + c)^2)\*sqrt(-a\*d^2/b)\*Ei(-d\*x - sqrt(-a\*d^2/b)))\*sinh(-c + sqrt(-a\*d^2/b)))/(a^2\*d\*x\*cosh(d\*x + c)^2 - a^2\*d\*x\*sinh(d\*x + c)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx)}{x^2(a + bx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x\*\*2/(b\*x\*\*2+a), x)

[Out] Integral(cosh(c + d\*x)/(x\*\*2\*(a + b\*x\*\*2)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x^2+a), x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((b\*x^2 + a)\*x^2), x)

$$3.64 \quad \int \frac{\cosh(c+dx)}{x^3(a+bx^2)} dx$$

**Optimal.** Leaf size=270

$$-\frac{b \cosh(c)\text{Chi}(dx)}{a^2} + \frac{b \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right)\text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^2} + \frac{b \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^2} - \frac{b \sinh(c)\text{Shi}(dx)}{a^2} - \frac{b \sinh(c)\text{Shi}(dx)}{a^2}$$

[Out] -Cosh[c + d\*x]/(2\*a\*x^2) - (b\*Cosh[c]\*CoshIntegral[d\*x])/a^2 + (d^2\*Cosh[c]\*CoshIntegral[d\*x])/(2\*a) + (b\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*a^2) + (b\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*a^2) - (d\*Sinh[c + d\*x])/(2\*a\*x) - (b\*Sinh[c]\*SinhIntegral[d\*x])/a^2 + (d^2\*Sinh[c]\*SinhIntegral[d\*x])/(2\*a) - (b\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*a^2) + (b\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*a^2)

**Rubi [A]** time = 0.504207, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {5293, 3297, 3303, 3298, 3301}

$$-\frac{b \cosh(c)\text{Chi}(dx)}{a^2} + \frac{b \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right)\text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^2} + \frac{b \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^2} - \frac{b \sinh(c)\text{Shi}(dx)}{a^2} - \frac{b \sinh(c)\text{Shi}(dx)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x^3\*(a + b\*x^2)), x]

[Out] -Cosh[c + d\*x]/(2\*a\*x^2) - (b\*Cosh[c]\*CoshIntegral[d\*x])/a^2 + (d^2\*Cosh[c]\*CoshIntegral[d\*x])/(2\*a) + (b\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*a^2) + (b\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*a^2) - (d\*Sinh[c + d\*x])/(2\*a\*x) - (b\*Sinh[c]\*SinhIntegral[d\*x])/a^2 + (d^2\*Sinh[c]\*SinhIntegral[d\*x])/(2\*a) - (b\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*a^2) + (b\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*a^2)

### Rule 5293

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], x^m\*(a + b\*x^n)^p, x], x] /; Fr

eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps



$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^3(a+bx^2)} dx &= \int \left( \frac{\cosh(c+dx)}{ax^3} - \frac{b \cosh(c+dx)}{a^2x} + \frac{b^2x \cosh(c+dx)}{a^2(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{x} dx}{a^2} + \frac{b^2 \int \frac{x \cosh(c+dx)}{a+bx^2} dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{2ax^2} + \frac{b^2 \int \left( -\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \right) dx}{a^2} + \frac{d \int \frac{\sinh(c+dx)}{x^2} dx}{2a} - \frac{(b \cosh(c)) \int \frac{\cosh(dx)}{a^2} dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b \cosh(c) \text{Chi}(dx)}{a^2} - \frac{d \sinh(c+dx)}{2ax} - \frac{b \sinh(c) \text{Shi}(dx)}{a^2} - \frac{b^{3/2} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2a^2} + \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b \cosh(c) \text{Chi}(dx)}{a^2} - \frac{d \sinh(c+dx)}{2ax} - \frac{b \sinh(c) \text{Shi}(dx)}{a^2} + \frac{(d^2 \cosh(c)) \int \frac{\cosh(dx)}{x} dx}{2a} \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b \cosh(c) \text{Chi}(dx)}{a^2} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{2a} + \frac{b \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^2} +
\end{aligned}$$

**Mathematica [C]** time = 0.530193, size = 257, normalized size = 0.95

$$x^2 \cosh(c) \left( -(2b - ad^2) \right) \text{Chi}(dx) + bx^2 \cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) + bx^2 \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d\*x]/(x^3\*(a + b\*x^2)), x]

[Out]  $-(a \text{Cosh}[c + d*x]) - (2*b - a*d^2)*x^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + b*x^2*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x] + b*x^2*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] - a*d*x*\text{Sinh}[c + d*x] - 2*b*x^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + a*d^2*x^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x] + I*b*x^2*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] - I*b*x^2*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])/(2*a^2*x^2)$

**Maple [A]** time = 0.063, size = 330, normalized size = 1.2

$$\frac{de^{-dx-c}}{4ax} - \frac{e^{-dx-c}}{4ax^2} - \frac{b}{4a^2} e^{-\frac{1}{b}(d\sqrt{-ab}+cb)} \text{Ei}\left(1, -\frac{1}{b}\left(d\sqrt{-ab} - (dx+c)b + cb\right)\right) - \frac{b}{4a^2} e^{\frac{1}{b}(d\sqrt{-ab}-cb)} \text{Ei}\left(1, \frac{1}{b}\left(d\sqrt{-ab} + (dx+c)b - cb\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/x^3/(b*x^2+a),x)`

[Out]  $\frac{1}{4}d\exp(-dx-c)/a/x - \frac{1}{4}\exp(-dx-c)/a/x^2 - \frac{1}{4}b/a^2\exp(-(d(-ab))^{1/2} + c*b)/b) * Ei(1, -(d(-ab))^{1/2} - (dx+c)*b+c*b)/b) - \frac{1}{4}b/a^2\exp((d(-ab))^{1/2} - c*b)/b) * Ei(1, (d(-ab))^{1/2} + (dx+c)*b-c*b)/b) - \frac{1}{4}d^2/a\exp(-c) * Ei(1, dx) + \frac{1}{2}a^2\exp(-c) * Ei(1, dx) * b - \frac{1}{4}d/a/x\exp(dx+c) - \frac{1}{4}a/x^2\exp(dx+c) - \frac{1}{4}b/a^2\exp(-(d(-ab))^{1/2} - c*b)/b) * Ei(1, -(d(-ab))^{1/2} + (dx+c)*b-c*b)/b) - \frac{1}{4}b/a^2\exp((d(-ab))^{1/2} + c*b)/b) * Ei(1, (d(-ab))^{1/2} - (dx+c)*b+c*b)/b) - \frac{1}{4}d^2/a\exp(c) * Ei(1, -dx) + \frac{1}{2}b/a^2\exp(c) * Ei(1, -dx)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^2+a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="maxima")`

[Out] `integrate(cosh(d*x + c)/((b*x^2 + a)*x^3), x)`

**Fricas [B]** time = 2.51016, size = 1283, normalized size = 4.75

$$2 adx \sinh(dx+c) + 2a \cosh(dx+c) - \left( (bx^2 \cosh(dx+c)^2 - bx^2 \sinh(dx+c)^2) Ei\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + (bx^2 \cosh(dx+c) + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="fricas")`

[Out]  $- \frac{1}{4} * (2*a*d*x*\sinh(dx+c) + 2*a*\cosh(dx+c) - ((b*x^2*\cosh(dx+c))^2 - b*x^2*\sinh(dx+c)^2)*Ei(dx - \sqrt{-a*d^2/b}) + (b*x^2*\cosh(dx+c)^2 - b*x^2*\sinh(dx+c)^2)*Ei(-dx + \sqrt{-a*d^2/b})) * \cosh(c + \sqrt{-a*d^2/b}) - ((a*d^2 - 2*b)*x^2*Ei(dx) + (a*d^2 - 2*b)*x^2*Ei(-dx)) * \cosh(c) - ((b*x^2*\cosh(dx+c))^2 - b*x^2*\sinh(dx+c)^2)*Ei(dx + \sqrt{-a*d^2/b}) + (b*x^2*\cosh(dx+c)^2 - b*x^2*\sinh(dx+c)^2)*Ei(-dx - \sqrt{-a*d^2/b})) * \cosh$

$$\begin{aligned} & (-c + \sqrt{-a*d^2/b}) - ((b*x^2*\cosh(d*x + c)^2 - b*x^2*\sinh(d*x + c)^2)*\text{Ei} \\ & (d*x - \sqrt{-a*d^2/b}) - (b*x^2*\cosh(d*x + c)^2 - b*x^2*\sinh(d*x + c)^2)*\text{Ei} \\ & (-d*x + \sqrt{-a*d^2/b}))*\sinh(c + \sqrt{-a*d^2/b}) - ((a*d^2 - 2*b)*x^2*\text{Ei}(d \\ & *x) - (a*d^2 - 2*b)*x^2*\text{Ei}(-d*x))*\sinh(c) + ((b*x^2*\cosh(d*x + c)^2 - b*x^2 \\ & *\sinh(d*x + c)^2)*\text{Ei}(d*x + \sqrt{-a*d^2/b}) - (b*x^2*\cosh(d*x + c)^2 - b*x^2 \\ & *\sinh(d*x + c)^2)*\text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))/ (a^ \\ & 2*x^2*\cosh(d*x + c)^2 - a^2*x^2*\sinh(d*x + c)^2) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx)}{x^3(a + bx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x\*\*3/(b\*x\*\*2+a),x)

[Out] Integral(cosh(c + d\*x)/(x\*\*3\*(a + b\*x\*\*2)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^3/(b\*x^2+a),x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((b\*x^2 + a)\*x^3), x)

$$3.65 \quad \int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=449

$$\frac{ad \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4b^3} - \frac{ad \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4b^3} + \frac{3\sqrt{-a} \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4b^{5/2}}$$

[Out] (x\*Cosh[c + d\*x])/(2\*b^2) - (x^3\*Cosh[c + d\*x])/(2\*b\*(a + b\*x^2)) + (3\*Sqrt[-a]\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(4\*b^(5/2)) - (3\*Sqrt[-a]\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(4\*b^(5/2)) - (a\*d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x]\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]])/(4\*b^3) - (a\*d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x]\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]])/(4\*b^3) + Sinh[c + d\*x]/(b^2\*d) + (a\*d\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(4\*b^3) - (3\*Sqrt[-a]\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(4\*b^(5/2)) - (a\*d\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(4\*b^3) - (3\*Sqrt[-a]\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(4\*b^(5/2))

**Rubi [A]** time = 0.859464, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {5291, 5293, 2637, 5281, 3303, 3298, 3301, 5292, 3296}

$$\frac{ad \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4b^3} - \frac{ad \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4b^3} + \frac{3\sqrt{-a} \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*Cosh[c + d\*x])/(a + b\*x^2)^2, x]

[Out] (x\*Cosh[c + d\*x])/(2\*b^2) - (x^3\*Cosh[c + d\*x])/(2\*b\*(a + b\*x^2)) + (3\*Sqrt[-a]\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(4\*b^(5/2)) - (3\*Sqrt[-a]\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(4\*b^(5/2)) - (a\*d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x]\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]])/(4\*b^3) - (a\*d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x]\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]])/(4\*b^3) + Sinh[c + d\*x]/(b^2\*d) + (a\*d\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(4\*b^3) - (3\*Sqrt[-a]\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(4\*b^(5/2)) - (a\*d\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(4\*b^3) - (3\*Sqrt[-a]\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(4\*b^(5/2))

```
hIntegral[(Sqrt[-a]*d)/Sqrt[b - d*x]]/(4*b^(5/2)) - (a*d*Cosh[c - (Sqrt[-a]
]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]]/(4*b^3) - (3*Sqrt[-
a]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]
)/(4*b^(5/2))
```

### Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x]]/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

### Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol]
:> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x]
/; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2} dx &= -\frac{x^3 \cosh(c+dx)}{2b(a+bx^2)} + \frac{3 \int \frac{x^2 \cosh(c+dx)}{a+bx^2} dx}{2b} + \frac{d \int \frac{x^3 \sinh(c+dx)}{a+bx^2} dx}{2b} \\
&= -\frac{x^3 \cosh(c+dx)}{2b(a+bx^2)} + \frac{3 \int \left( \frac{\cosh(c+dx)}{b} - \frac{a \cosh(c+dx)}{b(a+bx^2)} \right) dx}{2b} + \frac{d \int \left( \frac{x \sinh(c+dx)}{b} - \frac{ax \sinh(c+dx)}{b(a+bx^2)} \right) dx}{2b} \\
&= -\frac{x^3 \cosh(c+dx)}{2b(a+bx^2)} + \frac{3 \int \cosh(c+dx) dx}{2b^2} - \frac{(3a) \int \frac{\cosh(c+dx)}{a+bx^2} dx}{2b^2} + \frac{d \int x \sinh(c+dx) dx}{2b^2} - \frac{(ad)}{2b^2} \\
&= \frac{x \cosh(c+dx)}{2b^2} - \frac{x^3 \cosh(c+dx)}{2b(a+bx^2)} + \frac{3 \sinh(c+dx)}{2b^2 d} - \frac{\int \cosh(c+dx) dx}{2b^2} - \frac{(3a) \int \left( \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{bx})} \right) dx}{2b^2} \\
&= \frac{x \cosh(c+dx)}{2b^2} - \frac{x^3 \cosh(c+dx)}{2b(a+bx^2)} + \frac{\sinh(c+dx)}{b^2 d} - \frac{(3\sqrt{-a}) \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{4b^2} - \frac{(3\sqrt{-a}) \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{4b^2} \\
&= \frac{x \cosh(c+dx)}{2b^2} - \frac{x^3 \cosh(c+dx)}{2b(a+bx^2)} + \frac{\sinh(c+dx)}{b^2 d} - \frac{\left( 3\sqrt{-a} \cosh \left( c - \frac{\sqrt{-ad}}{\sqrt{b}} \right) \right) \int \frac{\cosh \left( \frac{\sqrt{-ad}}{\sqrt{b}} + dx \right)}{\sqrt{-a}+\sqrt{bx}} dx}{4b^2} \\
&= \frac{x \cosh(c+dx)}{2b^2} - \frac{x^3 \cosh(c+dx)}{2b(a+bx^2)} + \frac{3\sqrt{-a} \cosh \left( c + \frac{\sqrt{-ad}}{\sqrt{b}} \right) \text{Chi} \left( \frac{\sqrt{-ad}}{\sqrt{b}} - dx \right)}{4b^{5/2}} - \frac{3\sqrt{-a} \cosh \left( c - \frac{\sqrt{-ad}}{\sqrt{b}} \right) \text{Chi} \left( \frac{\sqrt{-ad}}{\sqrt{b}} + dx \right)}{4b^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 1.50657, size = 621, normalized size = 1.38

$$\frac{3\sqrt{a} \sinh(c) \left( \sin \left( \frac{\sqrt{ad}}{\sqrt{b}} \right) \text{CosIntegral} \left( -\frac{\sqrt{ad}}{\sqrt{b}} + idx \right) + \sin \left( \frac{\sqrt{ad}}{\sqrt{b}} \right) \text{CosIntegral} \left( \frac{\sqrt{ad}}{\sqrt{b}} + idx \right) - \cos \left( \frac{\sqrt{ad}}{\sqrt{b}} \right) \left( \text{Si} \left( \frac{\sqrt{ad}}{\sqrt{b}} - idx \right) + \text{Si} \left( idx + \frac{\sqrt{ad}}{\sqrt{b}} \right) \right) \right)}{\sqrt{b}} - \frac{ad \sinh(c) \left( \cos \left( \frac{\sqrt{ad}}{\sqrt{b}} \right) \text{CosIntegral} \left( -\frac{\sqrt{ad}}{\sqrt{b}} + idx \right) + \cos \left( \frac{\sqrt{ad}}{\sqrt{b}} \right) \text{CosIntegral} \left( \frac{\sqrt{ad}}{\sqrt{b}} + idx \right) - \sin \left( \frac{\sqrt{ad}}{\sqrt{b}} \right) \left( \text{Ci} \left( \frac{\sqrt{ad}}{\sqrt{b}} - idx \right) + \text{Ci} \left( idx + \frac{\sqrt{ad}}{\sqrt{b}} \right) \right) \right)}{\sqrt{b}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4\*Cosh[c + d\*x])/(a + b\*x^2)^2,x]

[Out] (2\*Cosh[d\*x]\*((a\*x\*Cosh[c])/(a + b\*x^2) + (2\*Sinh[c])/d) + 2\*((2\*Cosh[c])/d + (a\*x\*Sinh[c])/(a + b\*x^2))\*Sinh[d\*x] - ((3\*I)\*Sqrt[a]\*Cosh[c]\*(Cos[(Sqrt[a]\*d)/Sqrt[b]]\*CosIntegral[-((Sqrt[a]\*d)/Sqrt[b]) + I\*d\*x] - Cos[(Sqrt[a]\*d)/Sqrt[b]]\*CosIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x] + Sin[(Sqrt[a]\*d)/Sqrt[b]]\*(SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] - SinIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x])))/Sqrt[b] + (I\*a\*d\*Cosh[c]\*(CosIntegral[-((Sqrt[a]\*d)/Sqrt[b]) + I\*d\*x]\*Sin[(Sqrt[a]\*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]\*d)/Sqrt[b] + I

$$\begin{aligned} & *d*x]*\sin[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \cos[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*(-\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]))/b - \\ & (3*\text{Sqrt}[a]*\text{Sinh}[c]*(\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x]*\sin[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]*\sin[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] - \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*(\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]))/\text{Sqrt}[b] - (a*d*\text{Sinh}[c]*(\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x] + \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] + \sin[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*(\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])))/b)/(4*b^2) \end{aligned}$$

**Maple [A]** time = 0.319, size = 532, normalized size = 1.2

$$\frac{d^2 e^{-dx-c} ax}{4b^2 (bd^2 x^2 + ad^2)} - \frac{3a}{8b^2} e^{\frac{1}{b}(d\sqrt{-ab}-cb)} \text{Ei}\left(1, \frac{1}{b} \left(d\sqrt{-ab} + (dx+c)b - cb\right)\right) \frac{1}{\sqrt{-ab}} + \frac{3a}{8b^2} e^{-\frac{1}{b}(d\sqrt{-ab}+cb)} \text{Ei}\left(1, -\frac{1}{b} \left(d\sqrt{-ab} - (dx+c)b + cb\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*cosh(d\*x+c)/(b\*x^2+a)^2,x)

[Out]  $\frac{1}{4}d^2 \exp(-d*x-c) * a/b^2 / (b*d^2*x^2+a*d^2) * x - 3/8/b^2 * a / (-a*b)^{(1/2)} * \exp((d*(-a*b)^{(1/2)}-c*b)/b) * \text{Ei}(1, (d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b) + 3/8/b^2 * a / (-a*b)^{(1/2)} * \exp(-(d*(-a*b)^{(1/2)}+c*b)/b) * \text{Ei}(1, -(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b) - 1/2/d * \exp(-d*x-c) / b^2 - 1/8*d/b^3 * a * \exp((d*(-a*b)^{(1/2)}-c*b)/b) * \text{Ei}(1, (d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b) - 1/8*d/b^3 * a * \exp(-(d*(-a*b)^{(1/2)}+c*b)/b) * \text{Ei}(1, -(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b) + 1/2/d/b^2 * \exp(d*x+c) + 1/4*d^2 * \exp(d*x+c) * a/b^2 / (b*d^2*x^2+a*d^2) * x + 3/8/b^2 * a / (-a*b)^{(1/2)} * \exp((d*(-a*b)^{(1/2)}+c*b)/b) * \text{Ei}(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b) - 3/8/b^2 * a / (-a*b)^{(1/2)} * \exp(-(d*(-a*b)^{(1/2)}-c*b)/b) * \text{Ei}(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b) + 1/8*d/b^3 * a * \exp((d*(-a*b)^{(1/2)}+c*b)/b) * \text{Ei}(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b) + 1/8*d/b^3 * a * \exp(-(d*(-a*b)^{(1/2)}-c*b)/b) * \text{Ei}(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*cosh(d\*x+c)/(b\*x^2+a)^2,x, algorithm="maxima")



[Out] Exception raised: ValueError

**Fricas [B]** time = 2.31252, size = 2488, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>4</sup>\*cosh(d\*x+c)/(b\*x<sup>2</sup>+a)<sup>2</sup>,x, algorithm="fricas")

[Out] 
$$\frac{1}{8} \cdot (4 \cdot a \cdot b \cdot d \cdot x \cdot \cosh(dx + c) - ((a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \cosh(dx + c)^2 - (a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \sinh(dx + c)^2 - 3 \cdot ((b^2 \cdot x^2 + a \cdot b) \cdot \cosh(dx + c)^2 - (b^2 \cdot x^2 + a \cdot b) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2 / b}) \cdot \text{Ei}(dx - \sqrt{-a \cdot d^2 / b})) - ((a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \cosh(dx + c)^2 - (a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \sinh(dx + c)^2 + 3 \cdot ((b^2 \cdot x^2 + a \cdot b) \cdot \cosh(dx + c)^2 - (b^2 \cdot x^2 + a \cdot b) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2 / b}) \cdot \text{Ei}(-dx + \sqrt{-a \cdot d^2 / b})) \cdot \cosh(c + \sqrt{-a \cdot d^2 / b})) - (((a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \cosh(dx + c)^2 - (a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \sinh(dx + c)^2 + 3 \cdot ((b^2 \cdot x^2 + a \cdot b) \cdot \cosh(dx + c)^2 - (b^2 \cdot x^2 + a \cdot b) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2 / b}) \cdot \text{Ei}(dx + \sqrt{-a \cdot d^2 / b})) - ((a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \cosh(dx + c)^2 - (a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \sinh(dx + c)^2 - 3 \cdot ((b^2 \cdot x^2 + a \cdot b) \cdot \cosh(dx + c)^2 - (b^2 \cdot x^2 + a \cdot b) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2 / b})) \cdot \text{Ei}(-dx - \sqrt{-a \cdot d^2 / b})) \cdot \cosh(-c + \sqrt{-a \cdot d^2 / b})) + 8 \cdot (b^2 \cdot x^2 + a \cdot b) \cdot \sinh(dx + c) - (((a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \cosh(dx + c)^2 - (a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \sinh(dx + c)^2 - 3 \cdot ((b^2 \cdot x^2 + a \cdot b) \cdot \cosh(dx + c)^2 - (b^2 \cdot x^2 + a \cdot b) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2 / b}) \cdot \text{Ei}(dx - \sqrt{-a \cdot d^2 / b})) + ((a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \cosh(dx + c)^2 - (a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \sinh(dx + c)^2 + 3 \cdot ((b^2 \cdot x^2 + a \cdot b) \cdot \cosh(dx + c)^2 - (b^2 \cdot x^2 + a \cdot b) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2 / b}) \cdot \text{Ei}(-dx + \sqrt{-a \cdot d^2 / b})) \cdot \sinh(c + \sqrt{-a \cdot d^2 / b})) + (((a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \cosh(dx + c)^2 - (a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \sinh(dx + c)^2 + 3 \cdot ((b^2 \cdot x^2 + a \cdot b) \cdot \cosh(dx + c)^2 - (b^2 \cdot x^2 + a \cdot b) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2 / b}) \cdot \text{Ei}(dx + \sqrt{-a \cdot d^2 / b})) + ((a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \cosh(dx + c)^2 - (a \cdot b \cdot d^2 \cdot x^2 + a^2 \cdot d^2) \cdot \sinh(dx + c)^2 - 3 \cdot ((b^2 \cdot x^2 + a \cdot b) \cdot \cosh(dx + c)^2 - (b^2 \cdot x^2 + a \cdot b) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2 / b}) \cdot \text{Ei}(-dx - \sqrt{-a \cdot d^2 / b})) \cdot \sinh(-c + \sqrt{-a \cdot d^2 / b})) / ((b^4 \cdot d \cdot x^2 + a \cdot b^3 \cdot d) \cdot \cosh(dx + c)^2 - (b^4 \cdot d \cdot x^2 + a \cdot b^3 \cdot d) \cdot \sinh(dx + c)^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(dx + c)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

$$3.66 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=431

$$-\frac{\sqrt{-ad} \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4b^{5/2}} + \frac{\sqrt{-ad} \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4b^{5/2}} + \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2}$$

[Out] Cosh[c + d\*x]/(2\*b^2) - (x^2\*Cosh[c + d\*x])/(2\*b\*(a + b\*x^2)) + (Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b^2) + (Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b^2) - (Sqrt[-a]\*d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x]\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]])/(4\*b^(5/2)) + (Sqrt[-a]\*d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x]\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]])/(4\*b^(5/2)) - (Sqrt[-a]\*d\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(4\*b^(5/2)) - (Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b^2) - (Sqrt[-a]\*d\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(4\*b^(5/2)) + (Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b^2)

**Rubi [A]** time = 0.700681, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {5291, 5293, 3303, 3298, 3301, 5292, 2638, 5280}

$$-\frac{\sqrt{-ad} \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4b^{5/2}} + \frac{\sqrt{-ad} \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4b^{5/2}} + \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*Cosh[c + d\*x])/(a + b\*x^2)^2,x]

[Out] Cosh[c + d\*x]/(2\*b^2) - (x^2\*Cosh[c + d\*x])/(2\*b\*(a + b\*x^2)) + (Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b^2) + (Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b^2) - (Sqrt[-a]\*d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x]\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]])/(4\*b^(5/2)) + (Sqrt[-a]\*d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x]\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]])/(4\*b^(5/2)) - (Sqrt[-a]\*d\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(4\*b^(5/2)) - (Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(2\*b^2) - (Sqrt[-a]\*d\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(4\*b^(5/2)) + (Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(2\*b^2)

$$-a*d)/\text{Sqrt}[b + d*x]]/(4*b^{(5/2)}) + (\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b + d*x]]/(2*b^2)$$

### Rule 5291

$$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))}^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(x^{(m-n+1)}*(a + b*x^n)^{(p+1)}*\text{Cosh}[c + d*x])/(b*n*(p+1)), x] + (-\text{Dist}[(m-n+1)/(b*n*(p+1)), \text{Int}[x^{(m-n)}*(a + b*x^n)^{(p+1)}*\text{Cosh}[c + d*x], x], x] - \text{Dist}[d/(b*n*(p+1)), \text{Int}[x^{(m-n+1)}*(a + b*x^n)^{(p+1)}*\text{Sinh}[c + d*x], x], x]) /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{ILtQ}[p, -1] \&\& \text{IGtQ}[n, 0] \&\& \text{RationalQ}[m] \&\& (\text{GtQ}[m-n+1, 0] \parallel \text{GtQ}[n, 2])$$

### Rule 5293

$$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))}^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], x^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{ILtQ}[p, 0] \&\& \text{IntegerQ}[m] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n, 2] \parallel \text{EqQ}[p, -1])$$

### Rule 3303

$$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{NeQ}[d*e - c*f, 0]$$

### Rule 3298

$$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(I*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \text{EqQ}[d*e - c*f*fz*I, 0]$$

### Rule 3301

$$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \text{EqQ}[d*(e - \text{Pi}/2) - c*f*fz*I, 0]$$

### Rule 5292

$$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))}^{(p_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Sinh}[c + d*x], x^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{ILtQ}[p, 0] \&\& \text{IntegerQ}[m] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n, 2] \parallel \text{EqQ}[p, -1])$$

Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 5280

Int[((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_)\*Sinh[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Int[ExpandIntegrand[Sinh[c + d\*x], (a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \cosh(c + dx)}{(a + bx^2)^2} dx &= -\frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\int \frac{x \cosh(c+dx)}{a+bx^2} dx}{b} + \frac{d \int \frac{x^2 \sinh(c+dx)}{a+bx^2} dx}{2b} \\
 &= -\frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\int \left( -\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \right) dx}{b} + \frac{d \int \left( \frac{\sinh(c+dx)}{b} - \frac{a \sinh(c+dx)}{b(a+bx^2)} \right) dx}{2b} \\
 &= -\frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2b^{3/2}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2b^{3/2}} + \frac{d \int \sinh(c + dx) dx}{2b^2} - \frac{(ad) \int \frac{\sinh(c+dx)}{a+bx^2} dx}{2b^2} \\
 &= \frac{\cosh(c + dx)}{2b^2} - \frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} - \frac{(ad) \int \left( \frac{\sqrt{-a} \sinh(c+dx)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \sinh(c+dx)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx}{2b^2} + \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2b^2} \\
 &= \frac{\cosh(c + dx)}{2b^2} - \frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2} + \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(-\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2} \\
 &= \frac{\cosh(c + dx)}{2b^2} - \frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2} + \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(-\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2} \\
 &= \frac{\cosh(c + dx)}{2b^2} - \frac{x^2 \cosh(c + dx)}{2b(a + bx^2)} + \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2} + \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(-\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2b^2}
 \end{aligned}$$

**Mathematica [C]** time = 0.740256, size = 582, normalized size = 1.35

$$a^{3/2}d \cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} - idx\right) + a^{3/2}d \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(ixd + \frac{\sqrt{ad}}{\sqrt{b}}\right) + 2ib^{3/2}x^2 \sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} - idx\right) - 2i$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3\*Cosh[c + d\*x])/(a + b\*x^2)^2,x]

[Out] (2\*a\*Sqrt[b]\*Cosh[c + d\*x] + (a + b\*x^2)\*CosIntegral[-((Sqrt[a]\*d)/Sqrt[b]) + I\*d\*x]\*(2\*Sqrt[b]\*Cosh[c - (I\*Sqrt[a]\*d)/Sqrt[b]] - I\*Sqrt[a]\*d\*Sinh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]) + (a + b\*x^2)\*CosIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x]\*(2\*Sqrt[b]\*Cosh[c + (I\*Sqrt[a]\*d)/Sqrt[b]] + I\*Sqrt[a]\*d\*Sinh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]) + a^(3/2)\*d\*Cosh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] + Sqrt[a]\*b\*d\*x^2\*Cosh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] + (2\*I)\*a\*Sqrt[b]\*Sinh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] + (2\*I)\*b^(3/2)\*x^2\*Sinh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] + a^(3/2)\*d\*Cosh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x] + Sqrt[a]\*b\*d\*x^2\*Cosh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x] - (2\*I)\*a\*Sqrt[b]\*Sinh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x] - (2\*I)\*b^(3/2)\*x^2\*Sinh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x])/(4\*b^(5/2))\*(a + b\*x^2))

**Maple [A]** time = 0.095, size = 495, normalized size = 1.2

$$\frac{d^2 e^{-dx-ca}}{4b^2(bd^2x^2 + ad^2)} - \frac{1}{4b^2} e^{\frac{1}{b}(d\sqrt{-ab}-cb)} \text{Ei}\left(1, \frac{1}{b}\left(d\sqrt{-ab} + (dx+c)b - cb\right)\right) - \frac{1}{4b^2} e^{-\frac{1}{b}(d\sqrt{-ab}+cb)} \text{Ei}\left(1, -\frac{1}{b}\left(d\sqrt{-ab} - (dx+c)b + cb\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*cosh(d\*x+c)/(b\*x^2+a)^2,x)

[Out] 1/4\*d^2\*exp(-d\*x-c)\*a/b^2/(b\*d^2\*x^2+a\*d^2)-1/4/b^2\*exp((d\*(-a\*b)^(1/2)-c\*b)/b)\*Ei(1,(d\*(-a\*b)^(1/2)+(d\*x+c)\*b-c\*b)/b)-1/4/b^2\*exp(-(d\*(-a\*b)^(1/2)+c\*b)/b)\*Ei(1,-(d\*(-a\*b)^(1/2)-(d\*x+c)\*b+c\*b)/b)+1/8\*d/b^2\*a/(-a\*b)^(1/2)\*exp((d\*(-a\*b)^(1/2)-c\*b)/b)\*Ei(1,(d\*(-a\*b)^(1/2)+(d\*x+c)\*b-c\*b)/b)-1/8\*d/b^2\*a/(-a\*b)^(1/2)\*exp(-(d\*(-a\*b)^(1/2)+c\*b)/b)\*Ei(1,-(d\*(-a\*b)^(1/2)-(d\*x+c)\*b+c\*b)/b)-1/4/b^2\*exp((d\*(-a\*b)^(1/2)+c\*b)/b)\*Ei(1,(d\*(-a\*b)^(1/2)-(d\*x+c)\*b+c\*b)/b)-1/4/b^2\*exp(-(d\*(-a\*b)^(1/2)-c\*b)/b)\*Ei(1,-(d\*(-a\*b)^(1/2)+(d\*x+c)\*b-c\*b)/b)+1/4\*d^2\*exp(d\*x+c)\*a/b^2/(b\*d^2\*x^2+a\*d^2)+1/8\*d/b^2\*a/(-a\*b)^(1/2)\*exp((d\*(-a\*b)^(1/2)+c\*b)/b)\*Ei(1,(d\*(-a\*b)^(1/2)-(d\*x+c)\*b+c\*b)/b)-1/8\*d/b^2\*a/(-a\*b)^(1/2)\*exp(-(d\*(-a\*b)^(1/2)-c\*b)/b)\*Ei(1,-(d\*(-a\*b)^(1/2)+(d\*x+c)\*b-c\*b)/b)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.15299, size = 2099, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x^2+a)^2,x, algorithm="fricas")

[Out] 
$$\frac{1}{8} \cdot (4 \cdot a \cdot \cosh(d \cdot x + c) + ((2 \cdot (b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - 2 \cdot (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2 + ((b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2) \cdot \sqrt{-a \cdot d^2 / b} \cdot \text{Ei}(d \cdot x - \sqrt{-a \cdot d^2 / b}) + (2 \cdot (b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - 2 \cdot (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2 - ((b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2) \cdot \sqrt{-a \cdot d^2 / b} \cdot \text{Ei}(-d \cdot x + \sqrt{-a \cdot d^2 / b})) \cdot \cosh(c + \sqrt{-a \cdot d^2 / b}) + ((2 \cdot (b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - 2 \cdot (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2 - ((b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2) \cdot \sqrt{-a \cdot d^2 / b} \cdot \text{Ei}(d \cdot x + \sqrt{-a \cdot d^2 / b}) + (2 \cdot (b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - 2 \cdot (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2 + ((b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2) \cdot \sqrt{-a \cdot d^2 / b} \cdot \text{Ei}(-d \cdot x - \sqrt{-a \cdot d^2 / b})) \cdot \cosh(-c + \sqrt{-a \cdot d^2 / b}) + ((2 \cdot (b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - 2 \cdot (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2 + ((b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2) \cdot \sqrt{-a \cdot d^2 / b} \cdot \text{Ei}(d \cdot x - \sqrt{-a \cdot d^2 / b}) - (2 \cdot (b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - 2 \cdot (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2 - ((b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2) \cdot \sqrt{-a \cdot d^2 / b} \cdot \text{Ei}(-d \cdot x + \sqrt{-a \cdot d^2 / b})) \cdot \sinh(c + \sqrt{-a \cdot d^2 / b}) - ((2 \cdot (b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - 2 \cdot (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2 - ((b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2) \cdot \sqrt{-a \cdot d^2 / b} \cdot \text{Ei}(d \cdot x + \sqrt{-a \cdot d^2 / b}) - (2 \cdot (b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - 2 \cdot (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2 + ((b \cdot x^2 + a) \cdot \cosh(d \cdot x + c))^2 - (b \cdot x^2 + a) \cdot \sinh(d \cdot x + c))^2) \cdot \sqrt{-a \cdot d^2 / b} \cdot \text{Ei}(-d \cdot x - \sqrt{-a \cdot d^2 / b})) \cdot \sinh(-c + \sqrt{-a \cdot d^2 / b})) / ((b^3 \cdot x^2 + a \cdot b^2) \cdot \cosh(d \cdot x + c))^2 - (b^3 \cdot x^2 + a \cdot b^2) \cdot \sinh(d \cdot x + c))^2)$$

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*cosh(d\*x+c)/(b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(dx + c)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x^2+a)^2,x, algorithm="giac")

[Out] integrate(x^3\*cosh(d\*x + c)/(b\*x^2 + a)^2, x)



$$3.67 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=416

$$\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4b^2} + \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4b^2} + \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-ab^{3/2}}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4\sqrt{-ab^{3/2}}}$$

[Out]  $-(x*\text{Cosh}[c + d*x])/(2*b*(a + b*x^2)) + (\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*\text{Sqrt}[-a]*b^{(3/2)}) - (\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*\text{Sqrt}[-a]*b^{(3/2)}) + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*b^2) + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*b^2) - (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*b^2) - (\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*\text{Sqrt}[-a]*b^{(3/2)}) + (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*b^2) - (\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*\text{Sqrt}[-a]*b^{(3/2)})$

**Rubi [A]** time = 0.611313, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5291, 5281, 3303, 3298, 3301, 5292}

$$\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4b^2} + \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4b^2} + \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-ab^{3/2}}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4\sqrt{-ab^{3/2}}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*Cosh[c + d\*x])/(a + b\*x^2)^2,x]

[Out]  $-(x*\text{Cosh}[c + d*x])/(2*b*(a + b*x^2)) + (\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*\text{Sqrt}[-a]*b^{(3/2)}) - (\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*\text{Sqrt}[-a]*b^{(3/2)}) + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*b^2) + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*b^2) - (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*b^2) - (\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*\text{Sqrt}[-a]*b^{(3/2)}) + (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*b^2) - (\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*\text{Sqrt}[-a]*b^{(3/2)})$

$\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(4*\text{Sqrt}[-a]*b^{(3/2)})$

### Rule 5291

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))}^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(x^{(m-n+1)}*(a + b*x^n)^{(p+1)}*\text{Cosh}[c + d*x])/(b*n*(p+1)), x] + (-\text{Dist}[(m-n+1)/(b*n*(p+1)), \text{Int}[x^{(m-n)}*(a + b*x^n)^{(p+1)}*\text{Cosh}[c + d*x], x], x] - \text{Dist}[d/(b*n*(p+1)), \text{Int}[x^{(m-n+1)}*(a + b*x^n)^{(p+1)}*\text{Sinh}[c + d*x], x], x]) /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{ILtQ}[p, -1] \&\& \text{IGtQ}[n, 0] \&\& \text{RationalQ}[m] \&\& (\text{GtQ}[m-n+1, 0] \parallel \text{GtQ}[n, 2])$

### Rule 5281

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((a_.) + (b_.)*(x_.)^{(n_.))}^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{ILtQ}[p, 0] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n, 2] \parallel \text{EqQ}[p, -1])$

### Rule 3303

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{NeQ}[d*e - c*f, 0]$

### Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(I*\text{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \text{EqQ}[d*e - c*f*fz*I, 0]$

### Rule 3301

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \text{EqQ}[d*(e - \text{Pi}/2) - c*f*fz*I, 0]$

### Rule 5292

$\text{Int}[(x_.)^{(m_.)*((a_.) + (b_.)*(x_.)^{(n_.))}^{(p_.)}*\text{Sinh}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Sinh}[c + d*x], x^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{ILtQ}[p, 0] \&\& \text{IntegerQ}[m] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n, 2] \parallel \text{EqQ}[p, -1])$

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2} dx &= -\frac{x \cosh(c+dx)}{2b(a+bx^2)} + \frac{\int \frac{\cosh(c+dx)}{a+bx^2} dx}{2b} + \frac{d \int \frac{x \sinh(c+dx)}{a+bx^2} dx}{2b} \\
&= -\frac{x \cosh(c+dx)}{2b(a+bx^2)} + \frac{\int \left( \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx}{2b} + \frac{d \int \left( -\frac{\sinh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\sinh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \right) dx}{2b} \\
&= -\frac{x \cosh(c+dx)}{2b(a+bx^2)} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{4\sqrt{-ab}} - \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{4\sqrt{-ab}} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{4b^{3/2}} + \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{4b^{3/2}} \\
&= -\frac{x \cosh(c+dx)}{2b(a+bx^2)} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \int \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{bx}} dx}{4\sqrt{-ab}} + \frac{\left(d \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right)\right) \int \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{bx}} dx}{4b^{3/2}} \\
&= -\frac{x \cosh(c+dx)}{2b(a+bx^2)} + \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{4\sqrt{-ab}^{3/2}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{4\sqrt{-ab}^{3/2}} + \dots
\end{aligned}$$

**Mathematica [C]** time = 0.991712, size = 364, normalized size = 0.88

$$\frac{(a+bx^2) \text{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}}+idx\right) \left(\sqrt{ad} \sinh\left(c-\frac{i\sqrt{ad}}{\sqrt{b}}\right) + i\sqrt{b} \cosh\left(c-\frac{i\sqrt{ad}}{\sqrt{b}}\right)\right) + (a+bx^2) \text{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}}+idx\right)}{1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*Cosh[c + d\*x])/(a + b\*x^2)^2,x]

[Out]  $(-2*\text{Sqrt}[a]*b*x*\text{Cosh}[c + d*x] + (a + b*x^2)*\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x]*(I*\text{Sqrt}[b]*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{Sqrt}[a]*d*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]) + (a + b*x^2)*\text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]*((-I)*\text{Sqrt}[b]*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{Sqrt}[a]*d*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]) + (a + b*x^2)*(I*\text{Sqrt}[a]*d*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]] - \text{Sqrt}[b]*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]])*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] - (a + b*x^2)*(I*\text{Sqrt}[a]*d*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{Sqrt}[b]*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]])*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])/(4*\text{Sqrt}[a]*b^2*(a + b*x^2))$

**Maple [A]** time = 0.068, size = 491, normalized size = 1.2

$$-\frac{d^2 e^{-dx-c} x}{4b(bd^2x^2 + ad^2)} + \frac{d}{8b^2} e^{\frac{1}{b}(d\sqrt{-ab}-cb)} \operatorname{Ei}\left(1, \frac{1}{b}\left(d\sqrt{-ab} + (dx+c)b - cb\right)\right) + \frac{d}{8b^2} e^{-\frac{1}{b}(d\sqrt{-ab}+cb)} \operatorname{Ei}\left(1, -\frac{1}{b}\left(d\sqrt{-ab} - (dx+c)b + cb\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cosh(d\*x+c)/(b\*x^2+a)^2,x)

[Out] 
$$-1/4*d^2*\exp(-d*x-c)/b/(b*d^2*x^2+a*d^2)*x+1/8*d/b^2*\exp((d*(-a*b)^(1/2)-c*b)/b)*\operatorname{Ei}(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/8*d/b^2*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8/b/(-a*b)^(1/2)*\exp((d*(-a*b)^(1/2)-c*b)/b)*\operatorname{Ei}(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8/b/(-a*b)^(1/2)*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4*d^2*\exp(d*x+c)/b/(b*d^2*x^2+a*d^2)*x-1/8*d/b^2*\exp((d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/8*d/b^2*\exp(-(d*(-a*b)^(1/2)-c*b)/b)*\operatorname{Ei}(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/8/b/(-a*b)^(1/2)*\exp((d*(-a*b)^(1/2)+c*b)/b)*\operatorname{Ei}(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8/b/(-a*b)^(1/2)*\exp(-(d*(-a*b)^(1/2)-c*b)/b)*\operatorname{Ei}(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.19618, size = 2433, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x^2+a)^2,x, algorithm="fricas")

```
[Out] -1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 -
(a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2
- (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)
) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh
(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x +
c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
- (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(
d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x +
c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*c
osh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)
*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x
- sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*c
osh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)
*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x
- sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2
+ a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 +
a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c +
sqrt(-a*d^2/b)) + (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2
+ a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 +
a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b*d^2*
x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 -
((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a
*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^3*d*x^
2 + a^2*b^2*d)*cosh(d*x + c)^2 - (a*b^3*d*x^2 + a^2*b^2*d)*sinh(d*x + c)^2)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(dx + c)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

$$3.68 \quad \int \frac{x \cosh(c+dx)}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=239

$$-\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4\sqrt{-ab}^{3/2}} + \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-ab}^{3/2}} - \frac{d \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-ab}^{3/2}} - \frac{d \cos\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-ab}^{3/2}}$$

[Out] -Cosh[c + d\*x]/(2\*b\*(a + b\*x^2)) - (d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x]\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]])/(4\*Sqrt[-a]\*b^(3/2)) + (d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x]\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]])/(4\*Sqrt[-a]\*b^(3/2)) - (d\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(4\*Sqrt[-a]\*b^(3/2)) - (d\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(4\*Sqrt[-a]\*b^(3/2))

**Rubi [A]** time = 0.325431, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {5289, 5280, 3303, 3298, 3301}

$$-\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4\sqrt{-ab}^{3/2}} + \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-ab}^{3/2}} - \frac{d \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-ab}^{3/2}} - \frac{d \cos\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4\sqrt{-ab}^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x\*Cosh[c + d\*x])/(a + b\*x^2)^2,x]

[Out] -Cosh[c + d\*x]/(2\*b\*(a + b\*x^2)) - (d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x]\*Sinh[c - (Sqrt[-a]\*d)/Sqrt[b]])/(4\*Sqrt[-a]\*b^(3/2)) + (d\*CoshIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x]\*Sinh[c + (Sqrt[-a]\*d)/Sqrt[b]])/(4\*Sqrt[-a]\*b^(3/2)) - (d\*Cosh[c + (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] - d\*x])/(4\*Sqrt[-a]\*b^(3/2)) - (d\*Cosh[c - (Sqrt[-a]\*d)/Sqrt[b]]\*SinhIntegral[(Sqrt[-a]\*d)/Sqrt[b] + d\*x])/(4\*Sqrt[-a]\*b^(3/2))

**Rule 5289**

Int[Cosh[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(e^m\*(a + b\*x^n)^(p + 1)\*Cosh[c + d\*x])/(b\*n\*(p + 1)), x] - Dist[(d\*e^m)/(b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*Sinh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0]

] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])

### Rule 5280

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*Sinh[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Int[ExpandIntegrand[Sinh[c + d\*x], (a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps



$$\begin{aligned}
\int \frac{x \cosh(c+dx)}{(a+bx^2)^2} dx &= -\frac{\cosh(c+dx)}{2b(a+bx^2)} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^2} dx}{2b} \\
&= -\frac{\cosh(c+dx)}{2b(a+bx^2)} + \frac{d \int \left( \frac{\sqrt{-a} \sinh(c+dx)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \sinh(c+dx)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx}{2b} \\
&= -\frac{\cosh(c+dx)}{2b(a+bx^2)} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{4\sqrt{-ab}} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{4\sqrt{-ab}} \\
&= -\frac{\cosh(c+dx)}{2b(a+bx^2)} - \frac{\left( d \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \right) \int \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{bx}} dx}{4\sqrt{-ab}} + \frac{\left( d \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \right) \int \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{\sqrt{-a}-\sqrt{bx}} dx}{4\sqrt{-ab}} \\
&= -\frac{\cosh(c+dx)}{2b(a+bx^2)} - \frac{d \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right) \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4\sqrt{-ab}^{3/2}} + \frac{d \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right) \sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4\sqrt{-ab}^{3/2}} - \frac{d \cosh(c+dx)}{2b(a+bx^2)}
\end{aligned}$$

**Mathematica [C]** time = 0.513809, size = 239, normalized size = 1.

$$\frac{i \left( d(a+bx^2) \sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) - d(a+bx^2) \sinh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) + i \left( d(a+bx^2) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right) \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) - d(a+bx^2) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) \sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \right)}{4\sqrt{ab}^{3/2} (a+bx^2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x\*Cosh[c + d\*x])/(a + b\*x^2)^2,x]

[Out] ((I/4)\*(d\*(a + b\*x^2)\*CosIntegral[-((Sqrt[a]\*d)/Sqrt[b]) + I\*d\*x]\*Sinh[c - (I\*Sqrt[a]\*d)/Sqrt[b]] - d\*(a + b\*x^2)\*CosIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x]\*Sinh[c + (I\*Sqrt[a]\*d)/Sqrt[b]] + I\*(2\*Sqrt[a]\*Sqrt[b]\*Cosh[c + d\*x] + d\*(a + b\*x^2)\*Cosh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] + d\*(a + b\*x^2)\*Cosh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x]))/(Sqrt[a]\*b^(3/2)\*(a + b\*x^2))

**Maple [A]** time = 0.044, size = 291, normalized size = 1.2

$$-\frac{d^2 e^{-dx-c}}{4b(bd^2x^2 + ad^2)} - \frac{d}{8b} e^{\frac{1}{b}(d\sqrt{-ab}-cb)} \operatorname{Ei}\left(1, \frac{1}{b} \left( d\sqrt{-ab} + (dx+c)b - cb \right)\right) \frac{1}{\sqrt{-ab}} + \frac{d}{8b} e^{-\frac{1}{b}(d\sqrt{-ab}+cb)} \operatorname{Ei}\left(1, -\frac{1}{b} \left( d\sqrt{-ab} - (dx+c)b + cb \right)\right) \frac{1}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^2+a)^2,x)
```

```
[Out] -1/4*d^2*exp(-d*x-c)/b/(b*d^2*x^2+a*d^2)-1/8*d/b/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/8*d/b/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/4*d^2*exp(d*x+c)/b/(b*d^2*x^2+a*d^2)-1/8*d/b/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/8*d/b/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.16685, size = 1393, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*cosh(d*x + c) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b))) *cosh(c + sqrt(-a*d^2/b)) - (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b))) *cosh(-c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b))) *sinh(c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cosh(d*x + c)
```

$$\begin{aligned} &^2 - (b*x^2 + a)*\sinh(d*x + c)^2*\sqrt{-a*d^2/b}*Ei(d*x + \sqrt{-a*d^2/b}) + \\ &((b*x^2 + a)*\cosh(d*x + c)^2 - (b*x^2 + a)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b} \\ &*Ei(-d*x - \sqrt{-a*d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))/((a*b^2*x^2 + a^2*b) \\ &*\cosh(d*x + c)^2 - (a*b^2*x^2 + a^2*b)*\sinh(d*x + c)^2) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(dx + c)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x^2+a)^2,x, algorithm="giac")

[Out] integrate(x\*cosh(d\*x + c)/(b\*x^2 + a)^2, x)

$$3.69 \quad \int \frac{\cosh(c+dx)}{(a+bx^2)^2} dx$$

**Optimal.** Leaf size=476

$$\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4ab} - \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4ab} - \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4ab}$$

```
[Out] -Cosh[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]/(4*(-a)^(3/2)*Sqrt[b]) + (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]/(4*(-a)^(3/2)*Sqrt[b])) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*a*b) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*a*b) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]/(4*a*b) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]/(4*(-a)^(3/2)*Sqrt[b])) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]/(4*a*b) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]/(4*(-a)^(3/2)*Sqrt[b]))
```

**Rubi [A]** time = 0.817828, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5281, 3297, 3303, 3298, 3301}

$$\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4ab} - \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4ab} - \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4ab}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*x^2)^2, x]
```

```
[Out] -Cosh[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]/(4*(-a)^(3/2)*Sqrt[b]) + (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]/(4*(-a)^(3/2)*Sqrt[b])) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*a*b) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*a*b) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]/(4*a*b) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]/(4*(-a)^(3/2)*Sqrt[b])) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]/(4*a*b) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]/(4*(-a)^(3/2)*Sqrt[b]))
```

```

ntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]/(4*(-a)^(3/2)*Sqrt[b]) - (d*Cosh[c - (
Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]/(4*a*b) + (S
inh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]/(4*
(-a)^(3/2)*Sqrt[b])

```

### Rule 5281

```

Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

```

### Rule 3297

```

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]

```

### Rule 3303

```

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]

```

### Rule 3298

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

```

### Rule 3301

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{(a+bx^2)^2} dx &= \int \left( -\frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}-bx)^2} - \frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}+bx)^2} - \frac{b \cosh(c+dx)}{2a(-ab-b^2x^2)} \right) dx \\
&= -\frac{b \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^2} dx}{4a} - \frac{b \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}+bx)^2} dx}{4a} - \frac{b \int \frac{\cosh(c+dx)}{-ab-b^2x^2} dx}{2a} \\
&= -\frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}+\sqrt{bx})} - \frac{b \int \left( -\frac{\sqrt{-a} \cosh(c+dx)}{2ab(\sqrt{-a}-\sqrt{bx})} - \frac{\sqrt{-a} \cosh(c+dx)}{2ab(\sqrt{-a}+\sqrt{bx})} \right) dx}{2a} + \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}\sqrt{b}} dx}{4a} \\
&= -\frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}+\sqrt{bx})} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{4(-a)^{3/2}} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{4(-a)^{3/2}} - \frac{\left( d \cosh \left( c - \frac{\sqrt{-a}}{\sqrt{b}} \right) \right)}{4a} \\
&= -\frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}+\sqrt{bx})} - \frac{d \operatorname{Chi} \left( \frac{\sqrt{-ad}}{\sqrt{b}} + dx \right) \sinh \left( c - \frac{\sqrt{-ad}}{\sqrt{b}} \right)}{4ab} - \frac{d \operatorname{Chi} \left( \frac{\sqrt{-ad}}{\sqrt{b}} - dx \right)}{4ab} \\
&= -\frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{4a\sqrt{b}(\sqrt{-a}+\sqrt{bx})} - \frac{\cosh \left( c + \frac{\sqrt{-ad}}{\sqrt{b}} \right) \operatorname{Chi} \left( \frac{\sqrt{-ad}}{\sqrt{b}} - dx \right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\cosh \left( c - \frac{\sqrt{-ad}}{\sqrt{b}} \right)}{4(-a)^{3/2}\sqrt{b}}
\end{aligned}$$

**Mathematica [C]** time = 0.687951, size = 590, normalized size = 1.24

$$-\frac{ia^{3/2}d \cosh \left( c - \frac{i\sqrt{ad}}{\sqrt{b}} \right) \operatorname{Si} \left( \frac{\sqrt{ad}}{\sqrt{b}} - idx \right) + ia^{3/2}d \cosh \left( c + \frac{i\sqrt{ad}}{\sqrt{b}} \right) \operatorname{Si} \left( idx + \frac{\sqrt{ad}}{\sqrt{b}} \right) - b^{3/2}x^2 \sinh \left( c - \frac{i\sqrt{ad}}{\sqrt{b}} \right) \operatorname{Si} \left( \frac{\sqrt{ad}}{\sqrt{b}} - idx \right) - b^{3/2}x^2 \sinh \left( c + \frac{i\sqrt{ad}}{\sqrt{b}} \right) \operatorname{Si} \left( idx + \frac{\sqrt{ad}}{\sqrt{b}} \right)}{4(-a)^{3/2}\sqrt{b}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d\*x]/(a + b\*x^2)^2, x]

[Out] (2\*Sqrt[a]\*b\*x\*Cosh[c + d\*x] - (a + b\*x^2)\*CosIntegral[-((Sqrt[a]\*d)/Sqrt[b]) + I\*d\*x]\*((-I)\*Sqrt[b]\*Cosh[c - (I\*Sqrt[a]\*d)/Sqrt[b]] + Sqrt[a]\*d\*Sinh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]) - (a + b\*x^2)\*CosIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x]\*(I\*Sqrt[b]\*Cosh[c + (I\*Sqrt[a]\*d)/Sqrt[b]] + Sqrt[a]\*d\*Sinh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]) - I\*a^(3/2)\*d\*Cosh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] - I\*Sqrt[a]\*b\*d\*x^2\*Cosh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] - a\*Sqrt[b]\*Sinh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] - b^(3/2)\*x^2\*Sinh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] + I\*a^(3/2)\*d\*Cosh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b]]

+ I\*d\*x] + I\*Sqrt[a]\*b\*d\*x^2\*Cosh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x] - a\*Sqrt[b]\*Sinh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x] - b^(3/2)\*x^2\*Sinh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]\*SinIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x]/(4\*a^(3/2)\*b\*(a + b\*x^2))

**Maple [A]** time = 0.029, size = 503, normalized size = 1.1

$$\frac{d^2 e^{-dx-c} x}{4a(bd^2x^2 + ad^2)} - \frac{d}{8ab} e^{\frac{1}{b}(d\sqrt{-ab}-cb)} \operatorname{Ei}\left(1, \frac{1}{b}\left(d\sqrt{-ab} + (dx+c)b - cb\right)\right) - \frac{d}{8ab} e^{-\frac{1}{b}(d\sqrt{-ab}+cb)} \operatorname{Ei}\left(1, -\frac{1}{b}\left(d\sqrt{-ab} - (dx+c)b + cb\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/(b\*x^2+a)^2,x)

[Out]  $\frac{1}{4}d^2 \exp(-dx-c) x/a / (bd^2x^2 + ad^2) - \frac{1}{8}d/b/a \exp((d(-ab)^{1/2} - cb)/b) \operatorname{Ei}\left(1, \frac{d(-ab)^{1/2} + (dx+c)b - cb}{b}\right) - \frac{1}{8}d/b/a \exp(-\frac{d(-ab)^{1/2} + cb}{b}) \operatorname{Ei}\left(1, -\frac{d(-ab)^{1/2} - (dx+c)b + cb}{b}\right) + \frac{1}{8}a/(-ab)^{1/2} \exp((d(-ab)^{1/2} - cb)/b) \operatorname{Ei}\left(1, \frac{d(-ab)^{1/2} + (dx+c)b - cb}{b}\right) - \frac{1}{8}a/(-ab)^{1/2} \exp(-\frac{d(-ab)^{1/2} + cb}{b}) \operatorname{Ei}\left(1, -\frac{d(-ab)^{1/2} - (dx+c)b + cb}{b}\right) + \frac{1}{4}d^2 \exp(dx+c) x/a / (bd^2x^2 + ad^2) + \frac{1}{8}d/b/a \exp(-\frac{d(-ab)^{1/2} - cb}{b}) \operatorname{Ei}\left(1, -\frac{d(-ab)^{1/2} + (dx+c)b - cb}{b}\right) + \frac{1}{8}d/b/a \exp(\frac{d(-ab)^{1/2} + cb}{b}) \operatorname{Ei}\left(1, \frac{d(-ab)^{1/2} - (dx+c)b + cb}{b}\right) + \frac{1}{8}a/(-ab)^{1/2} \exp(-\frac{d(-ab)^{1/2} - cb}{b}) \operatorname{Ei}\left(1, -\frac{d(-ab)^{1/2} + (dx+c)b - cb}{b}\right) - \frac{1}{8}a/(-ab)^{1/2} \exp(\frac{d(-ab)^{1/2} + cb}{b}) \operatorname{Ei}\left(1, \frac{d(-ab)^{1/2} - (dx+c)b + cb}{b}\right)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.27362, size = 2431, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x^2+a)^2,x, algorithm="fricas")

[Out] 
$$\frac{1}{8} * (4 * a * b * d * x * \cosh(d * x + c) - (((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c))^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c))^2 + ((b^2 * x^2 + a * b) * \cosh(d * x + c))^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c))^2 * \sqrt{-a * d^2 / b}) * \operatorname{Ei}(d * x - \sqrt{-a * d^2 / b}) - ((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c))^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c))^2 - ((b^2 * x^2 + a * b) * \cosh(d * x + c))^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c))^2 * \sqrt{-a * d^2 / b}) * \operatorname{Ei}(-d * x + \sqrt{-a * d^2 / b})) * \cosh(c + \sqrt{-a * d^2 / b}) - (((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c))^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c))^2 - ((b^2 * x^2 + a * b) * \cosh(d * x + c))^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c))^2 * \sqrt{-a * d^2 / b}) * \operatorname{Ei}(d * x + \sqrt{-a * d^2 / b}) - ((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c))^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c))^2 + ((b^2 * x^2 + a * b) * \cosh(d * x + c))^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c))^2 * \sqrt{-a * d^2 / b}) * \operatorname{Ei}(-d * x - \sqrt{-a * d^2 / b})) * \cosh(-c + \sqrt{-a * d^2 / b}) - (((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c))^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c))^2 + ((b^2 * x^2 + a * b) * \cosh(d * x + c))^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c))^2 * \sqrt{-a * d^2 / b}) * \operatorname{Ei}(d * x - \sqrt{-a * d^2 / b}) + ((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c))^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c))^2 - ((b^2 * x^2 + a * b) * \cosh(d * x + c))^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c))^2 * \sqrt{-a * d^2 / b}) * \operatorname{Ei}(-d * x + \sqrt{-a * d^2 / b})) * \sinh(c + \sqrt{-a * d^2 / b}) + (((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c))^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c))^2 - ((b^2 * x^2 + a * b) * \cosh(d * x + c))^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c))^2 * \sqrt{-a * d^2 / b}) * \operatorname{Ei}(d * x + \sqrt{-a * d^2 / b}) + ((a * b * d^2 * x^2 + a^2 * d^2) * \cosh(d * x + c))^2 - (a * b * d^2 * x^2 + a^2 * d^2) * \sinh(d * x + c))^2 + ((b^2 * x^2 + a * b) * \cosh(d * x + c))^2 - (b^2 * x^2 + a * b) * \sinh(d * x + c))^2 * \sqrt{-a * d^2 / b}) * \operatorname{Ei}(-d * x - \sqrt{-a * d^2 / b})) * \sinh(-c + \sqrt{-a * d^2 / b})) / ((a^2 * b^2 * d * x^2 + a^3 * b * d) * \cosh(d * x + c))^2 - (a^2 * b^2 * d * x^2 + a^3 * b * d) * \sinh(d * x + c))^2$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x\*\*2+a)\*\*2,x)



[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x^2+a)^2,x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/(b\*x^2 + a)^2, x)

$$3.70 \quad \int \frac{\cosh(c+dx)}{x(a+bx^2)^2} dx$$

**Optimal.** Leaf size=435

$$\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^2} + \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^2}$$

```
[Out] Cosh[c + d*x]/(2*a*(a + b*x^2)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2)
```

**Rubi [A]** time = 0.839029, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5293, 3303, 3298, 3301, 5289, 5280}

$$\frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^2} + \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^2)^2), x]
```

```
[Out] Cosh[c + d*x]/(2*a*(a + b*x^2)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2)
```

$$[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a] * d) / \text{Sqrt}[b] - d * x] / (2 * a^2) - (d * \text{Cosh}[c - (\text{Sqrt}[-a] * d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a] * d) / \text{Sqrt}[b] + d * x]) / (4 * (-a)^{(3/2)} * \text{Sqrt}[b]) - (\text{Sinh}[c - (\text{Sqrt}[-a] * d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a] * d) / \text{Sqrt}[b] + d * x]) / (2 * a^2)$$

### Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] - Dist[(d*e^m)/(b*n*(p + 1)),
Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p]
&& EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

### Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Int[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx^2)^2} dx &= \int \left( \frac{\cosh(c+dx)}{a^2x} - \frac{bx \cosh(c+dx)}{a(a+bx^2)^2} - \frac{bx \cosh(c+dx)}{a^2(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a^2} - \frac{b \int \frac{x \cosh(c+dx)}{a+bx^2} dx}{a^2} - \frac{b \int \frac{x \cosh(c+dx)}{(a+bx^2)^2} dx}{a} \\
&= \frac{\cosh(c+dx)}{2a(a+bx^2)} - \frac{b \int \left( -\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \right) dx}{a^2} - \frac{d \int \frac{\sinh(c+dx)}{a+bx^2} dx}{2a} + \frac{\cosh(c) \int \frac{\cosh(dx)}{x} dx}{a^2} \\
&= \frac{\cosh(c+dx)}{2a(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} + \frac{\sqrt{b} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2a^2} - \frac{\sqrt{b} \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2a^2} - \frac{d \int \frac{\sinh(c+dx)}{a+bx^2} dx}{2a} \\
&= \frac{\cosh(c+dx)}{2a(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{4(-a)^{3/2}} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{4(-a)^{3/2}} - \frac{(\sqrt{b} \text{Chi}(\frac{\sqrt{-a}-\sqrt{bx}}{\sqrt{b}}) - \sqrt{b} \text{Chi}(\frac{\sqrt{-a}+\sqrt{bx}}{\sqrt{b}}))}{2a^2} \\
&= \frac{\cosh(c+dx)}{2a(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^2} \\
&= \frac{\cosh(c+dx)}{2a(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^2} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^2}
\end{aligned}$$

**Mathematica [C]** time = 4.80435, size = 363, normalized size = 0.83

$$4 \cosh(c)\text{Chi}(dx) + \frac{i \left( \text{Chi}\left(d\left(x + \frac{i\sqrt{a}}{\sqrt{b}}\right)\right) \left( 2i\sqrt{b} \cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) - \sqrt{ad} \sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \right) + \text{Chi}\left(d\left(x - \frac{i\sqrt{a}}{\sqrt{b}}\right)\right) \left( \sqrt{ad} \sinh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) + 2i\sqrt{b} \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \right) - \text{Shi}\left(d\left(x + \frac{i\sqrt{a}}{\sqrt{b}}\right)\right) \left( \sqrt{ad} \sinh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) + 2i\sqrt{b} \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \right) - \text{Shi}\left(d\left(x - \frac{i\sqrt{a}}{\sqrt{b}}\right)\right) \left( 2i\sqrt{b} \cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) - \sqrt{ad} \sinh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \right)}{4a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x\*(a + b\*x^2)^2), x]

[Out] (4\*Cosh[c]\*CoshIntegral[d\*x] + (I\*(((2\*I)\*a\*Sqrt[b]\*Cosh[c + d\*x])/(a + b\*x^2) + CoshIntegral[d\*((I\*Sqrt[a])/Sqrt[b] + x)]\*(((2\*I)\*Sqrt[b]\*Cosh[c - (I\*Sqrt[a]\*d)/Sqrt[b]] - Sqrt[a]\*d\*Sinh[c - (I\*Sqrt[a]\*d)/Sqrt[b]]) + CoshIntegral[d\*(((I)\*Sqrt[a])/Sqrt[b] + x)]\*(((2\*I)\*Sqrt[b]\*Cosh[c + (I\*Sqrt[a]\*d)

/Sqrt[b]] + Sqrt[a]\*d\*Sinh[c + (I\*Sqrt[a]\*d)/Sqrt[b]]) - (4\*I)\*Sqrt[b]\*Sinh[c]\*SinhIntegral[d\*x] - (Sqrt[a]\*d\*Cosh[c - (I\*Sqrt[a]\*d)/Sqrt[b]] - (2\*I)\*Sqrt[b]\*Sinh[c - (I\*Sqrt[a]\*d)/Sqrt[b]])\*SinhIntegral[d\*(I\*Sqrt[a])/Sqrt[b] + x] - (Sqrt[a]\*d\*Cosh[c + (I\*Sqrt[a]\*d)/Sqrt[b]] + (2\*I)\*Sqrt[b]\*Sinh[c + (I\*Sqrt[a]\*d)/Sqrt[b]])\*SinhIntegral[(I\*Sqrt[a]\*d)/Sqrt[b] - d\*x])/Sqrt[b])/(4\*a^2)

**Maple [A]** time = 0.074, size = 546, normalized size = 1.3

$$\frac{e^{-dx-c}d^2}{4a((dx+c)^2b-2(dx+c)bc+ad^2+bc^2)} - \frac{d}{8a}e^{-\frac{1}{b}(d\sqrt{-ab}+cb)}\text{Ei}\left(1, -\frac{1}{b}\left(d\sqrt{-ab} - (dx+c)b + cb\right)\right) \frac{1}{\sqrt{-ab}} + \frac{d}{8a}e^{\frac{1}{b}(d\sqrt{-ab}+cb)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/x/(b\*x^2+a)^2,x)

[Out] 1/4\*exp(-d\*x-c)\*d^2/a/((d\*x+c)^2\*b-2\*(d\*x+c)\*b\*c+a\*d^2+b\*c^2)-1/8/a/(-a\*b)^(1/2)\*exp(-(d\*(-a\*b)^(1/2)+c\*b)/b)\*Ei(1, -(d\*(-a\*b)^(1/2)-(d\*x+c)\*b+c\*b)/b)\*d+1/8/a/(-a\*b)^(1/2)\*exp((d\*(-a\*b)^(1/2)-c\*b)/b)\*Ei(1, (d\*(-a\*b)^(1/2)+(d\*x+c)\*b-c\*b)/b)\*d+1/4/a^2\*exp(-(d\*(-a\*b)^(1/2)+c\*b)/b)\*Ei(1, -(d\*(-a\*b)^(1/2)-(d\*x+c)\*b+c\*b)/b)+1/4/a^2\*exp((d\*(-a\*b)^(1/2)-c\*b)/b)\*Ei(1, (d\*(-a\*b)^(1/2)+(d\*x+c)\*b-c\*b)/b)-1/2/a^2\*exp(-c)\*Ei(1, d\*x)+1/4\*exp(d\*x+c)\*d^2/a/((d\*x+c)^2\*b-2\*(d\*x+c)\*b\*c+a\*d^2+b\*c^2)+1/8/a/(-a\*b)^(1/2)\*exp((d\*(-a\*b)^(1/2)+c\*b)/b)\*Ei(1, (d\*(-a\*b)^(1/2)-(d\*x+c)\*b+c\*b)/b)\*d-1/8/a/(-a\*b)^(1/2)\*exp(-(d\*(-a\*b)^(1/2)-c\*b)/b)\*Ei(1, -(d\*(-a\*b)^(1/2)+(d\*x+c)\*b-c\*b)/b)\*d+1/4/a^2\*exp((d\*(-a\*b)^(1/2)+c\*b)/b)\*Ei(1, (d\*(-a\*b)^(1/2)-(d\*x+c)\*b+c\*b)/b)+1/4/a^2\*exp(-(d\*(-a\*b)^(1/2)-c\*b)/b)\*Ei(1, -(d\*(-a\*b)^(1/2)+(d\*x+c)\*b-c\*b)/b)-1/2/a^2\*exp(c)\*Ei(1, -d\*x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.29714, size = 2253, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x^2+a)^2,x, algorithm="fricas")

[Out] 
$$\frac{1}{8} \cdot (4 \cdot a \cdot \cosh(dx + c) - ((2 \cdot (bx^2 + a) \cdot \cosh(dx + c)^2 - 2 \cdot (bx^2 + a) \cdot \sinh(dx + c)^2 - ((bx^2 + a) \cdot \cosh(dx + c)^2 - (bx^2 + a) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2/b}) \cdot \text{Ei}(dx - \sqrt{-a \cdot d^2/b}) + (2 \cdot (bx^2 + a) \cdot \cosh(dx + c)^2 - 2 \cdot (bx^2 + a) \cdot \sinh(dx + c)^2 + ((bx^2 + a) \cdot \cosh(dx + c)^2 - (bx^2 + a) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2/b}) \cdot \text{Ei}(-dx + \sqrt{-a \cdot d^2/b})) \cdot \cosh(c + \sqrt{-a \cdot d^2/b}) + 4 \cdot ((bx^2 + a) \cdot \text{Ei}(dx) + (bx^2 + a) \cdot \text{Ei}(-dx)) \cdot \cosh(c) - ((2 \cdot (bx^2 + a) \cdot \cosh(dx + c)^2 - 2 \cdot (bx^2 + a) \cdot \sinh(dx + c)^2 + ((bx^2 + a) \cdot \cosh(dx + c)^2 - (bx^2 + a) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2/b}) \cdot \text{Ei}(dx + \sqrt{-a \cdot d^2/b}) + (2 \cdot (bx^2 + a) \cdot \cosh(dx + c)^2 - 2 \cdot (bx^2 + a) \cdot \sinh(dx + c)^2 - ((bx^2 + a) \cdot \cosh(dx + c)^2 - (bx^2 + a) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2/b}) \cdot \text{Ei}(-dx - \sqrt{-a \cdot d^2/b})) \cdot \cosh(-c + \sqrt{-a \cdot d^2/b}) - ((2 \cdot (bx^2 + a) \cdot \cosh(dx + c)^2 - 2 \cdot (bx^2 + a) \cdot \sinh(dx + c)^2 - ((bx^2 + a) \cdot \cosh(dx + c)^2 - (bx^2 + a) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2/b}) \cdot \text{Ei}(dx - \sqrt{-a \cdot d^2/b})) - (2 \cdot (bx^2 + a) \cdot \cosh(dx + c)^2 - 2 \cdot (bx^2 + a) \cdot \sinh(dx + c)^2 + ((bx^2 + a) \cdot \cosh(dx + c)^2 - (bx^2 + a) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2/b}) \cdot \text{Ei}(-dx + \sqrt{-a \cdot d^2/b})) \cdot \sinh(c + \sqrt{-a \cdot d^2/b}) + 4 \cdot ((bx^2 + a) \cdot \text{Ei}(dx) - (bx^2 + a) \cdot \text{Ei}(-dx)) \cdot \sinh(c) + ((2 \cdot (bx^2 + a) \cdot \cosh(dx + c)^2 - 2 \cdot (bx^2 + a) \cdot \sinh(dx + c)^2 + ((bx^2 + a) \cdot \cosh(dx + c)^2 - (bx^2 + a) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2/b}) \cdot \text{Ei}(dx + \sqrt{-a \cdot d^2/b}) - (2 \cdot (bx^2 + a) \cdot \cosh(dx + c)^2 - 2 \cdot (bx^2 + a) \cdot \sinh(dx + c)^2 - ((bx^2 + a) \cdot \cosh(dx + c)^2 - (bx^2 + a) \cdot \sinh(dx + c)^2) \cdot \sqrt{-a \cdot d^2/b}) \cdot \text{Ei}(-dx - \sqrt{-a \cdot d^2/b})) \cdot \sinh(-c + \sqrt{-a \cdot d^2/b})) / ((a^2 \cdot bx^2 + a^3) \cdot \cosh(dx + c)^2 - (a^2 \cdot bx^2 + a^3) \cdot \sinh(dx + c)^2)$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x^2+a)^2,x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((b\*x^2 + a)^2\*x), x)

$$3.71 \quad \int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2} dx$$

**Optimal.** Leaf size=500

$$\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4a^2} + \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4a^2} - \frac{d \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4a^2} + \frac{d \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4a^2}$$

[Out]  $-(\text{Cosh}[c + d*x]/(a^2*x)) + (\text{Sqrt}[b]*\text{Cosh}[c + d*x])/(4*a^2*(\text{Sqrt}[-a] - \text{Sqrt}[b]*x)) - (\text{Sqrt}[b]*\text{Cosh}[c + d*x])/(4*a^2*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)) - (3*\text{Sqrt}[b]*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*(-a)^{(5/2)}) + (3*\text{Sqrt}[b]*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*(-a)^{(5/2)}) + (d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a^2 + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*a^2) + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*a^2) + (d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a^2 - (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*a^2) + (3*\text{Sqrt}[b]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*(-a)^{(5/2)}) + (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*a^2) + (3*\text{Sqrt}[b]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*(-a)^{(5/2)})$

**Rubi [A]** time = 1.25317, antiderivative size = 500, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4a^2} + \frac{d \sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4a^2} - \frac{d \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4a^2} + \frac{d \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4a^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cosh}[c + d*x]/(x^2*(a + b*x^2)^2), x]$

[Out]  $-(\text{Cosh}[c + d*x]/(a^2*x)) + (\text{Sqrt}[b]*\text{Cosh}[c + d*x])/(4*a^2*(\text{Sqrt}[-a] - \text{Sqrt}[b]*x)) - (\text{Sqrt}[b]*\text{Cosh}[c + d*x])/(4*a^2*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)) - (3*\text{Sqrt}[b]*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*(-a)^{(5/2)}) + (3*\text{Sqrt}[b]*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*(-a)^{(5/2)}) + (d*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/a^2 + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*a^2) + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(4*a^2) + (d*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/a^2 - (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*a^2) + (3*\text{Sqrt}[b]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(4*(-a)^{(5/2)}) + (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*a^2) + (3*\text{Sqrt}[b]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(4*(-a)^{(5/2)})$



$$\begin{aligned} & a] * d) / \text{Sqrt}[b]] / (4 * a^2) + (d * \text{Cosh}[c] * \text{SinhIntegral}[d * x]) / a^2 - (d * \text{Cosh}[c + (\text{Sqrt}[-a] * d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a] * d) / \text{Sqrt}[b] - d * x]) / (4 * a^2) + (3 * \text{Sqrt}[b] * \text{Sinh}[c + (\text{Sqrt}[-a] * d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a] * d) / \text{Sqrt}[b] - d * x]) / (4 * (-a)^{(5/2)}) + (d * \text{Cosh}[c - (\text{Sqrt}[-a] * d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a] * d) / \text{Sqrt}[b] + d * x]) / (4 * a^2) + (3 * \text{Sqrt}[b] * \text{Sinh}[c - (\text{Sqrt}[-a] * d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a] * d) / \text{Sqrt}[b] + d * x]) / (4 * (-a)^{(5/2)}) \end{aligned}$$

### Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*Sin[e + f*x]) / (d*(m + 1)), x] - Dist[f / (d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)] / ((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f) / d], Int[Sin[(c*f) / d + f*x] / (c + d*x), x], x] + Dist[Sin[(d*e - c*f) / d], Int[Cos[(c*f) / d + f*x] / (c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)] / ((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I * SinhIntegral[(c*f*fz) / d + f*fz*x]) / d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)] / ((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz) / d + f*fz*x] / d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
```

}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

### Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2} dx &= \int \left( \frac{\cosh(c+dx)}{a^2 x^2} - \frac{b \cosh(c+dx)}{a(a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^2(a+bx^2)} \right) dx \\
 &= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{a+bx^2} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx^2)^2} dx}{a} \\
 &= -\frac{\cosh(c+dx)}{a^2 x} - \frac{b \int \left( \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx}{a^2} - \frac{b \int \left( -\frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}-bx)^2} - \frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}+bx)^2} - \frac{b}{2a} \right) dx}{a} \\
 &= -\frac{\cosh(c+dx)}{a^2 x} + \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2(-a)^{5/2}} + \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2(-a)^{5/2}} + \frac{b^2 \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^2} dx}{4a^2} + \frac{b^2 \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}+bx)^2} dx}{4a^2} \\
 &= -\frac{\cosh(c+dx)}{a^2 x} + \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{bx})} - \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{bx})} + \frac{d \operatorname{Chi}(dx) \sinh(c)}{a^2} + \frac{d \cosh(c) \operatorname{Shi}(dx)}{a^2} \\
 &= -\frac{\cosh(c+dx)}{a^2 x} + \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{bx})} - \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{bx})} - \frac{\sqrt{b} \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2(-a)^{5/2}} + \\
 &= -\frac{\cosh(c+dx)}{a^2 x} + \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{bx})} - \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{bx})} - \frac{\sqrt{b} \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2(-a)^{5/2}} + \\
 &= -\frac{\cosh(c+dx)}{a^2 x} + \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}-\sqrt{bx})} - \frac{\sqrt{b} \cosh(c+dx)}{4a^2(\sqrt{-a}+\sqrt{bx})} - \frac{3\sqrt{b} \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{4(-a)^{5/2}}
 \end{aligned}$$

**Mathematica [C]** time = 1.03631, size = 675, normalized size = 1.35

$$\frac{ia^{3/2} dx \cosh\left(c - \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} - idx\right) - ia^{3/2} dx \cosh\left(c + \frac{i\sqrt{ad}}{\sqrt{b}}\right) \operatorname{Si}\left(ixd + \frac{\sqrt{ad}}{\sqrt{b}}\right) + 4a^{3/2} dx \sinh(c) \operatorname{Chi}(dx) + 4a^{3/2} dx \cosh(c)}{x^2(a+bx^2)^2}, x]$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d\*x]/(x^2\*(a + b\*x^2)^2), x]

```
[Out] (-4*a^(3/2)*Cosh[c + d*x] - 6*Sqrt[a]*b*x^2*Cosh[c + d*x] + 4*a^(3/2)*d*x*C
oshIntegral[d*x]*Sinh[c] + 4*Sqrt[a]*b*d*x^3*CoshIntegral[d*x]*Sinh[c] + x*
(a + b*x^2)*CosIntegral[-((Sqrt[a]*d)/Sqrt[b]) + I*d*x]*((-3*I)*Sqrt[b]*Cos
h[c - (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]) +
x*(a + b*x^2)*CosIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x]*((3*I)*Sqrt[b]*Cosh
[c + (I*Sqrt[a]*d)/Sqrt[b]] + Sqrt[a]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]) +
4*a^(3/2)*d*x*Cosh[c]*SinhIntegral[d*x] + 4*Sqrt[a]*b*d*x^3*Cosh[c]*SinhInt
egral[d*x] + I*a^(3/2)*d*x*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqr
t[a]*d)/Sqrt[b] - I*d*x] + I*Sqrt[a]*b*d*x^3*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]
]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + 3*a*Sqrt[b]*x*Sinh[c - (I*Sqrt
[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] + 3*b^(3/2)*x^3*Si
nh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] - I*d*x] - I*
a^(3/2)*d*x*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b]
+ I*d*x] - I*Sqrt[a]*b*d*x^3*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinIntegral[(
Sqrt[a]*d)/Sqrt[b] + I*d*x] + 3*a*Sqrt[b]*x*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]
*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x] + 3*b^(3/2)*x^3*Sinh[c + (I*Sqrt[
a]*d)/Sqrt[b]]*SinIntegral[(Sqrt[a]*d)/Sqrt[b] + I*d*x))/(4*a^(5/2)*x*(a +
b*x^2))
```

**Maple [A]** time = 0.086, size = 595, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/x^2/(b*x^2+a)^2,x)
```

```
[Out] -3/4*exp(-d*x-c)/a^2*x*d^2/(b*d^2*x^2+a*d^2)*b-1/2*exp(-d*x-c)/a/x*d^2/(b*d
^2*x^2+a*d^2)+1/8*d/a^2*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d
*x+c)*b-c*b)/b)+1/8*d/a^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2
)-(d*x+c)*b+c*b)/b)-3/8/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(
d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b+3/8/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2
)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b+1/2*d/a^2*exp(-c)*Ei(1,
d*x)-3/4*exp(d*x+c)/a^2*x*d^2/(b*d^2*x^2+a*d^2)*b-1/2*exp(d*x+c)/a/x*d^2/(b
*d^2*x^2+a*d^2)-1/8*d/a^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-
(d*x+c)*b+c*b)/b)-1/8*d/a^2*exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1
/2)+(d*x+c)*b-c*b)/b)+3/8/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1
,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-3/8/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1
/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)*b-1/2*d/a^2*exp(c)*Ei(1
,-d*x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x^2+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.33658, size = 2786, normalized size = 5.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x^2+a)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/8*(4*(3*a*b*d*x^2 + 2*a^2*d)*\cosh(d*x + c) - (((a*b*d^2*x^3 + a^2*d^2*x) \\ & * \cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*\sinh(d*x + c)^2 + 3*((b^2*x^3 \\ & + a*b*x)*\cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b} \\ & ))*Ei(d*x - \sqrt{-a*d^2/b}) - ((a*b*d^2*x^3 + a^2*d^2*x)*\cosh(d*x + c)^2 - \\ & (a*b*d^2*x^3 + a^2*d^2*x)*\sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*\cosh(d*x + \\ & c)^2 - (b^2*x^3 + a*b*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*Ei(-d*x + \sqrt{- \\ & a*d^2/b}))*\cosh(c + \sqrt{-a*d^2/b}) - 4*((a*b*d^2*x^3 + a^2*d^2*x)*Ei(d*x) \\ & - (a*b*d^2*x^3 + a^2*d^2*x)*Ei(-d*x))*\cosh(c) - (((a*b*d^2*x^3 + a^2*d^2*x) \\ & * \cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*\sinh(d*x + c)^2 - 3*((b^2*x^3 \\ & + a*b*x)*\cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b} \\ & ))*Ei(d*x + \sqrt{-a*d^2/b}) - ((a*b*d^2*x^3 + a^2*d^2*x)*\cosh(d*x + c)^2 - \\ & (a*b*d^2*x^3 + a^2*d^2*x)*\sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)*\cosh(d*x + \\ & c)^2 - (b^2*x^3 + a*b*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*Ei(-d*x - \sqrt{- \\ & a*d^2/b}))*\cosh(-c + \sqrt{-a*d^2/b}) - (((a*b*d^2*x^3 + a^2*d^2*x)*\cosh(d*x \\ & + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*\sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x)* \\ & \cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*Ei(d*x \\ & - \sqrt{-a*d^2/b}) + ((a*b*d^2*x^3 + a^2*d^2*x)*\cosh(d*x + c)^2 - (a*b*d^2*x \\ & ^3 + a^2*d^2*x)*\sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*\cosh(d*x + c)^2 - ( \\ & b^2*x^3 + a*b*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*Ei(-d*x + \sqrt{-a*d^2/b}) \\ & )*\sinh(c + \sqrt{-a*d^2/b}) - 4*((a*b*d^2*x^3 + a^2*d^2*x)*Ei(d*x) + (a*b*d^ \\ & 2*x^3 + a^2*d^2*x)*Ei(-d*x))*\sinh(c) + (((a*b*d^2*x^3 + a^2*d^2*x)*\cosh(d*x \\ & + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*\sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)* \\ & \cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*Ei(d*x \\ & + \sqrt{-a*d^2/b}) + ((a*b*d^2*x^3 + a^2*d^2*x)*\cosh(d*x + c)^2 - (a*b*d^2*x \end{aligned}$$

$$\frac{x^3 + a^2 d^2 x \sinh(dx + c)^2 + 3((b^2 x^3 + a b x) \cosh(dx + c)^2 - (b^2 x^3 + a b x) \sinh(dx + c)^2) \sqrt{-a d^2 / b} \operatorname{Ei}(-d x - \sqrt{-a d^2 / b}) \sinh(-c + \sqrt{-a d^2 / b})}{(a^3 b d x^3 + a^4 d x) \cosh(dx + c)^2 - (a^3 b d x^3 + a^4 d x) \sinh(dx + c)^2}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x\*\*2/(b\*x\*\*2+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x^2+a)^2,x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((b\*x^2 + a)^2\*x^2), x)

$$3.72 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=476

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16b^3} + \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16b^3} - \frac{3d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16\sqrt{-ab^{5/2}}} + \frac{3d \sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd - \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16\sqrt{-ab^{5/2}}}$$

[Out]  $-(x^2 \text{Cosh}[c + d*x]) / (4*b*(a + b*x^2)^2) - \text{Cosh}[c + d*x] / (4*b^2*(a + b*x^2)) + (d^2 \text{Cosh}[c + (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{CoshIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] - d*x]) / (16*b^3) + (d^2 \text{Cosh}[c - (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{CoshIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] + d*x]) / (16*b^3) - (3*d \text{CoshIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] + d*x] * \text{Sinh}[c - (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]]) / (16*\text{Sqrt}[-a]*b^{5/2}) + (3*d \text{CoshIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] - d*x] * \text{Sinh}[c + (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]]) / (16*\text{Sqrt}[-a]*b^{5/2}) - (d*x * \text{Sinh}[c + d*x]) / (8*b^2*(a + b*x^2)) - (3*d \text{Cosh}[c + (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] - d*x]) / (16*\text{Sqrt}[-a]*b^{5/2}) - (d^2 \text{Sinh}[c + (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] - d*x]) / (16*b^3) - (3*d \text{Cosh}[c - (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] + d*x]) / (16*\text{Sqrt}[-a]*b^{5/2}) + (d^2 \text{Sinh}[c - (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] + d*x]) / (16*b^3)$

**Rubi [A]** time = 1.06151, antiderivative size = 476, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {5291, 5289, 5280, 3303, 3298, 3301, 5290, 5293}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16b^3} + \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16b^3} - \frac{3d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16\sqrt{-ab^{5/2}}} + \frac{3d \sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd - \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16\sqrt{-ab^{5/2}}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3 \text{Cosh}[c + d*x]) / (a + b*x^2)^3, x]$

[Out]  $-(x^2 \text{Cosh}[c + d*x]) / (4*b*(a + b*x^2)^2) - \text{Cosh}[c + d*x] / (4*b^2*(a + b*x^2)) + (d^2 \text{Cosh}[c + (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{CoshIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] - d*x]) / (16*b^3) + (d^2 \text{Cosh}[c - (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{CoshIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] + d*x]) / (16*b^3) - (3*d \text{CoshIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] + d*x] * \text{Sinh}[c - (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]]) / (16*\text{Sqrt}[-a]*b^{5/2}) + (3*d \text{CoshIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] - d*x] * \text{Sinh}[c + (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]]) / (16*\text{Sqrt}[-a]*b^{5/2}) - (d*x * \text{Sinh}[c + d*x]) / (8*b^2*(a + b*x^2)) - (3*d \text{Cosh}[c + (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] - d*x]) / (16*\text{Sqrt}[-a]*b^{5/2}) - (d^2 \text{Sinh}[c + (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] - d*x]) / (16*b^3) - (3*d \text{Cosh}[c - (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] + d*x]) / (16*\text{Sqrt}[-a]*b^{5/2}) + (d^2 \text{Sinh}[c - (\text{Sqrt}[-a]*d) / \text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d) / \text{Sqrt}[b] + d*x]) / (16*b^3)$

```
] *d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]/(16*Sqrt[-a]*b^(5/2)) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]/(16*b^3) - (3*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]/(16*Sqrt[-a]*b^(5/2)) + (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]/(16*b^3)
```

### Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

### Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

### Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5290

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1)), x]
+ (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]
- Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x])
/; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

### Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rubi steps



$$\begin{aligned}
\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3} dx &= -\frac{x^2 \cosh(c+dx)}{4b(a+bx^2)^2} + \frac{\int \frac{x \cosh(c+dx)}{(a+bx^2)^2} dx}{2b} + \frac{d \int \frac{x^2 \sinh(c+dx)}{(a+bx^2)^2} dx}{4b} \\
&= -\frac{x^2 \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} - \frac{dx \sinh(c+dx)}{8b^2(a+bx^2)} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^2} dx}{8b^2} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^2} dx}{4b^2} + \dots \\
&= -\frac{x^2 \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} - \frac{dx \sinh(c+dx)}{8b^2(a+bx^2)} + \frac{d \int \left( \frac{\sqrt{-a} \sinh(c+dx)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \sinh(c+dx)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx}{8b^2} \\
&= -\frac{x^2 \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} - \frac{dx \sinh(c+dx)}{8b^2(a+bx^2)} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{16\sqrt{-ab^2}} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{16\sqrt{-ab^2}} - \dots \\
&= -\frac{x^2 \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} - \frac{dx \sinh(c+dx)}{8b^2(a+bx^2)} - \frac{\left( d \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \right) \int \frac{\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+dx\right)}{\sqrt{-a}+\sqrt{bx}} dx}{16\sqrt{-ab^2}} \\
&= -\frac{x^2 \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} + \frac{d^2 \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16b^3} + \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16b^3}
\end{aligned}$$

**Mathematica [C]** time = 1.82448, size = 648, normalized size = 1.36

$$\frac{id^2 \sinh(c) \left( \sin\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}}+idx\right) - \sin\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}}+idx\right) + \cos\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \left( \text{Si}\left(ixd+\frac{\sqrt{ad}}{\sqrt{b}}\right) - \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}}-idx\right) \right) \right)}{b} + \frac{d^2 \cosh(c) \left( \cos\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}}+idx\right) - \cos\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}}+idx\right) \right)}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3\*Cosh[c + d\*x])/(a + b\*x^2)^3,x]

[Out] ((-2\*Cosh[d\*x]\*(2\*(a + 2\*b\*x^2)\*Cosh[c] + d\*x\*(a + b\*x^2)\*Sinh[c]))/(a + b\*x^2)^2 - (2\*(d\*x\*(a + b\*x^2)\*Cosh[c] + 2\*(a + 2\*b\*x^2)\*Sinh[c])\*Sinh[d\*x])/(a + b\*x^2)^2 + ((3\*I)\*d\*Sinh[c]\*(Cos[(Sqrt[a]\*d)/Sqrt[b]]\*CosIntegral[-((Sqrt[a]\*d)/Sqrt[b]) + I\*d\*x] - Cos[(Sqrt[a]\*d)/Sqrt[b]]\*CosIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x] + Sin[(Sqrt[a]\*d)/Sqrt[b]]\*(SinIntegral[(Sqrt[a]\*d)/Sqrt[b] - I\*d\*x] - SinIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x]))/(Sqrt[a]\*Sqrt[b]) - (I\*d^2\*Sinh[c]\*(CosIntegral[-((Sqrt[a]\*d)/Sqrt[b]) + I\*d\*x]\*Sin[(Sqrt[a]\*d)/Sqrt[b]] - CosIntegral[(Sqrt[a]\*d)/Sqrt[b] + I\*d\*x]\*Sin[(Sqrt[a]\*d)/Sqrt[b]]))

$$\begin{aligned} & \text{Sqrt}[b]] + \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*(-\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d \\ & *x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]))/b + (3*d*\text{Cosh}[c]*(\text{CosInte} \\ & \text{gral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{CosIntegral} \\ & [(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] - \text{Cos}[(\text{Sqrt}[a]*d)/\text{S} \\ & \text{qrt}[b]]*(\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/ \\ & \text{Sqrt}[b] + I*d*x])))/(\text{Sqrt}[a]*\text{Sqrt}[b]) + (d^2*\text{Cosh}[c]*(\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[ \\ & b]]*\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x] + \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]* \\ & \text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] + \text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*(\text{SinInt} \\ & \text{egral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d* \\ & x])))/b)/(16*b^2) \end{aligned}$$

**Maple [B]** time = 0.376, size = 820, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*cosh(d\*x+c)/(b\*x^2+a)^3,x)

[Out] 
$$\begin{aligned} & -1/32*d^2/b^3*\exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}-(d*x+c)*b+ \\ & c*b)/b)-1/32*d^2/b^3*\exp((d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}+(d*x+ \\ & c)*b-c*b)/b)+3/32*d/b^2/(-a*b)^{(1/2)}*\exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, -(d* \\ & (-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)-3/32*d/b^2/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}- \\ & c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)-1/4*d^4*\exp(-d*x-c)/b/(b^2*d \\ & ^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2-1/8*d^4*\exp(-d*x-c)*a/b^2/(b^2*d^4*x^4+2* \\ & a*b*d^4*x^2+a^2*d^4)+1/16*d^5*\exp(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2* \\ & d^4)*x^3+1/16*d^5*\exp(-d*x-c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1 \\ & /16*d^5*\exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*\exp(d \\ & *x+c)*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/4*d^4*\exp(d*x+c)/b/(b^2 \\ & *d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2-1/8*d^4*\exp(d*x+c)*a/b^2/(b^2*d^4*x^4+2 \\ & *a*b*d^4*x^2+a^2*d^4)-1/32*d^2/b^3*\exp(-(d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, -(d*(- \\ & a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)-1/32*d^2/b^3*\exp((d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, \\ & (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)+3/32*d/b^2/(-a*b)^{(1/2)}*\exp(-(d*(-a*b)^{(1/2)} \\ & /2)-c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)-3/32*d/b^2/(-a*b)^{(1/2)} \\ & *\exp((d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.



$$\frac{x^4 + 2ab^2x^2 + a^2b \sinh(dx + c)^2 \sqrt{-ad^2/b} \operatorname{Ei}(-dx - \sqrt{-ad^2/b}) \sinh(-c + \sqrt{-ad^2/b})}{(ab^5x^4 + 2a^2b^4x^2 + a^3b^3) \cosh(dx + c)^2 - (ab^5x^4 + 2a^2b^4x^2 + a^3b^3) \sinh(dx + c)^2}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*cosh(d\*x+c)/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(dx + c)}{(bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x^2+a)^3,x, algorithm="giac")

[Out] integrate(x^3\*cosh(d\*x + c)/(b\*x^2 + a)^3, x)

$$3.73 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=746

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16\sqrt{-ab}^{5/2}} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16\sqrt{-ab}^{5/2}} - \frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16ab^2} - \dots$$

[Out]  $-\text{Cosh}[c + d*x]/(16*a*b^{(3/2)}*(\text{Sqrt}[-a] - \text{Sqrt}[b]*x)) + \text{Cosh}[c + d*x]/(16*a*b^{(3/2)}*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)) - (x*\text{Cosh}[c + d*x])/(4*b*(a + b*x^2)^2) - (\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*(-a)^{(3/2)}*b^{(3/2)}) + (d^2*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*\text{Sqrt}[-a]*b^{(5/2)}) + (\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*(-a)^{(3/2)}*b^{(3/2)}) - (d^2*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*\text{Sqrt}[-a]*b^{(5/2)}) - (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*a*b^2) - (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*a*b^2) - (d*\text{Sinh}[c + d*x])/(8*b^2*(a + b*x^2)) + (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*a*b^2) + (\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*(-a)^{(3/2)}*b^{(3/2)}) - (d^2*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*\text{Sqrt}[-a]*b^{(5/2)}) - (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*a*b^2) + (\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*(-a)^{(3/2)}*b^{(3/2)}) - (d^2*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*\text{Sqrt}[-a]*b^{(5/2)})$

**Rubi [A]** time = 1.10007, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {5291, 5281, 3297, 3303, 3298, 3301, 5288}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16\sqrt{-ab}^{5/2}} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16\sqrt{-ab}^{5/2}} - \frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16ab^2} - \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*\text{Cosh}[c + d*x])/(a + b*x^2)^3, x]$

[Out]  $-\text{Cosh}[c + d*x]/(16*a*b^{(3/2)}*(\text{Sqrt}[-a] - \text{Sqrt}[b]*x)) + \text{Cosh}[c + d*x]/(16*a*b^{(3/2)}*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)) - (x*\text{Cosh}[c + d*x])/(4*b*(a + b*x^2)^2) - (\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*(-a)^{(3/2)}*b^{(3/2)}) + (d^2*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*\text{Sqrt}[-a]*b^{(5/2)}) + (\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*(-a)^{(3/2)}*b^{(3/2)}) - (d^2*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*\text{Sqrt}[-a]*b^{(5/2)}) - (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*a*b^2) - (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*a*b^2) - (d*\text{Sinh}[c + d*x])/(8*b^2*(a + b*x^2)) + (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*a*b^2) + (\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*(-a)^{(3/2)}*b^{(3/2)}) - (d^2*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x])/(16*\text{Sqrt}[-a]*b^{(5/2)}) - (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*a*b^2) + (\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*(-a)^{(3/2)}*b^{(3/2)}) - (d^2*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*\text{Sqrt}[-a]*b^{(5/2)})$

$$\begin{aligned} & \text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*(-a)^{(3/2)}*b^{(3/2)}) + (d^2*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*\text{Sqrt}[-a]*b^{(5/2)}) + (\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*(-a)^{(3/2)}*b^{(3/2)}) - (d^2*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*\text{Sqrt}[-a]*b^{(5/2)}) - (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]/(16*a*b^2) - (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]/(16*a*b^2) - (d*\text{Sinh}[c + d*x]/(8*b^2*(a + b*x^2)) + (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*a*b^2) + (\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*(-a)^{(3/2)}*b^{(3/2)}) - (d^2*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*\text{Sqrt}[-a]*b^{(5/2)}) - (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*a*b^2) + (\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*(-a)^{(3/2)}*b^{(3/2)}) - (d^2*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*\text{Sqrt}[-a]*b^{(5/2)})) \end{aligned}$$

### Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

### Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 5288

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*Sinh[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[(e^m\*(a + b\*x^n)^(p + 1)\*Sinh[c + d\*x]/(b\*n\*(p + 1)), x] - Dist[(d\*e^m)/(b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*Cosh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3} dx &= -\frac{x \cosh(c+dx)}{4b(a+bx^2)^2} + \frac{\int \frac{\cosh(c+dx)}{(a+bx^2)^2} dx}{4b} + \frac{d \int \frac{x \sinh(c+dx)}{(a+bx^2)^2} dx}{4b} \\
&= -\frac{x \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d \sinh(c+dx)}{8b^2(a+bx^2)} + \frac{\int \left( -\frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}-bx)^2} - \frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}+bx)^2} - \frac{b \cosh(c+dx)}{2a(-ab-b^2x^2)} \right) dx}{4b} + \frac{d^2 \int}{4b} \\
&= -\frac{x \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d \sinh(c+dx)}{8b^2(a+bx^2)} - \frac{\int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^2} dx}{16a} - \frac{\int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}+bx)^2} dx}{16a} - \frac{\int \frac{\cosh(c+dx)}{-ab-b^2x^2} dx}{8a} + \frac{d^2 \int}{8a} \\
&= -\frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}+\sqrt{bx})} - \frac{x \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d \sinh(c+dx)}{8b^2(a+bx^2)} - \frac{\int \left( -\frac{\sqrt{-a} \cosh(c+dx)}{2ab(\sqrt{-a}-\sqrt{bx})} \right) dx}{16(-a)^{3/2}} \\
&= -\frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}+\sqrt{bx})} - \frac{x \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{d \sinh(c+dx)}{8b^2(a+bx^2)} + \frac{\int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{16(-a)^{3/2}} \\
&= -\frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}+\sqrt{bx})} - \frac{x \cosh(c+dx)}{4b(a+bx^2)^2} + \frac{d^2 \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} \\
&= -\frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{16ab^{3/2}(\sqrt{-a}+\sqrt{bx})} - \frac{x \cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 2.50882, size = 932, normalized size = 1.25

$$\frac{2\sqrt{ab^2} \cosh(c) \cosh(dx)x^3}{(bx^2+a)^2} + \frac{2\sqrt{ab^2} \sinh(c) \sinh(dx)x^3}{(bx^2+a)^2} - \frac{2a^{3/2}bd \cosh(dx) \sinh(c)x^2}{(bx^2+a)^2} - \frac{2a^{3/2}bd \cosh(c) \sinh(dx)x^2}{(bx^2+a)^2} - \frac{2a^{3/2}b \cosh(c) \cosh(dx)x}{(bx^2+a)^2} - \frac{2a^{3/2}b \sinh(c) \sinh(dx)x}{(bx^2+a)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*Cosh[c + d\*x])/(a + b\*x^2)^3,x]

[Out] ((-2\*a^(3/2)\*b\*x\*Cosh[c]\*Cosh[d\*x])/(a + b\*x^2)^2 + (2\*Sqrt[a]\*b^2\*x^3\*Cosh[c]\*Cosh[d\*x])/(a + b\*x^2)^2 - (2\*a^(5/2)\*d\*Cosh[d\*x]\*Sinh[c])/(a + b\*x^2)^2 - (2\*a^(3/2)\*b\*d\*x^2\*Cosh[d\*x]\*Sinh[c])/(a + b\*x^2)^2 + (I\*CosIntegral[-(



$$\begin{aligned}
& (\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x]*((b + a*d^2)*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]] \\
& + I*\text{Sqrt}[a]*\text{Sqrt}[b]*d*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]))/\text{Sqrt}[b] - (I*\text{CosInt} \\
& \text{egral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]*((b + a*d^2)*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt} \\
& [b]] - I*\text{Sqrt}[a]*\text{Sqrt}[b]*d*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/\text{Sqrt}[b]]))/\text{Sqrt}[b] - (2*a \\
& ^{(5/2)}*d*\text{Cosh}[c]*\text{Sinh}[d*x])/(a + b*x^2)^2 - (2*a^{(3/2)}*b*d*x^2*\text{Cosh}[c]*\text{Sinh} \\
& [d*x])/(a + b*x^2)^2 - (2*a^{(3/2)}*b*x*\text{Sinh}[c]*\text{Sinh}[d*x])/(a + b*x^2)^2 + (2 \\
& *\text{Sqrt}[a]*b^2*x^3*\text{Sinh}[c]*\text{Sinh}[d*x])/(a + b*x^2)^2 - I*\text{Sqrt}[a]*d*\text{Cos}[(\text{Sqrt}[a] \\
& ]*d)/\text{Sqrt}[b]]*\text{Cosh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + I*\text{Sqrt}[b]* \\
& \text{Cosh}[c]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + \\
& (I*a*d^2*\text{Cosh}[c]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] \\
& - I*d*x])/ \text{Sqrt}[b] - \text{Sqrt}[b]*\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Sinh}[c]*\text{SinIntegral}[(\text{S} \\
& \text{qrt}[a]*d)/\text{Sqrt}[b] - I*d*x] - (a*d^2*\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Sinh}[c]*\text{SinInt} \\
& \text{egral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x])/ \text{Sqrt}[b] - \text{Sqrt}[a]*d*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqr} \\
& \text{t}[b]]*\text{Sinh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + I*\text{Sqrt}[a]*d*\text{Cos}[(\text{S} \\
& \text{qrt}[a]*d)/\text{Sqrt}[b]]*\text{Cosh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d \\
& *x] - (I*a*d^2*\text{Cosh}[c]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqr} \\
& \text{t}[b] + I*d*x])/ \text{Sqrt}[b] - \text{Sqrt}[b]*\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Sinh}[c]*\text{SinIntegr} \\
& \text{al}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] - (a*d^2*\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{Sinh}[c]*\text{S} \\
& \text{inIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])/ \text{Sqrt}[b] - \text{Sqrt}[a]*d*\text{Sin}[(\text{Sqrt}[a]*d \\
& )/\text{Sqrt}[b]]*\text{Sinh}[c]*\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]/(16*a^{(3/2)}*b^ \\
& 2)
\end{aligned}$$


---

**Maple [A]** time = 0.108, size = 1064, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2*\cosh(dx+c)/(b*x^2+a)^3,x)$

[Out]  $\begin{aligned}
& 1/32/b/a/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1,(d*(-a*b)^{(1/2)}+(d*x \\
& +c)*b-c*b)/b)-1/32/b/a/(-a*b)^{(1/2)}*\exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1,-(d*( \\
& -a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)-1/32*d/b^2/a*\exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei} \\
& (1,-(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)+1/32*d^2/b^2/(-a*b)^{(1/2)}*\exp((d*(-a*b) \\
& )^{(1/2)}-c*b)/b)*\text{Ei}(1,(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)-1/32*d^2/b^2/(-a*b)^{(1/2)} \\
& *\exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1,-(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)- \\
& 1/32*d/b^2/a*\exp((d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1,(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b \\
& )/b)+1/16*d^5*\exp(-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2+1/16*d^ \\
& 4*\exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3+1/16*d^5*\exp(-d*x-c \\
& )*a/b^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-1/16*d^4*\exp(-d*x-c)/b/(b^2*d^4 \\
& *x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/32/b/a/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}+c*b
\end{aligned}$

$$\begin{aligned} & )/b) * \text{Ei}(1, (d * (-a * b)^{(1/2)} - (d * x + c) * b + c * b) / b) + 1/32 * b / a / (-a * b)^{(1/2)} * \exp(- (d * (-a * b)^{(1/2)} - c * b) / b) * \text{Ei}(1, - (d * (-a * b)^{(1/2)} + (d * x + c) * b - c * b) / b) - 1/32 * d^2 / b^2 / (-a * b)^{(1/2)} * \exp((d * (-a * b)^{(1/2)} + c * b) / b) * \text{Ei}(1, (d * (-a * b)^{(1/2)} - (d * x + c) * b + c * b) / b) + 1/32 * d^2 / b^2 / (-a * b)^{(1/2)} * \exp(- (d * (-a * b)^{(1/2)} - c * b) / b) * \text{Ei}(1, - (d * (-a * b)^{(1/2)} + (d * x + c) * b - c * b) / b) + 1/32 * d / b^2 / a * \exp((d * (-a * b)^{(1/2)} + c * b) / b) * \text{Ei}(1, (d * (-a * b)^{(1/2)} - (d * x + c) * b + c * b) / b) + 1/32 * d / b^2 / a * \exp(- (d * (-a * b)^{(1/2)} - c * b) / b) * \text{Ei}(1, - (d * (-a * b)^{(1/2)} + (d * x + c) * b - c * b) / b) - 1/16 * d^5 * \exp(d * x + c) / b / (b^2 * d^4 * x^4 + 2 * a * b * d^4 * x^2 + a^2 * d^4) * x^2 + 1/16 * d^4 * \exp(d * x + c) / a / (b^2 * d^4 * x^4 + 2 * a * b * d^4 * x^2 + a^2 * d^4) * x^3 - 1/16 * d^5 * \exp(d * x + c) * a / b^2 / (b^2 * d^4 * x^4 + 2 * a * b * d^4 * x^2 + a^2 * d^4) - 1/16 * d^4 * \exp(d * x + c) / b / (b^2 * d^4 * x^4 + 2 * a * b * d^4 * x^2 + a^2 * d^4) * x \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x^2+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.39734, size = 4169, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x^2+a)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/32 * (4 * (a * b^2 * d * x^3 - a^2 * b * d * x) * \cosh(d * x + c) - (((a * b^2 * d^2 * x^4 + 2 * a^2 * b * d^2 * x^2 + a^3 * d^2) * \cosh(d * x + c))^2 - (a * b^2 * d^2 * x^4 + 2 * a^2 * b * d^2 * x^2 + a^3 * d^2) * \sinh(d * x + c))^2 + ((a^3 * d^2 + (a * b^2 * d^2 + b^3) * x^4 + a^2 * b + 2 * (a^2 * b * d^2 + a * b^2) * x^2) * \cosh(d * x + c))^2 - (a^3 * d^2 + (a * b^2 * d^2 + b^3) * x^4 + a^2 * b + 2 * (a^2 * b * d^2 + a * b^2) * x^2) * \sinh(d * x + c))^2) * \sqrt{-a * d^2 / b}) * \text{Ei}(d * x - \sqrt{-a * d^2 / b}) - ((a * b^2 * d^2 * x^4 + 2 * a^2 * b * d^2 * x^2 + a^3 * d^2) * \cosh(d * x + c))^2 - (a * b^2 * d^2 * x^4 + 2 * a^2 * b * d^2 * x^2 + a^3 * d^2) * \sinh(d * x + c))^2 - ((a^3 * d^2 + (a * b^2 * d^2 + b^3) * x^4 + a^2 * b + 2 * (a^2 * b * d^2 + a * b^2) * x^2) * \cosh(d * x + c))^2 - (a^3 * d^2 + (a * b^2 * d^2 + b^3) * x^4 + a^2 * b + 2 * (a^2 * b * d^2 + a * b^2) * x^2) * \sinh(d * x + c))^2) * \sqrt{-a * d^2 / b}) * \text{Ei}(-d * x + \sqrt{-a * d^2 / b})) * \cosh(c + \sqrt{-a * d^2 / b}) - (((a * b^2 * d^2 * x^4 + 2 * a^2 * b * d^2 * x^2 + a^3 * d^2) * \cosh(d * x + c) \end{aligned}$$

$$\begin{aligned}
&^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2 - ((a^3 d^2 + (a^2 b^2 d^2 + b^3) x^4 + a^2 b + 2(a^2 b d^2 + a b^2) x^2) \cosh(dx + c))^2 - (a^3 d^2 + (a^2 b^2 d^2 + b^3) x^4 + a^2 b + 2(a^2 b d^2 + a b^2) x^2) \sinh(dx + c)^2 \sqrt{-a d^2 / b} \operatorname{Ei}(dx + \sqrt{-a d^2 / b}) - ((a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2 + ((a^3 d^2 + (a^2 b^2 d^2 + b^3) x^4 + a^2 b + 2(a^2 b d^2 + a b^2) x^2) \cosh(dx + c)^2 - (a^3 d^2 + (a^2 b^2 d^2 + b^3) x^4 + a^2 b + 2(a^2 b d^2 + a b^2) x^2) \sinh(dx + c)^2) \sqrt{-a d^2 / b}) \operatorname{Ei}(-dx - \sqrt{-a d^2 / b}) \cosh(-c + \sqrt{-a d^2 / b}) - 4(a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c) - (((a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2 + ((a^3 d^2 + (a^2 b^2 d^2 + b^3) x^4 + a^2 b + 2(a^2 b d^2 + a b^2) x^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2) \sqrt{-a d^2 / b}) \operatorname{Ei}(dx - \sqrt{-a d^2 / b}) + ((a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2 - ((a^3 d^2 + (a^2 b^2 d^2 + b^3) x^4 + a^2 b + 2(a^2 b d^2 + a b^2) x^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2) \sqrt{-a d^2 / b}) \operatorname{Ei}(-dx + \sqrt{-a d^2 / b}) \sinh(c + \sqrt{-a d^2 / b}) + (((a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2 - ((a^3 d^2 + (a^2 b^2 d^2 + b^3) x^4 + a^2 b + 2(a^2 b d^2 + a b^2) x^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2) \sqrt{-a d^2 / b}) \operatorname{Ei}(dx + \sqrt{-a d^2 / b}) + ((a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2 + ((a^3 d^2 + (a^2 b^2 d^2 + b^3) x^4 + a^2 b + 2(a^2 b d^2 + a b^2) x^2) \cosh(dx + c)^2 - (a^2 b^2 d^2 x^4 + 2 a^2 b d^2 x^2 + a^3 d^2) \sinh(dx + c)^2) \sqrt{-a d^2 / b}) \operatorname{Ei}(-dx - \sqrt{-a d^2 / b}) \sinh(-c + \sqrt{-a d^2 / b})) / ((a^2 b^4 d x^4 + 2 a^3 b^3 d x^2 + a^4 b^2 d) \cosh(dx + c)^2 - (a^2 b^4 d x^4 + 2 a^3 b^3 d x^2 + a^4 b^2 d) \sinh(dx + c)^2)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cosh(d\*x+c)/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(dx + c)}{(bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x^2+a)^3,x, algorithm="giac")

[Out] integrate(x^2\*cosh(d\*x + c)/(b\*x^2 + a)^3, x)

$$3.74 \quad \int \frac{x \cosh(c+dx)}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=512

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16ab^2} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16ab^2} + \frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \dots$$

[Out]  $-\text{Cosh}[c + d*x]/(4*b*(a + b*x^2)^2) - (d^2*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*a*b^2) - (d^2*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*a*b^2) + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*(-a)^{(3/2)*b^{(3/2)}}) - (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*(-a)^{(3/2)*b^{(3/2)}}) - (d*\text{Sinh}[c + d*x])/(16*a*b^{(3/2)*(\text{Sqrt}[-a] - \text{Sqrt}[b]*x)}) + (d*\text{Sinh}[c + d*x])/(16*a*b^{(3/2)*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)}) + (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*(-a)^{(3/2)*b^{(3/2)}}) + (d^2*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*a*b^2) + (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*(-a)^{(3/2)*b^{(3/2)}}) - (d^2*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*a*b^2)$

**Rubi [A]** time = 0.831389, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {5289, 5280, 3297, 3303, 3298, 3301}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16ab^2} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16ab^2} + \frac{d \sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}} - \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*\text{Cosh}[c + d*x])/(a + b*x^2)^3, x]$

[Out]  $-\text{Cosh}[c + d*x]/(4*b*(a + b*x^2)^2) - (d^2*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*a*b^2) - (d^2*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*a*b^2) + (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*(-a)^{(3/2)*b^{(3/2)}}) - (d*\text{CoshIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]])/(16*(-a)^{(3/2)*b^{(3/2)}}) - (d*\text{Sinh}[c + d*x])/(16*a*b^{(3/2)*(\text{Sqrt}[-a] - \text{Sqrt}[b]*x)}) + (d*\text{Sinh}[c + d*x])/(16*a*b^{(3/2)*(\text{Sqrt}[-a] + \text{Sqrt}[b]*x)}) + (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*(-a)^{(3/2)*b^{(3/2)}}) + (d^2*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]/(16*a*b^2) + (d*\text{Cosh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]/(16*(-a)^{(3/2)*b^{(3/2)}}) - (d^2*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]]*\text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x])/(16*a*b^2)$

$$\begin{aligned} & 3/2) * (\text{Sqrt}[-a] - \text{Sqrt}[b]*x) + (d*\text{Sinh}[c + d*x]) / (16*a*b^{3/2} * (\text{Sqrt}[-a] + \\ & \text{Sqrt}[b]*x)) + (d*\text{Cosh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{S} \\ & \text{qrt}[b] - d*x]) / (16*(-a)^{3/2} * b^{3/2}) + (d^2*\text{Sinh}[c + (\text{Sqrt}[-a]*d)/\text{Sqrt}[b] \\ & ] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] - d*x]) / (16*a*b^2) + (d*\text{Cosh}[c - (\text{Sqrt} \\ & -a]*d)/\text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt}[b] + d*x]) / (16*(-a)^{3/2} * b^ \\ & (3/2)) - (d^2*\text{Sinh}[c - (\text{Sqrt}[-a]*d)/\text{Sqrt}[b]] * \text{SinhIntegral}[(\text{Sqrt}[-a]*d)/\text{Sqrt} \\ & [b] + d*x]) / (16*a*b^2) \end{aligned}$$
Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] := Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x \cosh(c + dx)}{(a + bx^2)^3} dx &= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} + \frac{d \int \frac{\sinh(c+dx)}{(a+bx^2)^2} dx}{4b} \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} + \frac{d \int \left( -\frac{b \sinh(c+dx)}{4a(\sqrt{-a}\sqrt{b-bx})^2} - \frac{b \sinh(c+dx)}{4a(\sqrt{-a}\sqrt{b+bx})^2} - \frac{b \sinh(c+dx)}{2a(-ab-b^2x^2)} \right) dx}{4b} \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} - \frac{d \int \frac{\sinh(c+dx)}{(\sqrt{-a}\sqrt{b-bx})^2} dx}{16a} - \frac{d \int \frac{\sinh(c+dx)}{(\sqrt{-a}\sqrt{b+bx})^2} dx}{16a} - \frac{d \int \frac{\sinh(c+dx)}{-ab-b^2x^2} dx}{8a} \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} - \frac{d \sinh(c + dx)}{16ab^{3/2}(\sqrt{-a} - \sqrt{bx})} + \frac{d \sinh(c + dx)}{16ab^{3/2}(\sqrt{-a} + \sqrt{bx})} - \frac{d \int \left( \frac{\sqrt{-a} \sinh(c+dx)}{2ab(\sqrt{-a}-\sqrt{bx})} - \frac{\sqrt{-a} \sinh(c+dx)}{2ab(\sqrt{-a}+\sqrt{bx})} \right) dx}{8a} \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} - \frac{d \sinh(c + dx)}{16ab^{3/2}(\sqrt{-a} - \sqrt{bx})} + \frac{d \sinh(c + dx)}{16ab^{3/2}(\sqrt{-a} + \sqrt{bx})} + \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{16(-a)^{3/2}b} + \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{16(-a)^{3/2}b} \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} - \frac{d^2 \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16ab^2} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{16ab^2} \\
&= -\frac{\cosh(c + dx)}{4b(a + bx^2)^2} - \frac{d^2 \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16ab^2} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{16ab^2} + \dots
\end{aligned}$$

**Mathematica [C]** time = 1.71206, size = 637, normalized size = 1.24

$$\frac{id^2 \sinh(c) \left( \sin\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) - \sin\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) + \cos\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \left( \text{Si}\left(ixd + \frac{\sqrt{ad}}{\sqrt{b}}\right) - \text{Si}\left(\frac{\sqrt{ad}}{\sqrt{b}} - idx\right) \right) \right)}{b} - \frac{d^2 \cosh(c) \left( \cos\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(-\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) + \cos\left(\frac{\sqrt{ad}}{\sqrt{b}}\right) \text{CosIntegral}\left(\frac{\sqrt{ad}}{\sqrt{b}} + idx\right) \right)}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x\*Cosh[c + d\*x])/(a + b\*x^2)^3,x]

[Out] 
$$\begin{aligned} & ((2*\text{Cosh}[d*x]*(-2*a*\text{Cosh}[c] + d*x*(a + b*x^2)*\text{Sinh}[c]))/(a + b*x^2)^2 + (2* \\ & (d*x*(a + b*x^2)*\text{Cosh}[c] - 2*a*\text{Sinh}[c])*\text{Sinh}[d*x])/(a + b*x^2)^2 + (I*d*\text{Sin} \\ & h[c]*(\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x] \\ & - \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x] + \text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*(\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] - \text{SinIntegral} \\ & [(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])))/(\text{Sqrt}[a]*\text{Sqrt}[b]) + (I*d^2*\text{Sinh}[c]*(\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + I*d*x]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] - \text{CosIntegral} \\ & [(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*(-\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d) \\ & ]/\text{Sqrt}[b] + I*d*x)))/b + (d*\text{Cosh}[c]*(\text{CosIntegral}[-((\text{Sqrt}[a]*d)/\text{Sqrt}[b]) + \\ & I*d*x]*\text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] + \text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x]* \\ & \text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]] - \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*(\text{SinIntegral}[(\text{Sqrt}[a]*d) \\ & ]/\text{Sqrt}[b] - I*d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])))/(\text{Sqrt}[a]*\text{S} \\ & \text{qrt}[b]) - (d^2*\text{Cosh}[c]*(\text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[-((\text{Sqrt}[a]*d)/ \\ & \text{Sqrt}[b]) + I*d*x] + \text{Cos}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*\text{CosIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] \\ & ] + I*d*x] + \text{Sin}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b]]*(\text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] - I* \\ & d*x] + \text{SinIntegral}[(\text{Sqrt}[a]*d)/\text{Sqrt}[b] + I*d*x])))/b)/(16*a*b) \end{aligned}$$

---

**Maple [A]** time = 0.069, size = 743, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cosh(d\*x+c)/(b\*x^2+a)^3,x)

[Out] 
$$\begin{aligned} & -1/16*d^5*\exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*\exp \\ & (-d*x-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/8*d^4*\exp(-d*x-c)/b/(b^ \\ & 2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/32*d^2/b^2/a*\exp((d*(-a*b)^(1/2)-c*b)/b) \\ & *Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/32*d^2/b^2/a*\exp(-(d*(-a*b)^(1/2) \\ & +c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-1/32*d/b/a/(-a*b)^(1/2)*\exp \\ & ((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+1/32*d/b/a \\ & /(-a*b)^(1/2)*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+ \\ & c*b)/b)+1/16*d^5*\exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3+1/16* \\ & d^5*\exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x-1/8*d^4*\exp(d*x+c)/b \\ & /(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+1/32*d^2/b^2/a*\exp((d*(-a*b)^(1/2)+c*b) \\ & )/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d^2/b^2/a*\exp(-(d*(-a*b)^( \\ & 1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/32*d/b/a/(-a*b)^(1/2) \\ & )*\exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+1/32*d \\ & /b/a/(-a*b)^(1/2)*\exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c) \end{aligned}$$





$$\begin{aligned} &^2*x^2 + a^3*d^2)*\cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) \\ &d^2)*\sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\cosh(d*x + c)^2 - ( \\ &b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b))*\text{Ei}(-d*x + s \\ &\text{qrt}(-a*d^2/b)))*\sinh(c + \text{sqrt}(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x \\ &^2 + a^3*d^2)*\cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2) \\ &)*\sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\cosh(d*x + c)^2 - (b^3* \\ &x^4 + 2*a*b^2*x^2 + a^2*b)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b))*\text{Ei}(d*x + \text{sqrt}(- \\ &a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\cosh(d*x + c)^2 - \\ &(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\sinh(d*x + c)^2 + ((b^3*x^4 + 2 \\ &*a*b^2*x^2 + a^2*b)*\cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\sinh( \\ &d*x + c)^2)*\sqrt{-a*d^2/b))*\text{Ei}(-d*x - \text{sqrt}(-a*d^2/b)))*\sinh(-c + \text{sqrt}(-a*d^ \\ &2/b)))/((a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)*\cosh(d*x + c)^2 - (a^2*b^4*x \\ &x^4 + 2*a^3*b^3*x^2 + a^4*b^2)*\sinh(d*x + c)^2) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(dx + c)}{(bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x^2+a)^3,x, algorithm="giac")

[Out] integrate(x\*cosh(d\*x + c)/(b\*x^2 + a)^3, x)

$$3.75 \quad \int \frac{\cosh(c+dx)}{(a+bx^2)^3} dx$$

**Optimal.** Leaf size=856

result too large to display

```
[Out] -Cosh[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)^2) - (3*Cosh[c + d*x])/
(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*
(Sqrt[-a] + Sqrt[b]*x)^2) + (3*Cosh[c + d*x])/
(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) + (3*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(S
qrt[-a]*d)/Sqrt[b] - d*x])/
(16*(-a)^(5/2)*Sqrt[b]) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(16*(-a)^(3/2)*b^(3/2)) - (3*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x])/
(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*(-a)^(3/2)*b^(3/2)) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqr
t[b]])/
(16*a^2*b) + (d*Sinh[c + d*x])/
(16*(-a)^(3/2)*b*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh[c + d*x])/
(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]*x)) + (3*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(16*a^2*b) - (3*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(16*(-a)^(3/2)*b^(3/2)) - (3*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*a^2*b) - (3*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*(-a)^(3/2)*b^(3/2))
```

---

**Rubi [A]** time = 1.25523, antiderivative size = 856, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5281, 3297, 3303, 3298, 3301}

$$\frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{3/2} b^{3/2}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right) d^2}{16(-a)^{3/2} b^{3/2}} - \frac{\sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{3/2} b^{3/2}} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right) d^2}{16(-a)^{3/2} b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(a + b\*x^2)^3,x]

```
[Out] -Cosh[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)^2) - (3*Cosh[c + d*x])/
(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(16*(-a)^(3/2)*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)^2) + (3*Cosh[c + d*x])/
(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) + (3*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(16*(-a)^(5/2)*Sqrt[b]) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(16*(-a)^(3/2)*b^(3/2)) - (3*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*(-a)^(3/2)*b^(3/2)) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/
(16*a^2*b) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/
(16*a^2*b) + (d*Sinh[c + d*x])/
(16*(-a)^(3/2)*b*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh[c + d*x])/
(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]*x)) + (3*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(16*a^2*b) - (3*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(16*(-a)^(3/2)*b^(3/2)) - (3*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*a^2*b) - (3*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(16*(-a)^(3/2)*b^(3/2))
```

### Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
```

```
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\cosh(c+dx)}{(a+bx^2)^3} dx &= \int \left( -\frac{b^{3/2} \cosh(c+dx)}{8(-a)^{3/2} (\sqrt{-a}\sqrt{b}-bx)^3} - \frac{3b \cosh(c+dx)}{16a^2 (\sqrt{-a}\sqrt{b}-bx)^2} - \frac{b^{3/2} \cosh(c+dx)}{8(-a)^{3/2} (\sqrt{-a}\sqrt{b}+bx)^3} - \frac{3b \cosh(c+dx)}{16a^2 (\sqrt{-a}\sqrt{b}+bx)^2} \right) dx \\ &= -\frac{(3b) \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^2} dx}{16a^2} - \frac{(3b) \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}+bx)^2} dx}{16a^2} - \frac{(3b) \int \frac{\cosh(c+dx)}{-ab-b^2x^2} dx}{8a^2} - \frac{b^{3/2} \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^3} dx}{8(-a)^{3/2}} - \frac{b^{3/2} \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}+bx)^3} dx}{8(-a)^{3/2}} \\ &= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}-\sqrt{bx})^2} - \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}+\sqrt{bx})^2} + \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \\ &= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}-\sqrt{bx})^2} - \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}+\sqrt{bx})^2} + \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \\ &= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}-\sqrt{bx})^2} - \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}+\sqrt{bx})^2} + \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \\ &= -\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}-\sqrt{bx})^2} - \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}(\sqrt{-a}+\sqrt{bx})^2} + \frac{3 \cosh(c+dx)}{16a^2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \end{aligned}$$

**Mathematica [C]** time = 2.38417, size = 933, normalized size = 1.09

$$\frac{6b^{5/2} \cosh(c) \cosh(dx)x^3}{(bx^2+a)^2} + \frac{6b^{5/2} \sinh(c) \sinh(dx)x^3}{(bx^2+a)^2} + \frac{2ab^{3/2}d \cosh(dx) \sinh(c)x^2}{(bx^2+a)^2} + \frac{2ab^{3/2}d \cosh(c) \sinh(dx)x^2}{(bx^2+a)^2} + \frac{10ab^{3/2} \cosh(c) \cosh(dx)x}{(bx^2+a)^2} + \frac{10ab^{3/2} \sinh(c) \sinh(dx)x}{(bx^2+a)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d\*x]/(a + b\*x^2)^3, x]

[Out] 
$$\begin{aligned} & \left( \frac{10ab^{3/2}x \cosh[c] \cosh[dx]}{(a + bx^2)^2} + \frac{6b^{5/2}x^3 \cosh[c] \cosh[dx]}{(a + bx^2)^2} + \frac{2a^2 \sqrt{b} d \cosh[dx] \sinh[c]}{(a + bx^2)^2} + \frac{2ab^{3/2} dx^2 \cosh[dx] \sinh[c]}{(a + bx^2)^2} + \frac{\text{CosIntegral}\left[-\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) + Idx\right] \left( I(3b - a^2) \cosh\left[c - \frac{I\sqrt{a}d}{\sqrt{b}}\right] - 3\sqrt{a}\sqrt{b} d \sinh\left[c - \frac{I\sqrt{a}d}{\sqrt{b}}\right] \right)}{\sqrt{a}} + \frac{I \text{CosIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Idx\right] \left( (-3b + a^2) \cosh\left[c + \frac{I\sqrt{a}d}{\sqrt{b}}\right] + (3I)\sqrt{a}\sqrt{b} d \sinh\left[c + \frac{I\sqrt{a}d}{\sqrt{b}}\right] \right)}{\sqrt{a}} + \frac{2a^2 \sqrt{b} d \cosh[c] \sinh[dx]}{(a + bx^2)^2} + \frac{2ab^{3/2} dx^2 \cosh[c] \sinh[dx]}{(a + bx^2)^2} + \frac{10ab^{3/2} x \sinh[c] \sinh[dx]}{(a + bx^2)^2} + \frac{6b^{5/2} x^3 \sinh[c] \sinh[dx]}{(a + bx^2)^2} - \frac{(3I)\sqrt{b} d \cos\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \cosh[c] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Idx\right] + ((3I)b \cosh[c] \sin\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Idx\right])}{\sqrt{a}} - \frac{I\sqrt{a}d^2 \cosh[c] \sin\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Idx\right] - (3b \cos\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \sinh[c] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Idx\right])}{\sqrt{a}} + \frac{\sqrt{a}d^2 \cos\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \sinh[c] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Idx\right] - 3\sqrt{b} d \sin\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \sinh[c] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Idx\right] + (3I)\sqrt{b} d \cos\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \cosh[c] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Idx\right]}{\sqrt{b} + Idx} - \frac{((3I)b \cosh[c] \sin\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Idx\right])}{\sqrt{a}} + \frac{I\sqrt{a}d^2 \cosh[c] \sin\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Idx\right] - (3b \cos\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \sinh[c] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Idx\right])}{\sqrt{a}} + \frac{\sqrt{a}d^2 \cos\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \sinh[c] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Idx\right] - 3\sqrt{b} d \sin\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \sinh[c] \text{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Idx\right]}{\sqrt{b} + Idx} \right) / (16a^2 b^{3/2}) \end{aligned}$$

---

**Maple [A]** time = 0.037, size = 1064, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/(b\*x^2+a)^3, x)

[Out] 
$$\begin{aligned} & -\frac{1}{16}d^5 \exp(-dx-c)/a/(b^2d^4x^4+2ab^2d^4x^2+a^2d^4)x^2+3/16d^4 \exp(-dx-c)/a^2b/(b^2d^4x^4+2ab^2d^4x^2+a^2d^4)x^3-1/16d^5 \exp(-dx-c)/b/(b^2d^4x^4+2ab^2d^4x^2+a^2d^4)+5/16d^4 \exp(-dx-c)/a/(b^2d^4x^4+2ab^2d^4x^2+a^2d^4)x+1/32d^2/b/a/(-ab)^{1/2} \exp(-(d(-ab)^{1/2}+cb)/b)/b * \text{Ei}\left(1, \frac{-(d(-ab)^{1/2}-(dx+c)b+cb)}{b}\right) - 1/32d^2/b/a/(-ab)^{1/2} \exp((d(-ab)^{1/2}-cb)/b) * \text{Ei}\left(1, \frac{(d(-ab)^{1/2}+(dx+c)b-cb)}{b}\right) - 3/32d/b/a^2 \end{aligned}$$

$$2*\exp(-(d*(-a*b)^{(1/2)+c*b})/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)-3/32*d/b/a^2*\exp((d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)-3/32/a^2/(-a*b)^{(1/2)}*\exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)+3/32/a^2/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)+1/16*d^5*\exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2+3/16*d^4*\exp(d*x+c)/a^2*b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3+1/16*d^5*\exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+5/16*d^4*\exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x+1/32*d^2/b/a/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)-1/32*d^2/b/a/(-a*b)^{(1/2)}*\exp(-(d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)+3/32*d/b/a^2*\exp((d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)+3/32*d/b/a^2*\exp(-(d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)-3/32/a^2/(-a*b)^{(1/2)}*\exp((d*(-a*b)^{(1/2)}+c*b)/b)*\text{Ei}(1, (d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)+3/32/a^2/(-a*b)^{(1/2)}*\exp(-(d*(-a*b)^{(1/2)}-c*b)/b)*\text{Ei}(1, -(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x^2+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.33339, size = 4342, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x^2+a)^3,x, algorithm="fricas")

[Out]  $\frac{1}{32}*(4*(3*a*b^2*d*x^3 + 5*a^2*b*d*x)*\cosh(d*x + c) - ((3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x - \sqrt{-a*d^2/b}) - (3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*\sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x - \sqrt{-a*d^2/b})$

$$\begin{aligned}
& + a^3 d^2) \cosh(dx + c)^2 - 3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) \\
& * \sinh(dx + c)^2 + ((a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \cosh(dx + c)^2 - (a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \sinh(dx + c)^2) * \sqrt{-a d^2 / b}) * \text{Ei} \\
& (-dx + \sqrt{-a d^2 / b}) * \cosh(c + \sqrt{-a d^2 / b}) - ((3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \cosh(dx + c)^2 - 3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \sinh(dx + c)^2 + ((a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \cosh(dx + c)^2 - (a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \sinh(dx + c)^2) * \sqrt{-a d^2 / b}) * \text{Ei}(dx + \sqrt{-a d^2 / b}) - (3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \cosh(dx + c)^2 - 3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \sinh(dx + c)^2 - ((a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \cosh(dx + c)^2 - (a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \sinh(dx + c)^2) * \sqrt{-a d^2 / b}) * \text{Ei}(-dx - \sqrt{-a d^2 / b})) * \cosh(-c + \sqrt{-a d^2 / b}) + 4(a^2 b d^2 x^2 + a^3 d^2) * \sinh(dx + c) - ((3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \cosh(dx + c)^2 - 3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \sinh(dx + c)^2 - ((a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \cosh(dx + c)^2 - (a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \sinh(dx + c)^2) * \sqrt{-a d^2 / b}) * \text{Ei}(dx - \sqrt{-a d^2 / b}) + (3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \cosh(dx + c)^2 - 3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \sinh(dx + c)^2 + ((a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \cosh(dx + c)^2 - (a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \sinh(dx + c)^2) * \sqrt{-a d^2 / b}) * \text{Ei}(-dx + \sqrt{-a d^2 / b})) * \sinh(c + \sqrt{-a d^2 / b}) + ((3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \cosh(dx + c)^2 - 3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \sinh(dx + c)^2 + ((a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \cosh(dx + c)^2 - (a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \sinh(dx + c)^2) * \sqrt{-a d^2 / b}) * \text{Ei}(dx + \sqrt{-a d^2 / b}) + (3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \cosh(dx + c)^2 - 3(a^2 b^2 d^2 x^4 + 2a^2 b d^2 x^2 + a^3 d^2) * \sinh(dx + c)^2 - ((a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \cosh(dx + c)^2 - (a^3 d^2 + (a^2 b^2 d^2 - 3b^3) x^4 - 3a^2 b + 2(a^2 b d^2 - 3a^2 b^2) x^2) * \sinh(dx + c)^2) * \sqrt{-a d^2 / b}) * \text{Ei}(-dx - \sqrt{-a d^2 / b})) * \sinh(-c + \sqrt{-a d^2 / b})) / ((a^3 b^3 d^2 x^4 + 2a^4 b^2 d^2 x^2 + a^5 b d) * \cosh(dx + c)^2 - (a^3 b^3 d^2 x^4 + 2a^4 b^2 d^2 x^2 + a^5 b d) * \sinh(dx + c)^2)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(cosh(d*x+c)/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^2 + a)^3, x)
```

$$3.76 \quad \int \frac{\cosh(c+dx)}{x(a+bx^2)^3} dx$$

**Optimal.** Leaf size=730

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16a^2b} + \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16a^2b} - \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^3} - \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^3}$$

```
[Out] Cosh[c + d*x]/(4*a*(a + b*x^2)^2) + Cosh[c + d*x]/(2*a^2*(a + b*x^2)) + (Cosh[c]*CoshIntegral[d*x])/a^3 - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^3) + (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) + (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) + (d*Sinh[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Sinh[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 + (5*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) + (5*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^3) + (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b)
```

**Rubi [A]** time = 1.68183, antiderivative size = 730, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {5293, 3303, 3298, 3301, 5289, 5280, 3297}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16a^2b} + \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16a^2b} - \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^3} - \frac{\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x\*(a + b\*x^2)^3), x]

```
[Out] Cosh[c + d*x]/(4*a*(a + b*x^2)^2) + Cosh[c + d*x]/(2*a^2*(a + b*x^2)) + (Cosh[c]*CoshIntegral[d*x])/a^3 - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^3) + (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b) + (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) + (d*Sinh[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Sinh[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 + (5*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) + (5*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^3) + (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b)
```

### Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}
```

}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 5289

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(e^m\*(a + b\*x^n)^(p + 1)\*Cosh[c + d\*x])/(b\*n\*(p + 1)), x] - Dist[(d\*e^m)/(b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*Sinh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])

### Rule 5280

Int[((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_)\*Sinh[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Int[ExpandIntegrand[Sinh[c + d\*x], (a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx^2)^3} dx &= \int \left( \frac{\cosh(c+dx)}{a^3 x} - \frac{bx \cosh(c+dx)}{a(a+bx^2)^3} - \frac{bx \cosh(c+dx)}{a^2(a+bx^2)^2} - \frac{bx \cosh(c+dx)}{a^3(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a^3} - \frac{b \int \frac{x \cosh(c+dx)}{a+bx^2} dx}{a^3} - \frac{b \int \frac{x \cosh(c+dx)}{(a+bx^2)^2} dx}{a^2} - \frac{b \int \frac{x \cosh(c+dx)}{(a+bx^2)^3} dx}{a} \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} - \frac{b \int \left( -\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \right) dx}{a^3} - \frac{d \int \frac{\sinh(c+dx)}{a+bx^2} dx}{2a^2} - \frac{d \int \frac{\sinh(c+dx)}{(a+bx^2)^2} dx}{2a} \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{\sqrt{b} \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2a^3} - \frac{\sqrt{b} \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2a^3} \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} + \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{4(-a)^{5/2}} + \frac{d \int \frac{\sinh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{4(-a)^{5/2}} \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^3} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{2a^3} \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^3} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} + dx\right)}{2a^3} \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^3} + \frac{d^2 \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4a^3} \\
&= \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3} - \frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{2a^3} + \frac{d^2 \cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{4a^3}
\end{aligned}$$

**Mathematica [C]** time = 3.262, size = 1558, normalized size = 2.13

result too large to display

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x\*(a + b\*x^2)^3), x]

```

[Out] (12*a^2*b*Cosh[c + d*x] + 8*a*b^2*x^2*Cosh[c + d*x] + 16*b*(a + b*x^2)^2*Co
sh[c]*CoshIntegral[d*x] - 8*a^2*b*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CoshInteg
ral[d*((I*Sqrt[a])/Sqrt[b] + x)] + a^3*d^2*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*
CoshIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - 16*a*b^2*x^2*Cosh[c - (I*Sqrt[a
]*d)/Sqrt[b]]*CoshIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + 2*a^2*b*d^2*x^2*C
osh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CoshIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] -
8*b^3*x^4*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CoshIntegral[d*((I*Sqrt[a])/Sqrt[
b] + x)] + a*b^2*d^2*x^4*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*CoshIntegral[d*((I
*Sqrt[a])/Sqrt[b] + x)] - (5*I)*a^(5/2)*Sqrt[b]*d*CoshIntegral[d*((I*Sqrt[a
])/Sqrt[b] + x)]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]] - (10*I)*a^(3/2)*b^(3/2)*d
*x^2*CoshIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]*Sinh[c - (I*Sqrt[a]*d)/Sqrt[
b]] - (5*I)*Sqrt[a]*b^(5/2)*d*x^4*CoshIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)]
*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]] + (a + b*x^2)^2*CoshIntegral[d*((-I)*Sqrt
[a])/Sqrt[b] + x)]*((-8*b + a*d^2)*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]] + (5*I)*
Sqrt[a]*Sqrt[b]*d*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]) - 2*a^2*b*d*x*Sinh[c + d
*x] - 2*a*b^2*d*x^3*Sinh[c + d*x] + 16*a^2*b*Sinh[c]*SinhIntegral[d*x] + 32
*a*b^2*x^2*Sinh[c]*SinhIntegral[d*x] + 16*b^3*x^4*Sinh[c]*SinhIntegral[d*x]
- (5*I)*a^(5/2)*Sqrt[b]*d*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*(
(I*Sqrt[a])/Sqrt[b] + x)] - (10*I)*a^(3/2)*b^(3/2)*d*x^2*Cosh[c - (I*Sqrt[a
]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - (5*I)*Sqrt[a]*b^(
5/2)*d*x^4*Cosh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt
[b] + x)] - 8*a^2*b*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt
[a])/Sqrt[b] + x)] + a^3*d^2*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d
*((I*Sqrt[a])/Sqrt[b] + x)] - 16*a*b^2*x^2*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*
SinhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + 2*a^2*b*d^2*x^2*Sinh[c - (I*Sqr
t[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] - 8*b^3*x^4*Sinh
[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt[b] + x)] + a*b
^2*d^2*x^4*Sinh[c - (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[d*((I*Sqrt[a])/Sqrt
[b] + x)] - (5*I)*a^(5/2)*Sqrt[b]*d*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhInt
egral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (10*I)*a^(3/2)*b^(3/2)*d*x^2*Cosh[c +
(I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - (5*I)*Sq
rt[a]*b^(5/2)*d*x^4*Cosh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x] + 8*a^2*b*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(
I*Sqrt[a]*d)/Sqrt[b] - d*x] - a^3*d^2*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhI
ntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + 16*a*b^2*x^2*Sinh[c + (I*Sqrt[a]*d)/
Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] - 2*a^2*b*d^2*x^2*Sinh[c
+ (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d*x] + 8*b^3
*x^4*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]*d)/Sqrt[b] - d
*x] - a*b^2*d^2*x^4*Sinh[c + (I*Sqrt[a]*d)/Sqrt[b]]*SinhIntegral[(I*Sqrt[a]
*d)/Sqrt[b] - d*x))/(16*a^3*b*(a + b*x^2)^2)

```

**Maple [A]** time = 0.105, size = 1090, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/x/(b*x^2+a)^3,x)`

[Out] 
$$\frac{1}{16} \exp(-d*x-c) * d^2 * ((d*x+c)^3 * b - 3 * (d*x+c)^2 * b * c + (d*x+c) * a * d^2 + 3 * (d*x+c) * b * c^2 - a * c * d^2 - b * c^3 + 4 * (d*x+c)^2 * b - 8 * (d*x+c) * b * c + 6 * a * d^2 + 4 * b * c^2) / a^2 / ((d*x+c)^4 * b^2 - 4 * (d*x+c)^3 * b^2 * c + 2 * (d*x+c)^2 * a * b * d^2 + 6 * (d*x+c)^2 * c^2 * b^2 - 4 * (d*x+c) * a * b * c * d^2 - 4 * (d*x+c) * b^2 * c^3 + a^2 * d^4 + 2 * a * b * c^2 * d^2 + b^2 * c^4) - 1/32 / b / a^2 * \exp(-(d * (-a * b)^{(1/2)} + c * b) / b) * \text{Ei}(1, -(d * (-a * b)^{(1/2)} - (d * x + c) * b + c * b) / b) * d^2 - 1/32 / b / a^2 * \exp((d * (-a * b)^{(1/2)} - c * b) / b) * \text{Ei}(1, (d * (-a * b)^{(1/2)} + (d * x + c) * b - c * b) / b) * d^2 - 5/32 / a^2 / (-a * b)^{(1/2)} * \exp(-(d * (-a * b)^{(1/2)} + c * b) / b) * \text{Ei}(1, -(d * (-a * b)^{(1/2)} - (d * x + c) * b + c * b) / b) * d + 5/32 / a^2 / (-a * b)^{(1/2)} * \exp((d * (-a * b)^{(1/2)} - c * b) / b) * \text{Ei}(1, (d * (-a * b)^{(1/2)} + (d * x + c) * b - c * b) / b) * d + 1/4 / a^3 * \exp(-(d * (-a * b)^{(1/2)} + c * b) / b) * \text{Ei}(1, -(d * (-a * b)^{(1/2)} - (d * x + c) * b + c * b) / b) + 1/4 / a^3 * \exp((d * (-a * b)^{(1/2)} - c * b) / b) * \text{Ei}(1, (d * (-a * b)^{(1/2)} + (d * x + c) * b - c * b) / b) - 1/2 / a^3 * \exp(-c) * \text{Ei}(1, d * x) - 1/16 * \exp(d * x + c) * d^2 * ((d*x+c)^3 * b - 3 * (d*x+c)^2 * b * c + (d*x+c) * a * d^2 + 3 * (d*x+c) * b * c^2 - a * c * d^2 - b * c^3 - 4 * (d*x+c)^2 * b + 8 * (d*x+c) * b * c - 6 * a * d^2 - 4 * b * c^2) / a^2 / ((d*x+c)^4 * b^2 - 4 * (d*x+c)^3 * b^2 * c + 2 * (d*x+c)^2 * a * b * d^2 + 6 * (d*x+c)^2 * c^2 * b^2 - 4 * (d*x+c) * a * b * c * d^2 - 4 * (d*x+c) * b^2 * c^3 + a^2 * d^4 + 2 * a * b * c^2 * d^2 + b^2 * c^4) - 1/32 / b / a^2 * \exp((d * (-a * b)^{(1/2)} + c * b) / b) * \text{Ei}(1, (d * (-a * b)^{(1/2)} - (d * x + c) * b + c * b) / b) * d^2 - 1/32 / b / a^2 * \exp(-(d * (-a * b)^{(1/2)} - c * b) / b) * \text{Ei}(1, -(d * (-a * b)^{(1/2)} + (d * x + c) * b - c * b) / b) * d^2 + 5/32 / a^2 / (-a * b)^{(1/2)} * \exp((d * (-a * b)^{(1/2)} + c * b) / b) * \text{Ei}(1, (d * (-a * b)^{(1/2)} - (d * x + c) * b + c * b) / b) * d - 5/32 / a^2 / (-a * b)^{(1/2)} * \exp(-(d * (-a * b)^{(1/2)} - c * b) / b) * \text{Ei}(1, -(d * (-a * b)^{(1/2)} + (d * x + c) * b - c * b) / b) * d + 1/4 / a^3 * \exp((d * (-a * b)^{(1/2)} + c * b) / b) * \text{Ei}(1, (d * (-a * b)^{(1/2)} - (d * x + c) * b + c * b) / b) + 1/4 / a^3 * \exp(-(d * (-a * b)^{(1/2)} - c * b) / b) * \text{Ei}(1, -(d * (-a * b)^{(1/2)} + (d * x + c) * b - c * b) / b) - 1/2 / a^3 * \exp(c) * \text{Ei}(1, -d * x)$$

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.15226, size = 4301, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(8*(2*a*b^2*x^2 + 3*a^2*b)*cosh(d*x + c) + (((a^3*d^2 + (a*b^2*d^2 - 8
*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d
^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(
d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4
+ 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^
2/b)) + ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*
b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b +
2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 +
a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*s
qrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))*cosh(c + sqrt(-a*d^2/b)) + 16*((b
^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(d*x) + (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(
-d*x))*cosh(c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*
d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 -
8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*
b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x
+ c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a^3*d^2 + (a*b^2*d^2
- 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^
3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*si
nh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x
^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-
a*d^2/b))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a*b^2*d*x^3 + a^2*b*d*x)*sinh(d*x
+ c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a
*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b +
2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 +
a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*
sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*
x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (
a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x +
c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*
b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))
)*sinh(c + sqrt(-a*d^2/b)) + 16*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(d*x) -
(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(-d*x))*sinh(c) - (((a^3*d^2 + (a*b^2*d^2
- 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a
^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*s
inh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*
```



$$x^4 + 2ab^2x^2 + a^2b) \sinh(dx + c)^2 \sqrt{-ad^2/b}) \operatorname{Ei}(dx + \sqrt{-ad^2/b}) - ((a^3d^2 + (ab^2d^2 - 8b^3)x^4 - 8a^2b + 2(a^2bd^2 - 8ab^2)x^2) \cosh(dx + c)^2 - (a^3d^2 + (ab^2d^2 - 8b^3)x^4 - 8a^2b + 2(a^2bd^2 - 8ab^2)x^2) \sinh(dx + c)^2 + 5((b^3x^4 + 2ab^2x^2 + a^2b) \cosh(dx + c)^2 - (b^3x^4 + 2ab^2x^2 + a^2b) \sinh(dx + c)^2) \sqrt{-ad^2/b}) \operatorname{Ei}(-dx - \sqrt{-ad^2/b})) \sinh(-c + \sqrt{-ad^2/b})) / ((a^3b^3x^4 + 2a^4b^2x^2 + a^5b) \cosh(dx + c)^2 - (a^3b^3x^4 + 2a^4b^2x^2 + a^5b) \sinh(dx + c)^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)/x/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)/x/(b\*x^2+a)^3,x, algorithm="giac")

[Out] integrate(cosh(dx + c)/((b\*x^2 + a)^3\*x), x)

$$3.77 \quad \int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3} dx$$

**Optimal.** Leaf size=874

result too large to display

```
[Out] -(Cosh[c + d*x]/(a^3*x)) - (Sqrt[b]*Cosh[c + d*x])/((16*(-a)^(5/2)*(Sqrt[-a]
- Sqrt[b]*x)^2) + (7*Sqrt[b]*Cosh[c + d*x])/((16*a^3*(Sqrt[-a] - Sqrt[b]*x)
) + (Sqrt[b]*Cosh[c + d*x])/((16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (7*S
qrt[b]*Cosh[c + d*x])/((16*a^3*(Sqrt[-a] + Sqrt[b]*x)) + (15*Sqrt[b]*Cosh[c
+ (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/((16*(-a)^(
7/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/((16*(-a)^(5/2)*Sqrt[b]) - (15*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqr
t[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/((16*(-a)^(7/2)) - (d^2*Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/((16*(-
a)^(5/2)*Sqrt[b]) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 + (7*d*CoshIntegral[(
Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3) + (7*d*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(1
6*a^3) + (d*Sinh[c + d*x])/((16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh
[c + d*x])/((16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cosh[c]*SinhIntegral
[d*x])/a^3 - (7*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/((16*a^3) - (15*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/((16*(-a)^(7/2)) - (d^2*Sinh[c + (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/((16*(-a)^(5/2)*Sq
rt[b]) + (7*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/((16*a^3) - (15*Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/((16*(-a)^(7/2)) - (d^2*Sinh[c - (Sqrt[-a]*
d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/((16*(-a)^(5/2)*Sqrt[b
])]
```

**Rubi [A]** time = 2.68917, antiderivative size = 874, normalized size of antiderivative = 1., number of steps used = 60, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{\cosh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{5/2}\sqrt{b}} - \frac{\cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right) d^2}{16(-a)^{5/2}\sqrt{b}} - \frac{\sinh\left(c + \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right) d^2}{16(-a)^{5/2}\sqrt{b}} - \frac{\sinh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Shi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right) d^2}{16(-a)^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x^2\*(a + b\*x^2)^3), x]

```
[Out] -(Cosh[c + d*x]/(a^3*x)) - (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a]
- Sqrt[b]*x)^2) + (7*Sqrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)
) + (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (7*S
qrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) + (15*Sqrt[b]*Cosh[c
+ (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(
7/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (15*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqr
t[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (d^2*Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-
a)^(5/2)*Sqrt[b]) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 + (7*d*CoshIntegral[(
Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a^3) + (7*d*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(1
6*a^3) + (d*Sinh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*x)) + (d*Sinh
[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cosh[c]*SinhIntegral
[d*x])/a^3 - (7*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (d^2*Sinh[c + (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sq
rt[b]) + (7*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (d^2*Sinh[c - (Sqrt[-a]*
d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b
])
```

### Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

### Rule 3297

```
Int[(((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3} dx &= \int \left( \frac{\cosh(c+dx)}{a^3 x^2} - \frac{b \cosh(c+dx)}{a(a+bx^2)^3} - \frac{b \cosh(c+dx)}{a^2(a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a^3} - \frac{b \int \frac{\cosh(c+dx)}{a+bx^2} dx}{a^3} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx^2)^2} dx}{a^2} - \frac{b \int \frac{\cosh(c+dx)}{(a+bx^2)^3} dx}{a} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{b \int \left( \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{-a} \cosh(c+dx)}{2a(\sqrt{-a}+\sqrt{bx})} \right) dx}{a^3} - \frac{b \int \left( -\frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}-bx)^2} - \frac{b \cosh(c+dx)}{4a(\sqrt{-a}\sqrt{b}+bx)^2} \right) dx}{a^2} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}-\sqrt{bx}} dx}{2(-a)^{7/2}} - \frac{b \int \frac{\cosh(c+dx)}{\sqrt{-a}+\sqrt{bx}} dx}{2(-a)^{7/2}} + \frac{(3b^2) \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}-bx)^2} dx}{16a^3} + \frac{(3b^2) \int \frac{\cosh(c+dx)}{(\sqrt{-a}\sqrt{b}+bx)^2} dx}{16a^3} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}-\sqrt{bx})^2} + \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}+\sqrt{bx})^2} - \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}+\sqrt{bx})} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}-\sqrt{bx})^2} + \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}+\sqrt{bx})^2} - \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}+\sqrt{bx})} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}-\sqrt{bx})^2} + \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}+\sqrt{bx})^2} - \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}+\sqrt{bx})} \\
&= -\frac{\cosh(c+dx)}{a^3 x} - \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}-\sqrt{bx})^2} + \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}-\sqrt{bx})} + \frac{\sqrt{b} \cosh(c+dx)}{16(-a)^{5/2}(\sqrt{-a}+\sqrt{bx})^2} - \frac{7\sqrt{b} \cosh(c+dx)}{16a^3(\sqrt{-a}+\sqrt{bx})}
\end{aligned}$$

**Mathematica [C]** time = 3.37809, size = 1359, normalized size = 1.55

result too large to display

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x^2\*(a + b\*x^2)^3), x]

[Out] (-16\*a^(5/2)\*Sqrt[b]\*Cosh[c + d\*x] - 50\*a^(3/2)\*b^(3/2)\*x^2\*Cosh[c + d\*x] - 30\*Sqrt[a]\*b^(5/2)\*x^4\*Cosh[c + d\*x] + 16\*a^(5/2)\*Sqrt[b]\*d\*x\*CoshIntegral[d\*x]\*Sinh[c] + 32\*a^(3/2)\*b^(3/2)\*d\*x^3\*CoshIntegral[d\*x]\*Sinh[c] + 16\*Sqrt[a]\*b^(5/2)\*d\*x^5\*CoshIntegral[d\*x]\*Sinh[c] + x\*(a + b\*x^2)^2\*CoshIntegral

$$\begin{aligned}
& [d*((I*\text{Sqrt}[a])/ \text{Sqrt}[b] + x)]*((-I)*(15*b - a*d^2)*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]] + 7*\text{Sqrt}[a]*\text{Sqrt}[b]*d*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]) + x*(a + b*x^2)^2*\text{CoshIntegral}[d*((-I)*\text{Sqrt}[a])/ \text{Sqrt}[b] + x)]*(I*(15*b - a*d^2)*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]] + 7*\text{Sqrt}[a]*\text{Sqrt}[b]*d*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]) - 2*a^(5/2)*\text{Sqrt}[b]*d*x*\text{Sinh}[c + d*x] - 2*a^(3/2)*b^(3/2)*d*x^3*\text{Sinh}[c + d*x] + 16*a^(5/2)*\text{Sqrt}[b]*d*x*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + 32*a^(3/2)*b^(3/2)*d*x^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + 16*\text{Sqrt}[a]*b^(5/2)*d*x^5*\text{Cosh}[c]*\text{SinhIntegral}[d*x] + 7*a^(5/2)*\text{Sqrt}[b]*d*x*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[d*((I*\text{Sqrt}[a])/ \text{Sqrt}[b] + x)] + 14*a^(3/2)*b^(3/2)*d*x^3*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[d*((I*\text{Sqrt}[a])/ \text{Sqrt}[b] + x)] + 7*\text{Sqrt}[a]*b^(5/2)*d*x^5*\text{Cosh}[c - (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[d*((I*\text{Sqrt}[a])/ \text{Sqrt}[b] + x)] - (15*I)*a^2*b*x*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[d*((I*\text{Sqrt}[a])/ \text{Sqrt}[b] + x)] + I*a^3*d^2*x*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[d*((I*\text{Sqrt}[a])/ \text{Sqrt}[b] + x)] - (30*I)*a*b^2*x^3*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[d*((I*\text{Sqrt}[a])/ \text{Sqrt}[b] + x)] + (2*I)*a^2*b*d^2*x^3*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[d*((I*\text{Sqrt}[a])/ \text{Sqrt}[b] + x)] - (15*I)*b^3*x^5*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[d*((I*\text{Sqrt}[a])/ \text{Sqrt}[b] + x)] + I*a*b^2*d^2*x^5*\text{Sinh}[c - (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[d*((I*\text{Sqrt}[a])/ \text{Sqrt}[b] + x)] - 7*a^(5/2)*\text{Sqrt}[b]*d*x*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[(I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b] - d*x] - 14*a^(3/2)*b^(3/2)*d*x^3*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[(I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b] - d*x] - 7*\text{Sqrt}[a]*b^(5/2)*d*x^5*\text{Cosh}[c + (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[(I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b] - d*x] - (15*I)*a^2*b*x*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[(I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b] - d*x] + I*a^3*d^2*x*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[(I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b] - d*x] - (30*I)*a*b^2*x^3*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[(I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b] - d*x] + (2*I)*a^2*b*d^2*x^3*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[(I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b] - d*x] - (15*I)*b^3*x^5*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[(I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b] - d*x] + I*a*b^2*d^2*x^5*\text{Sinh}[c + (I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b]]*\text{SinhIntegral}[(I*\text{Sqrt}[a]*d)/ \text{Sqrt}[b] - d*x)]/(16*a^(7/2)*\text{Sqrt}[b]*x*(a + b*x^2)^2)
\end{aligned}$$


---

**Maple [A]** time = 0.128, size = 1178, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/x^2/(b*x^2+a)^3,x)`

[Out] `1/16*exp(-d*x-c)/a^2*x^2*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-15/16*exp(-d*x-c)/a^3*x^3*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+1/16*exp(-d*x`

$$\begin{aligned}
& -c)/a*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-25/16*\exp(-d*x-c)/a^2*x*d^4/( \\
& b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-1/2*\exp(-d*x-c)/a/x*d^4/(b^2*d^4*x^4+2 \\
& *a*b*d^4*x^2+a^2*d^4)+1/32/a^2*d^2/(-a*b)^(1/2)*\exp((d*(-a*b)^(1/2)-c*b)/b) \\
& *Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-1/32/a^2*d^2/(-a*b)^(1/2)*\exp(-(d*( \\
& -a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)+7/32*d/a^3*\exp( \\
& (d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+7/32*d/a^3*\exp( \\
& -(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-15/32/a \\
& ^3/(-a*b)^(1/2)*\exp((d*(-a*b)^(1/2)-c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+(d*x+c)*b- \\
& c*b)/b)*b+15/32/a^3/(-a*b)^(1/2)*\exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a* \\
& b)^(1/2)-(d*x+c)*b+c*b)/b)*b+1/2*d/a^3*\exp(-c)*Ei(1,d*x)-1/16*\exp(d*x+c)/a^ \\
& 2*x^2*d^5/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-15/16*\exp(d*x+c)/a^3*x^3*d^ \\
& 4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-1/16*\exp(d*x+c)/a*d^5/(b^2*d^4*x^ \\
& 4+2*a*b*d^4*x^2+a^2*d^4)-25/16*\exp(d*x+c)/a^2*x*d^4/(b^2*d^4*x^4+2*a*b*d^4* \\
& x^2+a^2*d^4)*b-1/2*\exp(d*x+c)/a/x*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-1 \\
& /32/a^2*d^2/(-a*b)^(1/2)*\exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-( \\
& d*x+c)*b+c*b)/b)+1/32/a^2*d^2/(-a*b)^(1/2)*\exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei( \\
& 1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)-7/32*d/a^3*\exp((d*(-a*b)^(1/2)+c*b)/b) \\
& *Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)-7/32*d/a^3*\exp(-(d*(-a*b)^(1/2)-c*b \\
& )/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b)/b)+15/32/a^3/(-a*b)^(1/2)*\exp((d* \\
& (-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-(d*x+c)*b+c*b)/b)*b-15/32/a^3/(-a \\
& *b)^(1/2)*\exp(-(d*(-a*b)^(1/2)-c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+(d*x+c)*b-c*b) \\
& /b)*b-1/2*d/a^3*\exp(c)*Ei(1,-d*x)
\end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x^2+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.17239, size = 4894, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x^2+a)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& -1/32*(4*(15*a*b^2*d*x^4 + 25*a^2*b*d*x^2 + 8*a^3*d)*\cosh(d*x + c) - ((7*(a \\
& *b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\cosh(d*x + c)^2 - 7*(a*b^2*d^2* \\
& x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\sinh(d*x + c)^2 - (((a*b^2*d^2 - 15*b^3) \\
& *x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*\cosh(d*x + c) \\
& ^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - \\
& 15*a^2*b)*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x - \sqrt{-a*d^2/b}) - (7 \\
& *(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\cosh(d*x + c)^2 - 7*(a*b^2*d^2 \\
& ^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\sinh(d*x + c)^2 + (((a*b^2*d^2 - 15*b \\
& ^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*\cosh(d*x + \\
& c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 \\
& - 15*a^2*b)*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x + \sqrt{-a*d^2/b})) \\
& *\cosh(c + \sqrt{-a*d^2/b}) - 16*((a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x) \\
& * \text{Ei}(d*x) - (a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\text{Ei}(-d*x))*\cosh(c \\
& ) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\cosh(d*x + c)^2 - 7*( \\
& a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\sinh(d*x + c)^2 + (((a*b^2*d^2 \\
& - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*\cos \\
& h(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + ( \\
& a^3*d^2 - 15*a^2*b)*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x + \sqrt{-a*d^ \\
& 2/b}) - (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\cosh(d*x + c)^2 - \\
& 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\sinh(d*x + c)^2 - (((a*b^2* \\
& d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)* \\
& \cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 \\
& + (a^3*d^2 - 15*a^2*b)*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x - \sqrt{- \\
& a*d^2/b}))*\cosh(-c + \sqrt{-a*d^2/b}) + 4*(a^2*b*d^2*x^3 + a^3*d^2*x)*\sinh(d \\
& *x + c) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\cosh(d*x + c)^2 \\
& - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\sinh(d*x + c)^2 - (((a*b \\
& ^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)* \\
& x)*\cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x \\
& ^3 + (a^3*d^2 - 15*a^2*b)*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(d*x - \sqrt{ \\
& -a*d^2/b}) + (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\cosh(d*x + c \\
& )^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\sinh(d*x + c)^2 + ((( \\
& a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2* \\
& b)*x)*\cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2 \\
& )*x^3 + (a^3*d^2 - 15*a^2*b)*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei}(-d*x + \\
& \sqrt{-a*d^2/b}))*\sinh(c + \sqrt{-a*d^2/b}) - 16*((a*b^2*d^2*x^5 + 2*a^2*b*d^ \\
& 2*x^3 + a^3*d^2*x)*\text{Ei}(d*x) + (a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)* \\
& \text{Ei}(-d*x))*\sinh(c) + ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\cosh( \\
& d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\sinh(d*x + c)^ \\
& 2 + (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - \\
& 15*a^2*b)*x)*\cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 1 \\
& 5*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*\sinh(d*x + c)^2)*\sqrt{-a*d^2/b})*\text{Ei} \\
& (d*x + \sqrt{-a*d^2/b}) + (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\co \\
& sh(d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*\sinh(d*x + \\
& c)^2 - (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 \\
& - 15*a^2*b)*x)*\cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2
\end{aligned}$$



$$- 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*\sinh(d*x + c)^2*\sqrt{-a*d^2/b))*\operatorname{Ei}(-d*x - \sqrt{-a*d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))/((a^4*b^2*d*x^5 + 2*a^5*b*d*x^3 + a^6*d*x)*\cosh(d*x + c)^2 - (a^4*b^2*d*x^5 + 2*a^5*b*d*x^3 + a^6*d*x)*\sinh(d*x + c)^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x\*\*2/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x^2+a)^3,x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((b\*x^2 + a)^3\*x^2), x)

$$3.78 \quad \int \frac{\cosh(c+dx)}{x^3(a+bx^2)^3} dx$$

**Optimal.** Leaf size=791

result too large to display

```
[Out] -Cosh[c + d*x]/(2*a^3*x^2) - (b*Cosh[c + d*x])/(4*a^2*(a + b*x^2)^2) - (b*Cosh[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a^3) + (3*b*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) - (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (3*b*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3) + (9*Sqrt[b]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) - (9*Sqrt[b]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) - (d*Sinh[c + d*x])/(2*a^3*x) - (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)) + (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) - (3*b*Sinh[c]*SinhIntegral[d*x])/a^4 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a^3) + (9*Sqrt[b]*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (3*b*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) + (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (9*Sqrt[b]*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) + (3*b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3)
```

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**Rubi [A]** time = 1.86841, antiderivative size = 791, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {5293, 3297, 3303, 3298, 3301, 5289, 5280}

$$\frac{d^2 \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right) \text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}} - dx\right)}{16a^3} - \frac{d^2 \cosh\left(c - \frac{\sqrt{-ad}}{\sqrt{b}}\right) \text{Chi}\left(xd + \frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16a^3} - \frac{3b \cosh(c) \text{Chi}(dx)}{a^4} + \frac{3b \cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}} + c\right)}{2a^4}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x^3\*(a + b\*x^2)^3), x]

```
[Out] -Cosh[c + d*x]/(2*a^3*x^2) - (b*Cosh[c + d*x])/(4*a^2*(a + b*x^2)^2) - (b*Cosh[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a^3) + (3*b*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) - (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (3*b*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3) + (9*Sqrt[b]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) - (9*Sqrt[b]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) - (d*Sinh[c + d*x])/(2*a^3*x) - (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*x)) + (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) - (3*b*Sinh[c]*SinhIntegral[d*x])/a^4 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a^3) + (9*Sqrt[b]*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (3*b*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) + (d^2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (9*Sqrt[b]*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) + (3*b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4) - (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3)
```

### Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol]
:> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x]/(b*n*(p + 1)), x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

### Rule 5280

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Int[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^3(a+bx^2)^3} dx &= \int \left( \frac{\cosh(c+dx)}{a^3x^3} - \frac{3b \cosh(c+dx)}{a^4x} + \frac{b^2x \cosh(c+dx)}{a^2(a+bx^2)^3} + \frac{2b^2x \cosh(c+dx)}{a^3(a+bx^2)^2} + \frac{3b^2x \cosh(c+dx)}{a^4(a+bx^2)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{a^3} - \frac{(3b) \int \frac{\cosh(c+dx)}{x} dx}{a^4} + \frac{(3b^2) \int \frac{x \cosh(c+dx)}{a+bx^2} dx}{a^4} + \frac{(2b^2) \int \frac{x \cosh(c+dx)}{(a+bx^2)^2} dx}{a^3} + \frac{b^2 \int \frac{x \cosh(c+dx)}{(a+bx^2)} dx}{a^4} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b \cosh(c+dx)}{4a^2(a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3(a+bx^2)} + \frac{(3b^2) \int \left( -\frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}-\sqrt{bx})} + \frac{\cosh(c+dx)}{2\sqrt{b}(\sqrt{-a}+\sqrt{bx})} \right) dx}{a^4} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b \cosh(c+dx)}{4a^2(a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3(a+bx^2)} - \frac{3b \cosh(c)\text{Chi}(dx)}{a^4} - \frac{d \sinh(c+dx)}{2a^3x} - \frac{3b \sinh(c)\text{Chi}(dx)}{a^4} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b \cosh(c+dx)}{4a^2(a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3(a+bx^2)} - \frac{3b \cosh(c)\text{Chi}(dx)}{a^4} - \frac{d \sinh(c+dx)}{2a^3x} - \frac{3b \sinh(c)\text{Chi}(dx)}{a^4} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b \cosh(c+dx)}{4a^2(a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3(a+bx^2)} - \frac{3b \cosh(c)\text{Chi}(dx)}{a^4} + \frac{d^2 \cosh(c)\text{Chi}(dx)}{2a^3} + \frac{d \sinh(c)\text{Chi}(dx)}{2a^3} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b \cosh(c+dx)}{4a^2(a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3(a+bx^2)} - \frac{3b \cosh(c)\text{Chi}(dx)}{a^4} + \frac{d^2 \cosh(c)\text{Chi}(dx)}{2a^3} + \frac{d \sinh(c)\text{Chi}(dx)}{2a^3} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b \cosh(c+dx)}{4a^2(a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3(a+bx^2)} - \frac{3b \cosh(c)\text{Chi}(dx)}{a^4} + \frac{d^2 \cosh(c)\text{Chi}(dx)}{2a^3} + \frac{d \sinh(c)\text{Chi}(dx)}{2a^3} \\
&= -\frac{\cosh(c+dx)}{2a^3x^2} - \frac{b \cosh(c+dx)}{4a^2(a+bx^2)^2} - \frac{b \cosh(c+dx)}{a^3(a+bx^2)} - \frac{3b \cosh(c)\text{Chi}(dx)}{a^4} + \frac{d^2 \cosh(c)\text{Chi}(dx)}{2a^3} + \frac{d \sinh(c)\text{Chi}(dx)}{2a^3}
\end{aligned}$$

**Mathematica [C]** time = 3.71688, size = 998, normalized size = 1.26

$$-ia \sinh(c) \left( \text{CosIntegral} \left( idx - \frac{\sqrt{ad}}{\sqrt{b}} \right) \sin \left( \frac{\sqrt{ad}}{\sqrt{b}} \right) - \text{CosIntegral} \left( idx + \frac{\sqrt{ad}}{\sqrt{b}} \right) \sin \left( \frac{\sqrt{ad}}{\sqrt{b}} \right) + \cos \left( \frac{\sqrt{ad}}{\sqrt{b}} \right) \left( \text{Si} \left( idx + \frac{\sqrt{ad}}{\sqrt{b}} \right) - \text{Si} \left( idx - \frac{\sqrt{ad}}{\sqrt{b}} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d\*x]/(x^3\*(a + b\*x^2)^3), x]

[Out] 
$$-\left(\frac{2a \operatorname{Cosh}[d*x] \left(2(2a^2 + 9abx^2 + 6b^2x^4) \operatorname{Cosh}[c] + dx(4a^2 + 7abx^2 + 3b^2x^4) \operatorname{Sinh}[c]\right)}{x^2(a + bx^2)^2} + \frac{2a(dx(4a^2 + 7abx^2 + 3b^2x^4) \operatorname{Cosh}[c] + 2(2a^2 + 9abx^2 + 6b^2x^4) \operatorname{Sinh}[c]) \operatorname{Sinh}[d*x]}{x^2(a + bx^2)^2} + 8(6b - ad^2) \left(\operatorname{Cosh}[c] \operatorname{CoshIntegral}[d*x] + \operatorname{Sinh}[c] \operatorname{SinhIntegral}[d*x]\right) - (9I) \sqrt{a} \sqrt{b} dx \operatorname{Sinh}[c] \left(\operatorname{Cos}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[-\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) + Id*x\right] - \operatorname{Cos}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right] + \operatorname{Sin}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Id*x\right] - \operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right]\right)\right) + (24I) b \operatorname{Sinh}[c] \left(\operatorname{CosIntegral}\left[-\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) + Id*x\right] \operatorname{Sin}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] - \operatorname{CosIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right] \operatorname{Sin}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] + \operatorname{Cos}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \left(-\operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Id*x\right] + \operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right]\right) - Id^2 \operatorname{Sinh}[c] \left(\operatorname{CosIntegral}\left[-\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) + Id*x\right] \operatorname{Sin}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] - \operatorname{CosIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right] \operatorname{Sin}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] + \operatorname{Cos}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \left(-\operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Id*x\right] + \operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right]\right) - 9 \sqrt{a} \sqrt{b} dx \operatorname{Cosh}[c] \left(\operatorname{CosIntegral}\left[-\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) + Id*x\right] \operatorname{Sin}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] + \operatorname{CosIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right] \operatorname{Sin}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] - \operatorname{Cos}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Id*x\right] + \operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right]\right) - 24b \operatorname{Cosh}[c] \left(\operatorname{Cos}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[-\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) + Id*x\right] + \operatorname{Cos}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right] + \operatorname{Sin}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Id*x\right] + \operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right]\right) + ad^2 \operatorname{Cosh}[c] \left(\operatorname{Cos}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[-\left(\frac{\sqrt{a}d}{\sqrt{b}}\right) + Id*x\right] + \operatorname{Cos}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \operatorname{CosIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right] + \operatorname{Sin}\left[\frac{\sqrt{a}d}{\sqrt{b}}\right] \left(\operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} - Id*x\right] + \operatorname{SinIntegral}\left[\frac{\sqrt{a}d}{\sqrt{b}} + Id*x\right]\right)\right)\right) / (16a^4)$$

---

**Maple [B]** time = 0.152, size = 1294, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/x^3/(b\*x^2+a)^3, x)

[Out] 
$$\frac{3}{16} d^5 \exp(-dx-c) / a^3 x^3 / (b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4) b^2 + 7/16 d^5 \exp(-dx-c) / a^2 x / (b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4) b^{-3/4} d^4 \exp(-dx-c) / a^3 x^2 / (b^2 d^4 x^4 + 2ab d^4 x^2 + a^2 d^4) b^2 + 9/32 d / a^3 / (-ab)^{1/2} \exp(-(d(-ab)^{1/2} + cb)/b) \operatorname{Ei}\left(1, -(d(-ab)^{1/2} - (dx+c) * b + cb)/b\right) b^{-9/3}$$

$$2*d/a^3/(-a*b)^{(1/2)}*exp((d*(-a*b)^{(1/2)}-c*b)/b)*Ei(1,(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)*b+1/32*d^2/a^3*exp((d*(-a*b)^{(1/2)}+c*b)/b)*Ei(1,(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)+1/32*d^2/a^3*exp(-(d*(-a*b)^{(1/2)}-c*b)/b)*Ei(1,-(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)-3/4/a^4*exp((d*(-a*b)^{(1/2)}+c*b)/b)*Ei(1,(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)*b-3/4/a^4*exp(-(d*(-a*b)^{(1/2)}-c*b)/b)*Ei(1,-(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)*b-1/4*d^2/a^3*exp(-c)*Ei(1,d*x)-3/16*d^5*exp(d*x+c)/a^3*x^3/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-7/16*d^5*exp(d*x+c)/a^2*x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^3/4*d^4*exp(d*x+c)/a^3*x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-9/32*d/a^3/(-a*b)^{(1/2)}*exp((d*(-a*b)^{(1/2)}+c*b)/b)*Ei(1,(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)*b+3/2/a^4*b*exp(c)*Ei(1,-d*x)+9/32*d/a^3/(-a*b)^{(1/2)}*exp(-(d*(-a*b)^{(1/2)}-c*b)/b)*Ei(1,-(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)*b+1/32*d^2/a^3*exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*Ei(1,-(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)+1/32*d^2/a^3*exp((d*(-a*b)^{(1/2)}-c*b)/b)*Ei(1,(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)-3/4/a^4*exp(-(d*(-a*b)^{(1/2)}+c*b)/b)*Ei(1,-(d*(-a*b)^{(1/2)}-(d*x+c)*b+c*b)/b)*b-3/4/a^4*exp((d*(-a*b)^{(1/2)}-c*b)/b)*Ei(1,(d*(-a*b)^{(1/2)}+(d*x+c)*b-c*b)/b)*b+1/4*d^5*exp(-d*x-c)/a/x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-9/8*d^4*exp(-d*x-c)/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-1/4*d^4*exp(-d*x-c)/a/x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+3/2/a^4*exp(-c)*Ei(1,d*x)*b-1/4*d^2/a^3*exp(c)*Ei(1,-d*x)-1/4*d^5*exp(d*x+c)/a/x/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-9/8*d^4*exp(d*x+c)/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-1/4*d^4*exp(d*x+c)/a/x^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^3/(b\*x^2+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.20699, size = 4891, normalized size = 6.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^3/(b\*x^2+a)^3,x, algorithm="fricas")

```
[Out] -1/32*(8*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)*cosh(d*x + c) + (((a*b^2*d^2
- 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*co
sh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 +
(a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 + 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2
*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)
^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (((a*b^2*d^2 - 24*b^3)*x^6 +
2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 -
((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a
^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*
x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2
/b))*Ei(-d*x + sqrt(-a*d^2/b))*cosh(c + sqrt(-a*d^2/b)) - 8*(((a*b^2*d^2 -
6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a^2*b)*x^2)*Ei(d*x
) + ((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a
^2*b)*x^2)*Ei(-d*x))*cosh(c) + (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 -
24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 -
24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh
(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3
*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + s
qrt(-a*d^2/b)) + (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4
+ (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2
*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 + 9
*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*
x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b))
)*cosh(-c + sqrt(-a*d^2/b)) + 4*(3*a*b^2*d*x^5 + 7*a^2*b*d*x^3 + 4*a^3*d*x)
*sinh(d*x + c) + (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4
+ (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 +
2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 +
9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2
*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b))
- (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 2
4*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 -
24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2
*a*b^2*x^4 + a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^
2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b))*sinh(c + sqr
t(-a*d^2/b)) - 8*(((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 +
(a^3*d^2 - 6*a^2*b)*x^2)*Ei(d*x) - ((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2
- 6*a*b^2)*x^4 + (a^3*d^2 - 6*a^2*b)*x^2)*Ei(-d*x))*sinh(c) - (((a*b^2*d^2
- 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*c
osh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 +
(a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^
2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c
)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (((a*b^2*d^2 - 24*b^3)*x^6
+ 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2
- ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*
a^2*b)*x^2)*sinh(d*x + c)^2 + 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(d
```



$$*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*\sinh(d*x + c)^2*\sqrt{-a*d^2/b))*\text{Ei}(-d*x - \sqrt{-a*d^2/b}))*\sinh(-c + \sqrt{-a*d^2/b}))/((a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)*\cosh(d*x + c)^2 - (a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)*\sinh(d*x + c)^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x\*\*3/(b\*x\*\*2+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^2 + a)^3 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^3/(b\*x^2+a)^3,x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((b\*x^2 + a)^3\*x^3), x)

### 3.79 $\int x^3 (a + bx^3) \cosh(c + dx) dx$

**Optimal.** Leaf size=154

$$-\frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{6a \cosh(c + dx)}{d^4} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{30bx^4 \sinh(c + dx)}{d^3} + \frac{360bx^2 \sinh(c + dx)}{d^5}$$

[Out]  $(-6*a*Cosh[c + d*x])/d^4 - (720*b*x*Cosh[c + d*x])/d^6 - (3*a*x^2*Cosh[c + d*x])/d^2 - (120*b*x^3*Cosh[c + d*x])/d^4 - (6*b*x^5*Cosh[c + d*x])/d^2 + (720*b*Sinh[c + d*x])/d^7 + (6*a*x*Sinh[c + d*x])/d^3 + (360*b*x^2*Sinh[c + d*x])/d^5 + (a*x^3*Sinh[c + d*x])/d + (30*b*x^4*Sinh[c + d*x])/d^3 + (b*x^6*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.297123, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {5287, 3296, 2638, 2637}

$$-\frac{3ax^2 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} - \frac{6a \cosh(c + dx)}{d^4} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{30bx^4 \sinh(c + dx)}{d^3} + \frac{360bx^2 \sinh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(a + b*x^3)*Cosh[c + d*x], x]$

[Out]  $(-6*a*Cosh[c + d*x])/d^4 - (720*b*x*Cosh[c + d*x])/d^6 - (3*a*x^2*Cosh[c + d*x])/d^2 - (120*b*x^3*Cosh[c + d*x])/d^4 - (6*b*x^5*Cosh[c + d*x])/d^2 + (720*b*Sinh[c + d*x])/d^7 + (6*a*x*Sinh[c + d*x])/d^3 + (360*b*x^2*Sinh[c + d*x])/d^5 + (a*x^3*Sinh[c + d*x])/d + (30*b*x^4*Sinh[c + d*x])/d^3 + (b*x^6*Sinh[c + d*x])/d$

#### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m, n\}, x$  &&  $\text{IGtQ}[p, 0]$

#### Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /;$   $\text{FreeQ}\{c, d, e, f\}, x$  &&  $\text{GtQ}[m, 0]$

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 (a + bx^3) \cosh(c + dx) dx &= \int (ax^3 \cosh(c + dx) + bx^6 \cosh(c + dx)) dx \\
&= a \int x^3 \cosh(c + dx) dx + b \int x^6 \cosh(c + dx) dx \\
&= \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^6 \sinh(c + dx)}{d} - \frac{(3a) \int x^2 \sinh(c + dx) dx}{d} - \frac{(6b) \int x^5 \sinh(c + dx) dx}{d} \\
&= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{6bx^5 \cosh(c + dx)}{d^2} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{bx^6 \sinh(c + dx)}{d} + \frac{3a}{d} \\
&= -\frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{6bx^5 \cosh(c + dx)}{d^2} + \frac{6ax \sinh(c + dx)}{d^3} + \frac{ax^3 \sinh(c + dx)}{d} + \frac{3a}{d} \\
&= -\frac{6a \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx)}{d^4} - \frac{6bx^5 \cosh(c + dx)}{d^2} + \frac{3a}{d} \\
&= -\frac{6a \cosh(c + dx)}{d^4} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx)}{d^4} - \frac{6bx^5 \cosh(c + dx)}{d^2} + \frac{3a}{d} \\
&= -\frac{6a \cosh(c + dx)}{d^4} - \frac{720bx \cosh(c + dx)}{d^6} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx)}{d^4} + \frac{3a}{d} \\
&= -\frac{6a \cosh(c + dx)}{d^4} - \frac{720bx \cosh(c + dx)}{d^6} - \frac{3ax^2 \cosh(c + dx)}{d^2} - \frac{120bx^3 \cosh(c + dx)}{d^4} + \frac{3a}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.160777, size = 100, normalized size = 0.65

$$\frac{(ad^4x(d^2x^2 + 6) + b(d^6x^6 + 30d^4x^4 + 360d^2x^2 + 720)) \sinh(c + dx) - 3d(ad^2(d^2x^2 + 2) + 2bx(d^4x^4 + 20d^2x^2 + 120))}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)*Cosh[c + d*x], x]
```

```
[Out] (-3*d*(a*d^2*(2 + d^2*x^2) + 2*b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c + d
*x] + (a*d^4*x*(6 + d^2*x^2) + b*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6))
```

) \* Sinh[c + d\*x]) / d^7

**Maple [B]** time = 0.008, size = 551, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^3+a)\*cosh(d\*x+c), x)

[Out]  $\frac{1}{d^4} \left( \frac{1}{d^3} b \left( (d*x+c)^6 \sinh(d*x+c) - 6(d*x+c)^5 \cosh(d*x+c) + 30(d*x+c)^4 \sinh(d*x+c) - 120(d*x+c)^3 \cosh(d*x+c) + 360(d*x+c)^2 \sinh(d*x+c) - 720(d*x+c) \cosh(d*x+c) + 720 \sinh(d*x+c) \right) - \frac{6}{d^3} b c \left( (d*x+c)^5 \sinh(d*x+c) - 5(d*x+c)^4 \cosh(d*x+c) + 20(d*x+c)^3 \sinh(d*x+c) - 60(d*x+c)^2 \cosh(d*x+c) + 120(d*x+c) \sinh(d*x+c) - 120 \cosh(d*x+c) \right) + \frac{15}{d^3} b c^2 \left( (d*x+c)^4 \sinh(d*x+c) - 4(d*x+c)^3 \cosh(d*x+c) + 12(d*x+c)^2 \sinh(d*x+c) - 24(d*x+c) \cosh(d*x+c) + 24 \sinh(d*x+c) \right) - \frac{20}{d^3} b c^3 \left( (d*x+c)^3 \sinh(d*x+c) - 3(d*x+c)^2 \cosh(d*x+c) + 6(d*x+c) \sinh(d*x+c) - 6 \cosh(d*x+c) \right) + \frac{15}{d^3} b c^4 \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) - \frac{6}{d^3} b c^5 \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) + \frac{1}{d^3} b c^6 \sinh(d*x+c) + a \left( (d*x+c)^3 \sinh(d*x+c) - 3(d*x+c)^2 \cosh(d*x+c) + 6(d*x+c) \sinh(d*x+c) - 6 \cosh(d*x+c) \right) - 3 a c \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) + 3 a c^2 \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) - a c^3 \sinh(d*x+c) \right)$

**Maxima [A]** time = 1.04636, size = 362, normalized size = 2.35

$$-\frac{1}{56} d \left( \frac{7(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 24 d x e^c + 24 e^c) a e^{(d x)}}{d^5} + \frac{7(d^4 x^4 + 4 d^3 x^3 + 12 d^2 x^2 + 24 d x + 24) a e^{(-d x - c)}}{d^5} + \frac{4(d^7 x^7 e^c - 7 d^6 x^6 e^c + 42 d^5 x^5 e^c - 210 d^4 x^4 e^c + 840 d^3 x^3 e^c - 2520 d^2 x^2 e^c + 5040 d x e^c - 5040 e^c) b e^{(d x)}}{d^8} + \frac{4(d^7 x^7 + 7 d^6 x^6 + 42 d^5 x^5 + 210 d^4 x^4 + 840 d^3 x^3 + 2520 d^2 x^2 + 5040 d x + 5040) b e^{(-d x - c)}}{d^8} + \frac{1}{28} (4 b x^7 + \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x^3+a)\*cosh(d\*x+c), x, algorithm="maxima")

[Out] 
$$-\frac{1}{56} d \left( \frac{7(d^4 x^4 e^c - 4 d^3 x^3 e^c + 12 d^2 x^2 e^c - 24 d x e^c + 24 e^c) a e^{(d x)}}{d^5} + \frac{7(d^4 x^4 + 4 d^3 x^3 + 12 d^2 x^2 + 24 d x + 24) a e^{(-d x - c)}}{d^5} + \frac{4(d^7 x^7 e^c - 7 d^6 x^6 e^c + 42 d^5 x^5 e^c - 210 d^4 x^4 e^c + 840 d^3 x^3 e^c - 2520 d^2 x^2 e^c + 5040 d x e^c - 5040 e^c) b e^{(d x)}}{d^8} + \frac{4(d^7 x^7 + 7 d^6 x^6 + 42 d^5 x^5 + 210 d^4 x^4 + 840 d^3 x^3 + 2520 d^2 x^2 + 5040 d x + 5040) b e^{(-d x - c)}}{d^8} + \frac{1}{28} (4 b x^7 + \right)$$

$7*a*x^4)*\cosh(d*x + c)$

**Fricas [A]** time = 1.76403, size = 240, normalized size = 1.56

$$\frac{3(2bd^5x^5 + ad^5x^2 + 40bd^3x^3 + 2ad^3 + 240bdx)\cosh(dx + c) - (bd^6x^6 + ad^6x^3 + 30bd^4x^4 + 6ad^4x + 360bd^2x^2 + 720b)\sinh(dx + c)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x^3+a)\*cosh(d\*x+c),x, algorithm="fricas")

[Out]  $-(3*(2*b*d^5*x^5 + a*d^5*x^2 + 40*b*d^3*x^3 + 2*a*d^3 + 240*b*d*x)*\cosh(d*x + c) - (b*d^6*x^6 + a*d^6*x^3 + 30*b*d^4*x^4 + 6*a*d^4*x + 360*b*d^2*x^2 + 720*b)*\sinh(d*x + c))/d^7$

**Sympy [A]** time = 9.67527, size = 185, normalized size = 1.2

$$\left\{ \frac{ax^3 \sinh(c+dx)}{d} - \frac{3ax^2 \cosh(c+dx)}{d^2} + \frac{6ax \sinh(c+dx)}{d^3} - \frac{6a \cosh(c+dx)}{d^4} + \frac{bx^6 \sinh(c+dx)}{d} - \frac{6bx^5 \cosh(c+dx)}{d^2} + \frac{30bx^4 \sinh(c+dx)}{d^3} - \frac{120bx^3 \cosh(c+dx)}{d^4} \right\} \left( \frac{ax^4}{4} + \frac{bx^7}{7} \right) \cosh(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(b\*x\*\*3+a)\*cosh(d\*x+c),x)

[Out] Piecewise((a\*x\*\*3\*sinh(c + d\*x)/d - 3\*a\*x\*\*2\*cosh(c + d\*x)/d\*\*2 + 6\*a\*x\*sinh(c + d\*x)/d\*\*3 - 6\*a\*cosh(c + d\*x)/d\*\*4 + b\*x\*\*6\*sinh(c + d\*x)/d - 6\*b\*x\*\*5\*cosh(c + d\*x)/d\*\*2 + 30\*b\*x\*\*4\*sinh(c + d\*x)/d\*\*3 - 120\*b\*x\*\*3\*cosh(c + d\*x)/d\*\*4 + 360\*b\*x\*\*2\*sinh(c + d\*x)/d\*\*5 - 720\*b\*x\*cosh(c + d\*x)/d\*\*6 + 720\*b\*sinh(c + d\*x)/d\*\*7, Ne(d, 0)), ((a\*x\*\*4/4 + b\*x\*\*7/7)\*cosh(c), True))

**Giac [A]** time = 1.14007, size = 259, normalized size = 1.68

$$\frac{(bd^6x^6 - 6bd^5x^5 + ad^6x^3 + 30bd^4x^4 - 3ad^5x^2 - 120bd^3x^3 + 6ad^4x + 360bd^2x^2 - 6ad^3 - 720bdx + 720b)e^{(dx+c)}}{2d^7} - (b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^6*x^6 - 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 - 3*a*d^5*x^2 - 120
*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 - 6*a*d^3 - 720*b*d*x + 720*b)*e^(d*
x + c)/d^7 - 1/2*(b*d^6*x^6 + 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 + 3*a*
d^5*x^2 + 120*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 + 6*a*d^3 + 720*b*d*x +
720*b)*e^(-d*x - c)/d^7
```

### 3.80 $\int x^2 (a + bx^3) \cosh(c + dx) dx$

**Optimal.** Leaf size=124

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{20bx^3 \sinh(c + dx)}{d^3} - \frac{5bx^4 \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4}$$

[Out]  $(-120*b*Cosh[c + d*x])/d^6 - (2*a*x*Cosh[c + d*x])/d^2 - (60*b*x^2*Cosh[c + d*x])/d^4 - (5*b*x^4*Cosh[c + d*x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (120*b*x*Sinh[c + d*x])/d^5 + (a*x^2*Sinh[c + d*x])/d + (20*b*x^3*Sinh[c + d*x])/d^3 + (b*x^5*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.228484, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {5287, 3296, 2637, 2638}

$$\frac{2a \sinh(c + dx)}{d^3} - \frac{2ax \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{20bx^3 \sinh(c + dx)}{d^3} - \frac{5bx^4 \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*x^3)*Cosh[c + d*x], x]$

[Out]  $(-120*b*Cosh[c + d*x])/d^6 - (2*a*x*Cosh[c + d*x])/d^2 - (60*b*x^2*Cosh[c + d*x])/d^4 - (5*b*x^4*Cosh[c + d*x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (120*b*x*Sinh[c + d*x])/d^5 + (a*x^2*Sinh[c + d*x])/d + (20*b*x^3*Sinh[c + d*x])/d^3 + (b*x^5*Sinh[c + d*x])/d$

#### Rule 5287

$\text{Int}[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_))^{(n_)}]^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x \ \&\& \text{IGtQ}[p, 0]$

#### Rule 3296

$\text{Int}[((c_.) + (d_.)*(x_))^{(m_.)}*\sin[(e_.) + (f_.)*(x_)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \text{GtQ}[m, 0]$

#### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int x^2 (a + bx^3) \cosh(c + dx) dx &= \int (ax^2 \cosh(c + dx) + bx^5 \cosh(c + dx)) dx \\
 &= a \int x^2 \cosh(c + dx) dx + b \int x^5 \cosh(c + dx) dx \\
 &= \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^5 \sinh(c + dx)}{d} - \frac{(2a) \int x \sinh(c + dx) dx}{d} - \frac{(5b) \int x^4 \sinh(c + dx) dx}{d} \\
 &= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{bx^5 \sinh(c + dx)}{d} + \frac{(2a)}{d} \\
 &= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{ax^2 \sinh(c + dx)}{d} + \frac{20bx}{d} \\
 &= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{ax}{d} \\
 &= -\frac{2ax \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{12bx}{d} \\
 &= -\frac{120b \cosh(c + dx)}{d^6} - \frac{2ax \cosh(c + dx)}{d^2} - \frac{60bx^2 \cosh(c + dx)}{d^4} - \frac{5bx^4 \cosh(c + dx)}{d^2} + \frac{2a \sinh(c + dx)}{d^3} + \frac{12bx}{d}
 \end{aligned}$$

**Mathematica [A]** time = 0.133114, size = 84, normalized size = 0.68

$$\frac{d(ad^2(d^2x^2 + 2) + bx(d^4x^4 + 20d^2x^2 + 120)) \sinh(c + dx) - (2ad^4x + 5b(d^4x^4 + 12d^2x^2 + 24)) \cosh(c + dx)}{d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*Cosh[c + d*x],x]
```

```
[Out] (-((2*a*d^4*x + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x]) + d*(a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6
```



**Maple [B]** time = 0.007, size = 389, normalized size = 3.1

$$\frac{1}{d^3} \left( \frac{b((dx+c)^5 \sinh(dx+c) - 5(dx+c)^4 \cosh(dx+c) + 20(dx+c)^3 \sinh(dx+c) - 60(dx+c)^2 \cosh(dx+c) + 120(dx+c) \sinh(dx+c) - 60 \cosh(dx+c) + 20 \sinh(dx+c) - 5 \cosh(dx+c) + 2 \sinh(dx+c) - \cosh(dx+c) + a \cosh(dx+c))}{d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^3+a)\*cosh(d\*x+c), x)

[Out]  $\frac{1}{d^3} \left( \frac{1}{d^3} b \left( (dx+c)^5 \sinh(dx+c) - 5(dx+c)^4 \cosh(dx+c) + 20(dx+c)^3 \sinh(dx+c) - 60(dx+c)^2 \cosh(dx+c) + 120(dx+c) \sinh(dx+c) - 60 \cosh(dx+c) + 20 \sinh(dx+c) - 5 \cosh(dx+c) + 2 \sinh(dx+c) - \cosh(dx+c) + a \cosh(dx+c) \right) - \frac{5}{d^3} b \left( (dx+c)^4 \sinh(dx+c) - 4(dx+c)^3 \cosh(dx+c) + 12(dx+c)^2 \sinh(dx+c) - 24(dx+c) \cosh(dx+c) + 24 \sinh(dx+c) \right) + \frac{10}{d^3} b \left( (dx+c)^3 \sinh(dx+c) - 3(dx+c)^2 \cosh(dx+c) + 6(dx+c) \sinh(dx+c) - 6 \cosh(dx+c) \right) - \frac{10}{d^3} b \left( (dx+c)^2 \sinh(dx+c) - 2(dx+c) \cosh(dx+c) + 2 \sinh(dx+c) \right) + \frac{5}{d^3} b \left( (dx+c) \sinh(dx+c) - \cosh(dx+c) \right) - \frac{1}{d^3} b \left( \sinh(dx+c) + a \cosh(dx+c) \right) \right)$

**Maxima [B]** time = 1.04248, size = 360, normalized size = 2.9

$$\frac{(bx^3 + a)^2 \cosh(dx + c)}{6b} - \left( \frac{a^2 e^{(dx+c)}}{d} + \frac{a^2 e^{(-dx-c)}}{d} + \frac{2(d^3 x^3 e^c - 3d^2 x^2 e^c + 6dx e^c - 6e^c) a b e^{(dx)}}{d^4} + \frac{2(d^3 x^3 + 3d^2 x^2 + 6dx + 6) a b e^{(-dx-c)}}{d^4} + \frac{(d^6 x^6 e^c - 6d^5 x^5 e^c + 30d^4 x^4 e^c - 120d^3 x^3 e^c + 360d^2 x^2 e^c - 720d x e^c + 720e^c) b^2 e^{(dx)}}{d^7} + \frac{(d^6 x^6 + 6d^5 x^5 + 30d^4 x^4 + 120d^3 x^3 + 360d^2 x^2 + 720dx + 720) b^2 e^{(-dx-c)}}{d^7} \right) \frac{d}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^3+a)\*cosh(d\*x+c), x, algorithm="maxima")

[Out]  $\frac{1}{6} (bx^3 + a)^2 \cosh(dx + c) / b - \frac{1}{12} (a^2 e^{(dx+c)} / d + a^2 e^{(-dx-c)} / d + 2(d^3 x^3 e^c - 3d^2 x^2 e^c + 6dx e^c - 6e^c) a b e^{(dx)} / d^4 + 2(d^3 x^3 + 3d^2 x^2 + 6dx + 6) a b e^{(-dx-c)} / d^4 + (d^6 x^6 e^c - 6d^5 x^5 e^c + 30d^4 x^4 e^c - 120d^3 x^3 e^c + 360d^2 x^2 e^c - 720d x e^c + 720e^c) b^2 e^{(dx)} / d^7 + (d^6 x^6 + 6d^5 x^5 + 30d^4 x^4 + 120d^3 x^3 + 360d^2 x^2 + 720dx + 720) b^2 e^{(-dx-c)} / d^7) \frac{d}{b}$

**Fricas [A]** time = 1.73341, size = 200, normalized size = 1.61

$$\frac{(5bd^4x^4 + 2ad^4x + 60bd^2x^2 + 120b) \cosh(dx + c) - (bd^5x^5 + ad^5x^2 + 20bd^3x^3 + 2ad^3 + 120bdx) \sinh(dx + c)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^3+a)\*cosh(d\*x+c),x, algorithm="fricas")

[Out] -((5\*b\*d^4\*x^4 + 2\*a\*d^4\*x + 60\*b\*d^2\*x^2 + 120\*b)\*cosh(d\*x + c) - (b\*d^5\*x^5 + a\*d^5\*x^2 + 20\*b\*d^3\*x^3 + 2\*a\*d^3 + 120\*b\*d\*x)\*sinh(d\*x + c))/d^6

**Sympy [A]** time = 6.04663, size = 151, normalized size = 1.22

$$\left\{ \begin{array}{l} \frac{ax^2 \sinh(c+dx)}{d} - \frac{2ax \cosh(c+dx)}{d^2} + \frac{2a \sinh(c+dx)}{d^3} + \frac{bx^5 \sinh(c+dx)}{d} - \frac{5bx^4 \cosh(c+dx)}{d^2} + \frac{20bx^3 \sinh(c+dx)}{d^3} - \frac{60bx^2 \cosh(c+dx)}{d^4} + \frac{120bx \sinh(c+dx)}{d^5} \\ \left( \frac{ax^3}{3} + \frac{bx^6}{6} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*3+a)\*cosh(d\*x+c),x)

[Out] Piecewise((a\*x\*\*2\*sinh(c + d\*x)/d - 2\*a\*x\*cosh(c + d\*x)/d\*\*2 + 2\*a\*sinh(c + d\*x)/d\*\*3 + b\*x\*\*5\*sinh(c + d\*x)/d - 5\*b\*x\*\*4\*cosh(c + d\*x)/d\*\*2 + 20\*b\*x\*\*3\*sinh(c + d\*x)/d\*\*3 - 60\*b\*x\*\*2\*cosh(c + d\*x)/d\*\*4 + 120\*b\*x\*sinh(c + d\*x)/d\*\*5 - 120\*b\*cosh(c + d\*x)/d\*\*6, Ne(d, 0)), ((a\*x\*\*3/3 + b\*x\*\*6/6)\*cosh(c), True))

**Giac [A]** time = 1.18935, size = 211, normalized size = 1.7

$$\frac{(bd^5x^5 - 5bd^4x^4 + ad^5x^2 + 20bd^3x^3 - 2ad^4x - 60bd^2x^2 + 2ad^3 + 120bdx - 120b)e^{(dx+c)}}{2d^6} - \frac{(bd^5x^5 + 5bd^4x^4 + ad^5x^2 + 20bd^3x^3 - 2ad^4x - 60bd^2x^2 + 2ad^3 + 120bdx - 120b)e^{-(dx+c)}}{2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^3+a)\*cosh(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(b\*d^5\*x^5 - 5\*b\*d^4\*x^4 + a\*d^5\*x^2 + 20\*b\*d^3\*x^3 - 2\*a\*d^4\*x - 60\*b\*d^2\*x^2 + 2\*a\*d^3 + 120\*b\*d\*x - 120\*b)\*e^(d\*x + c)/d^6 - 1/2\*(b\*d^5\*x^5 + 5\*b\*d^4\*x^4 + a\*d^5\*x^2 + 20\*b\*d^3\*x^3 + 2\*a\*d^4\*x + 60\*b\*d^2\*x^2 + 2\*a\*d^3 + 120\*b\*d\*x + 120\*b)\*e^(-d\*x - c)/d^6

### 3.81 $\int x (a + bx^3) \cosh(c + dx) dx$

**Optimal.** Leaf size=94

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{24b \sinh(c + dx)}{d^5} - \frac{24bx \cosh(c + dx)}{d^4}$$

[Out]  $-\frac{(a \cosh[c + d*x])}{d^2} - \frac{(24*b*x*\cosh[c + d*x])}{d^4} - \frac{(4*b*x^3*\cosh[c + d*x])}{d^2} + \frac{(24*b*\sinh[c + d*x])}{d^5} + \frac{(a*x*\sinh[c + d*x])}{d} + \frac{(12*b*x^2*\sinh[c + d*x])}{d^3} + \frac{(b*x^4*\sinh[c + d*x])}{d}$

**Rubi [A]** time = 0.159169, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {5287, 3296, 2638, 2637}

$$-\frac{a \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{24b \sinh(c + dx)}{d^5} - \frac{24bx \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*x^3)*\text{Cosh}[c + d*x], x]$

[Out]  $-\frac{(a \cosh[c + d*x])}{d^2} - \frac{(24*b*x*\cosh[c + d*x])}{d^4} - \frac{(4*b*x^3*\cosh[c + d*x])}{d^2} + \frac{(24*b*\sinh[c + d*x])}{d^5} + \frac{(a*x*\sinh[c + d*x])}{d} + \frac{(12*b*x^2*\sinh[c + d*x])}{d^3} + \frac{(b*x^4*\sinh[c + d*x])}{d}$

#### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;$  FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\cos[c + d*x]/d, x] /;$  FreeQ[{c, d}, x]

Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /;  
FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int x(a + bx^3) \cosh(c + dx) dx &= \int (ax \cosh(c + dx) + bx^4 \cosh(c + dx)) dx \\
 &= a \int x \cosh(c + dx) dx + b \int x^4 \cosh(c + dx) dx \\
 &= \frac{ax \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} - \frac{a \int \sinh(c + dx) dx}{d} - \frac{(4b) \int x^3 \sinh(c + dx) dx}{d} \\
 &= -\frac{a \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{bx^4 \sinh(c + dx)}{d} + \frac{(12b) \int x^2 \sinh(c + dx) dx}{d^3} \\
 &= -\frac{a \cosh(c + dx)}{d^2} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} + \frac{bx^4 \sinh(c + dx)}{d} \\
 &= -\frac{a \cosh(c + dx)}{d^2} - \frac{24bx \cosh(c + dx)}{d^4} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{ax \sinh(c + dx)}{d} + \frac{12bx^2 \sinh(c + dx)}{d^3} \\
 &= -\frac{a \cosh(c + dx)}{d^2} - \frac{24bx \cosh(c + dx)}{d^4} - \frac{4bx^3 \cosh(c + dx)}{d^2} + \frac{24b \sinh(c + dx)}{d^5} + \frac{ax \sinh(c + dx)}{d}
 \end{aligned}$$

**Mathematica [A]** time = 0.110483, size = 66, normalized size = 0.7

$$\frac{(ad^4x + b(d^4x^4 + 12d^2x^2 + 24)) \sinh(c + dx) - d(ad^2 + 4bx(d^2x^2 + 6)) \cosh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^3)\*Cosh[c + d\*x], x]

[Out]  $(-(d*(a*d^2 + 4*b*x*(6 + d^2*x^2))*Cosh[c + d*x]) + (a*d^4*x + b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5$

**Maple [B]** time = 0.007, size = 257, normalized size = 2.7

$$\frac{1}{d^2} \left( \frac{b((dx + c)^4 \sinh(dx + c) - 4(dx + c)^3 \cosh(dx + c) + 12(dx + c)^2 \sinh(dx + c) - 24(dx + c) \cosh(dx + c) + 24 \sinh(dx + c))}{d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^3+a)*cosh(d*x+c),x)`

[Out]  $\frac{1}{d^2} \left( \frac{1}{d^3} b \left( (d*x+c)^4 \sinh(d*x+c) - 4(d*x+c)^3 \cosh(d*x+c) + 12(d*x+c)^2 \sinh(d*x+c) - 24(d*x+c) \cosh(d*x+c) + 24 \sinh(d*x+c) \right) - \frac{4}{d^3} b * c \left( (d*x+c)^3 \sinh(d*x+c) - 3(d*x+c)^2 \cosh(d*x+c) + 6(d*x+c) \sinh(d*x+c) - 6 \cosh(d*x+c) \right) + \frac{6}{d^3} b * c^2 \left( (d*x+c)^2 \sinh(d*x+c) - 2(d*x+c) \cosh(d*x+c) + 2 \sinh(d*x+c) \right) - \frac{4}{d^3} b * c^3 \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) + a \left( (d*x+c) \sinh(d*x+c) - \cosh(d*x+c) \right) + \frac{1}{d^3} b * c^4 \sinh(d*x+c) - c * a \sinh(d*x+c) \right)$

**Maxima [B]** time = 1.04164, size = 265, normalized size = 2.82

$$-\frac{1}{20} d \left( \frac{5(d^2 x^2 e^c - 2 dx e^c + 2 e^c) a e^{(dx)}}{d^3} + \frac{5(d^2 x^2 + 2 dx + 2) a e^{(-dx-c)}}{d^3} + \frac{2(d^5 x^5 e^c - 5 d^4 x^4 e^c + 20 d^3 x^3 e^c - 60 d^2 x^2 e^c + 120 d x e^c - 120 e^c)}{d^6} + \frac{2(d^5 x^5 e^{-c} - 5 d^4 x^4 e^{-c} + 20 d^3 x^3 e^{-c} - 60 d^2 x^2 e^{-c} + 120 d x e^{-c} - 120 e^{-c}) b e^{(dx)}}{d^6} + \frac{2(d^5 x^5 + 5 d^4 x^4 + 20 d^3 x^3 + 60 d^2 x^2 + 120 d x + 120) b e^{(-dx-c)}}{d^6} + \frac{1}{10} (2 b x^5 + 5 a x^2) \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")`

[Out]  $-1/20*d*(5*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*e^{(d*x)}/d^3 + 5*(d^2*x^2 + 2*d*x + 2)*a*e^{(-d*x - c)}/d^3 + 2*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^{(d*x)}/d^6 + 2*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^{(-d*x - c)}/d^6 + 1/10*(2*b*x^5 + 5*a*x^2)*cosh(d*x + c)$

**Fricas [A]** time = 1.71885, size = 155, normalized size = 1.65

$$\frac{(4bd^3x^3 + ad^3 + 24bdx) \cosh(dx+c) - (bd^4x^4 + ad^4x + 12bd^2x^2 + 24b) \sinh(dx+c)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")`

[Out]  $-((4*b*d^3*x^3 + a*d^3 + 24*b*d*x)*cosh(d*x + c) - (b*d^4*x^4 + a*d^4*x + 12*b*d^2*x^2 + 24*b)*sinh(d*x + c))/d^5$

**Sympy [A]** time = 3.26191, size = 116, normalized size = 1.23

$$\begin{cases} \frac{ax \sinh(c+dx)}{d} - \frac{a \cosh(c+dx)}{d^2} + \frac{bx^4 \sinh(c+dx)}{d} - \frac{4bx^3 \cosh(c+dx)}{d^2} + \frac{12bx^2 \sinh(c+dx)}{d^3} - \frac{24bx \cosh(c+dx)}{d^4} + \frac{24b \sinh(c+dx)}{d^5} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^5}{5}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*3+a)\*cosh(d\*x+c),x)

[Out] Piecewise((a\*x\*sinh(c + d\*x)/d - a\*cosh(c + d\*x)/d\*\*2 + b\*x\*\*4\*sinh(c + d\*x)/d - 4\*b\*x\*\*3\*cosh(c + d\*x)/d\*\*2 + 12\*b\*x\*\*2\*sinh(c + d\*x)/d\*\*3 - 24\*b\*x\*cosh(c + d\*x)/d\*\*4 + 24\*b\*sinh(c + d\*x)/d\*\*5, Ne(d, 0)), ((a\*x\*\*2/2 + b\*x\*\*5/5)\*cosh(c), True))

**Giac [A]** time = 1.36826, size = 161, normalized size = 1.71

$$\frac{(bd^4x^4 - 4bd^3x^3 + ad^4x + 12bd^2x^2 - ad^3 - 24bdx + 24b)e^{(dx+c)}}{2d^5} - \frac{(bd^4x^4 + 4bd^3x^3 + ad^4x + 12bd^2x^2 + ad^3 + 24bdx + 24b)e^{-(dx+c)}}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^3+a)\*cosh(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(b\*d^4\*x^4 - 4\*b\*d^3\*x^3 + a\*d^4\*x + 12\*b\*d^2\*x^2 - a\*d^3 - 24\*b\*d\*x + 24\*b)\*e^(d\*x + c)/d^5 - 1/2\*(b\*d^4\*x^4 + 4\*b\*d^3\*x^3 + a\*d^4\*x + 12\*b\*d^2\*x^2 + a\*d^3 + 24\*b\*d\*x + 24\*b)\*e^(-d\*x - c)/d^5

### 3.82 $\int (a + bx^3) \cosh(c + dx) dx$

**Optimal.** Leaf size=66

$$\frac{a \sinh(c + dx)}{d} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{6b \cosh(c + dx)}{d^4} + \frac{bx^3 \sinh(c + dx)}{d}$$

[Out]  $(-6*b*Cosh[c + d*x])/d^4 - (3*b*x^2*Cosh[c + d*x])/d^2 + (a*Sinh[c + d*x])/d + (6*b*x*Sinh[c + d*x])/d^3 + (b*x^3*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.103047, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {5277, 2637, 3296, 2638}

$$\frac{a \sinh(c + dx)}{d} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{6b \cosh(c + dx)}{d^4} + \frac{bx^3 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)\*Cosh[c + d\*x],x]

[Out]  $(-6*b*Cosh[c + d*x])/d^4 - (3*b*x^2*Cosh[c + d*x])/d^2 + (a*Sinh[c + d*x])/d + (6*b*x*Sinh[c + d*x])/d^3 + (b*x^3*Sinh[c + d*x])/d$

#### Rule 5277

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[Cosh[c + d\*x], (a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[p, 0]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int (a + bx^3) \cosh(c + dx) dx &= \int (a \cosh(c + dx) + bx^3 \cosh(c + dx)) dx \\
 &= a \int \cosh(c + dx) dx + b \int x^3 \cosh(c + dx) dx \\
 &= \frac{a \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} - \frac{(3b) \int x^2 \sinh(c + dx) dx}{d} \\
 &= -\frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{a \sinh(c + dx)}{d} + \frac{bx^3 \sinh(c + dx)}{d} + \frac{(6b) \int x \cosh(c + dx) dx}{d^2} \\
 &= -\frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{a \sinh(c + dx)}{d} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{bx^3 \sinh(c + dx)}{d} - \frac{(6b) \int \sin}{d} \\
 &= -\frac{6b \cosh(c + dx)}{d^4} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{a \sinh(c + dx)}{d} + \frac{6bx \sinh(c + dx)}{d^3} + \frac{bx^3 \sinh(c + dx)}{d}
 \end{aligned}$$

**Mathematica [A]** time = 0.0808538, size = 49, normalized size = 0.74

$$\frac{d(ad^2 + bx(d^2x^2 + 6)) \sinh(c + dx) - 3b(d^2x^2 + 2) \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*Cosh[c + d*x], x]
```

```
[Out] (-3*b*(2 + d^2*x^2)*Cosh[c + d*x] + d*(a*d^2 + b*x*(6 + d^2*x^2))*Sinh[c +
d*x])/d^4
```

**Maple [B]** time = 0.007, size = 158, normalized size = 2.4

$$\frac{1}{d} \left( \frac{b((dx + c)^3 \sinh(dx + c) - 3(dx + c)^2 \cosh(dx + c) + 6(dx + c) \sinh(dx + c) - 6 \cosh(dx + c))}{d^3} - 3 \frac{cb((dx + c)^2 \sinh(dx + c))}{d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*cosh(d*x+c), x)
```



[Out]  $1/d*(1/d^3*b*((d*x+c)^3*\sinh(d*x+c)-3*(d*x+c)^2*\cosh(d*x+c)+6*(d*x+c)*\sinh(d*x+c)-6*\cosh(d*x+c))-3/d^3*b*c*((d*x+c)^2*\sinh(d*x+c)-2*(d*x+c)*\cosh(d*x+c)+2*\sinh(d*x+c))+3/d^3*b*c^2*((d*x+c)*\sinh(d*x+c)-\cosh(d*x+c))-1/d^3*b*c^3*\sinh(d*x+c)+a*\sinh(d*x+c))$

**Maxima [A]** time = 1.02864, size = 140, normalized size = 2.12

$$\frac{ae^{(dx+c)}}{2d} - \frac{ae^{(-dx-c)}}{2d} + \frac{(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)be^{(dx)}}{2d^4} - \frac{(d^3x^3 + 3d^2x^2 + 6dx + 6)be^{(-dx-c)}}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")`

[Out]  $1/2*a*e^{(d*x + c)}/d - 1/2*a*e^{(-d*x - c)}/d + 1/2*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*b*e^{(d*x)}/d^4 - 1/2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b*e^{(-d*x - c)}/d^4$

**Fricas [A]** time = 1.81258, size = 119, normalized size = 1.8

$$\frac{3(bd^2x^2 + 2b)\cosh(dx + c) - (bd^3x^3 + ad^3 + 6bdx)\sinh(dx + c)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")`

[Out]  $-(3*(b*d^2*x^2 + 2*b)*\cosh(d*x + c) - (b*d^3*x^3 + a*d^3 + 6*b*d*x)*\sinh(d*x + c))/d^4$

**Sympy [A]** time = 1.40829, size = 82, normalized size = 1.24

$$\begin{cases} \frac{a \sinh(c+dx)}{d} + \frac{bx^3 \sinh(c+dx)}{d} - \frac{3bx^2 \cosh(c+dx)}{d^2} + \frac{6bx \sinh(c+dx)}{d^3} - \frac{6b \cosh(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^4}{4}\right) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)), ((a*x + b*x**4/4)*cosh(c), True))
```

**Giac [A]** time = 1.22369, size = 119, normalized size = 1.8

$$\frac{(bd^3x^3 - 3bd^2x^2 + ad^3 + 6bdx - 6b)e^{(dx+c)}}{2d^4} - \frac{(bd^3x^3 + 3bd^2x^2 + ad^3 + 6bdx + 6b)e^{(-dx-c)}}{2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^3*x^3 - 3*b*d^2*x^2 + a*d^3 + 6*b*d*x - 6*b)*e^(d*x + c)/d^4 - 1/2*(b*d^3*x^3 + 3*b*d^2*x^2 + a*d^3 + 6*b*d*x + 6*b)*e^(-d*x - c)/d^4
```

$$3.83 \quad \int \frac{(a+bx^3) \cosh(c+dx)}{x} dx$$

**Optimal.** Leaf size=56

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) + \frac{2b \sinh(c+dx)}{d^3} - \frac{2bx \cosh(c+dx)}{d^2} + \frac{bx^2 \sinh(c+dx)}{d}$$

[Out]  $(-2*b*x*Cosh[c + d*x])/d^2 + a*Cosh[c]*CoshIntegral[d*x] + (2*b*Sinh[c + d*x])/d^3 + (b*x^2*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[d*x]$

**Rubi [A]** time = 0.128231, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {5287, 3303, 3298, 3301, 3296, 2637}

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) + \frac{2b \sinh(c+dx)}{d^3} - \frac{2bx \cosh(c+dx)}{d^2} + \frac{bx^2 \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^3)\*Cosh[c + d\*x])/x, x]

[Out]  $(-2*b*x*Cosh[c + d*x])/d^2 + a*Cosh[c]*CoshIntegral[d*x] + (2*b*Sinh[c + d*x])/d^3 + (b*x^2*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[d*x]$

#### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f}

, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^3) \cosh(c + dx)}{x} dx &= \int \left( \frac{a \cosh(c + dx)}{x} + bx^2 \cosh(c + dx) \right) dx \\
 &= a \int \frac{\cosh(c + dx)}{x} dx + b \int x^2 \cosh(c + dx) dx \\
 &= \frac{bx^2 \sinh(c + dx)}{d} - \frac{(2b) \int x \sinh(c + dx) dx}{d} + (a \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (a \sinh(c)) \int \frac{\cosh(dx)}{x} dx \\
 &= -\frac{2bx \cosh(c + dx)}{d^2} + a \cosh(c) \text{Chi}(dx) + \frac{bx^2 \sinh(c + dx)}{d} + a \sinh(c) \text{Shi}(dx) + \frac{(2b) \int \cosh(dx)}{d} \\
 &= -\frac{2bx \cosh(c + dx)}{d^2} + a \cosh(c) \text{Chi}(dx) + \frac{2b \sinh(c + dx)}{d^3} + \frac{bx^2 \sinh(c + dx)}{d} + a \sinh(c) \text{Shi}(dx)
 \end{aligned}$$

**Mathematica [A]** time = 0.164428, size = 49, normalized size = 0.88

$$a \cosh(c) \text{Chi}(dx) + a \sinh(c) \text{Shi}(dx) + \frac{b \left( (d^2 x^2 + 2) \sinh(c + dx) - 2dx \cosh(c + dx) \right)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^3)\*Cosh[c + d\*x])/x,x]

[Out]  $a \operatorname{Cosh}[c] \operatorname{CoshIntegral}[d*x] + (b*(-2*d*x*\operatorname{Cosh}[c + d*x] + (2 + d^2*x^2)*\operatorname{Sinh}[c + d*x]))/d^3 + a*\operatorname{Sinh}[c]*\operatorname{SinhIntegral}[d*x]$

**Maple [A]** time = 0.032, size = 113, normalized size = 2.

$$\frac{be^{dx+c}x^2}{2d} - \frac{be^{dx+c}x}{d^2} - \frac{be^{-dx-c}x^2}{2d} - \frac{be^{-dx-c}x}{d^2} - \frac{ae^c\operatorname{Ei}(1, -dx)}{2} - \frac{ae^{-c}\operatorname{Ei}(1, dx)}{2} + \frac{be^{dx+c}}{d^3} - \frac{be^{-dx-c}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)*cosh(d*x+c)/x,x)`

[Out]  $1/2/d*b*\exp(d*x+c)*x^2-1/d^2*b*\exp(d*x+c)*x-1/2/d*b*\exp(-d*x-c)*x^2-1/d^2*b*\exp(-d*x-c)*x-1/2*a*\exp(c)*\operatorname{Ei}(1, -d*x)-1/2*a*\exp(-c)*\operatorname{Ei}(1, d*x)+1/d^3*b*\exp(d*x+c)-1/d^3*b*\exp(-d*x-c)$

**Maxima [B]** time = 1.23097, size = 189, normalized size = 3.38

$$-\frac{1}{6} \left( b \left( \frac{(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)e^{(dx)}}{d^4} + \frac{(d^3x^3 + 3d^2x^2 + 6dx + 6)e^{(-dx-c)}}{d^4} \right) + \frac{2a \cosh(dx + c) \log(x^3)}{d} - \frac{3(\operatorname{Ei}(-dx)*e^{-c} + \operatorname{Ei}(dx)*e^c)*a}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="maxima")`

[Out]  $-1/6*(b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^{(d*x)}/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^{(-d*x - c)}/d^4) + 2*a*\cosh(d*x + c)*\log(x^3)/d - 3*(\operatorname{Ei}(-d*x)*e^{-c} + \operatorname{Ei}(d*x)*e^c)*a/d)*d + 1/3*(b*x^3 + a*\log(x^3))*\cosh(d*x + c)$

**Fricas [A]** time = 1.69971, size = 211, normalized size = 3.77

$$\frac{4bdx \cosh(dx + c) - (ad^3\operatorname{Ei}(dx) + ad^3\operatorname{Ei}(-dx)) \cosh(c) - 2(bd^2x^2 + 2b) \sinh(dx + c) - (ad^3\operatorname{Ei}(dx) - ad^3\operatorname{Ei}(-dx))}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*cosh(d\*x+c)/x,x, algorithm="fricas")

[Out] 
$$\frac{-1/2*(4*b*d*x*cosh(d*x + c) - (a*d^3*Ei(d*x) + a*d^3*Ei(-d*x))*cosh(c) - 2*(b*d^2*x^2 + 2*b)*sinh(d*x + c) - (a*d^3*Ei(d*x) - a*d^3*Ei(-d*x))*sinh(c))}{d^3}$$

**Sympy [A]** time = 4.8626, size = 66, normalized size = 1.18

$$a \sinh(c) \operatorname{Shi}(dx) + a \cosh(c) \operatorname{Chi}(dx) + b \begin{cases} \frac{x^2 \sinh(c+dx)}{d} - \frac{2x \cosh(c+dx)}{d^2} + \frac{2 \sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \frac{x^3 \cosh(c)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*cosh(d\*x+c)/x,x)

[Out] 
$$a*\sinh(c)*\operatorname{Shi}(d*x) + a*\cosh(c)*\operatorname{Chi}(d*x) + b*\operatorname{Piecewise}((x**2*\sinh(c + d*x)/d - 2*x*\cosh(c + d*x)/d**2 + 2*\sinh(c + d*x)/d**3, \operatorname{Ne}(d, 0)), (x**3*\cosh(c)/3, \operatorname{True}))$$

**Giac [A]** time = 1.27269, size = 147, normalized size = 2.62

$$\frac{bd^2x^2e^{(dx+c)} - bd^2x^2e^{(-dx-c)} + ad^3Ei(-dx)e^{(-c)} + ad^3Ei(dx)e^c - 2bdxe^{(dx+c)} - 2bdxe^{(-dx-c)} + 2be^{(dx+c)} - 2be^{(-dx-c)}}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*cosh(d\*x+c)/x,x, algorithm="giac")

[Out] 
$$\frac{1/2*(b*d^2*x^2*e^{(d*x + c)} - b*d^2*x^2*e^{(-d*x - c)} + a*d^3*Ei(-d*x)*e^{(-c)} + a*d^3*Ei(d*x)*e^c - 2*b*d*x*e^{(d*x + c)} - 2*b*d*x*e^{(-d*x - c)} + 2*b*e^{(d*x + c)} - 2*b*e^{(-d*x - c)})}{d^3}$$

$$3.84 \quad \int \frac{(a+bx^3) \cosh(c+dx)}{x^2} dx$$

**Optimal.** Leaf size=55

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{x} - \frac{b \cosh(c+dx)}{d^2} + \frac{bx \sinh(c+dx)}{d}$$

[Out]  $-\left(\frac{b \cosh[c + d*x]}{d^2}\right) - \frac{a \cosh[c + d*x]}{x} + a*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + \frac{b*x*\text{Sinh}[c + d*x]}{d} + a*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x]$

**Rubi [A]** time = 0.129105, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638}

$$ad \sinh(c)\text{Chi}(dx) + ad \cosh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{x} - \frac{b \cosh(c+dx)}{d^2} + \frac{bx \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + b*x^3)*\text{Cosh}[c + d*x]}{x^2}, x]$

[Out]  $-\left(\frac{b \cosh[c + d*x]}{d^2}\right) - \frac{a \cosh[c + d*x]}{x} + a*d*\text{CoshIntegral}[d*x]*\text{Sinh}[c] + \frac{b*x*\text{Sinh}[c + d*x]}{d} + a*d*\text{Cosh}[c]*\text{SinhIntegral}[d*x]$

#### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x\} \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 3297

$\text{Int}[\frac{(c_.) + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)]}{x\_Symbol}, x\_Symbol] \rightarrow \text{Simp}[\frac{(c + d*x)^{(m+1)}*\sin[e + f*x]}{(d*(m+1))}, x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\cos[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{LtQ}[m, -1]$

#### Rule 3303

$\text{Int}[\frac{\sin[(e_.) + (f_.)*(x_.)]}{(c_.) + (d_.)*(x_.)}, x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\frac{\sin[(c*f)/d + f*x]}{(c + d*x)}, x], x] + \text{Dist}[\frac{\sin[(d*e - c*f)}{d}], \text{Int}[\frac{\cos[(c*f)/d + f*x]}{(c + d*x)}, x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\&$

NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^3) \cosh(c + dx)}{x^2} dx &= \int \left( \frac{a \cosh(c + dx)}{x^2} + bx \cosh(c + dx) \right) dx \\
 &= a \int \frac{\cosh(c + dx)}{x^2} dx + b \int x \cosh(c + dx) dx \\
 &= -\frac{a \cosh(c + dx)}{x} + \frac{bx \sinh(c + dx)}{d} - \frac{b \int \sinh(c + dx) dx}{d} + (ad) \int \frac{\sinh(c + dx)}{x} dx \\
 &= -\frac{b \cosh(c + dx)}{d^2} - \frac{a \cosh(c + dx)}{x} + \frac{bx \sinh(c + dx)}{d} + (ad \cosh(c)) \int \frac{\sinh(dx)}{x} dx + (ad \sinh(c)) \int \frac{\cosh(dx)}{x} dx \\
 &= -\frac{b \cosh(c + dx)}{d^2} - \frac{a \cosh(c + dx)}{x} + ad \operatorname{Chi}(dx) \sinh(c) + \frac{bx \sinh(c + dx)}{d} + ad \cosh(c) \operatorname{Shi}(dx)
 \end{aligned}$$

**Mathematica [A]** time = 0.131198, size = 55, normalized size = 1.

$$ad \sinh(c) \operatorname{Chi}(dx) + ad \cosh(c) \operatorname{Shi}(dx) - \frac{a \cosh(c + dx)}{x} - \frac{b \cosh(c + dx)}{d^2} + \frac{bx \sinh(c + dx)}{d}$$



Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^3)\*Cosh[c + d\*x])/x^2,x]

[Out] -((b\*Cosh[c + d\*x])/d^2) - (a\*Cosh[c + d\*x])/x + a\*d\*CoshIntegral[d\*x]\*Sinh[c] + (b\*x\*Sinh[c + d\*x])/d + a\*d\*Cosh[c]\*SinhIntegral[d\*x]

**Maple [A]** time = 0.054, size = 110, normalized size = 2.

$$-\frac{ae^{-dx-c}}{2x} + \frac{dae^{-c}Ei(1,dx)}{2} - \frac{be^{-dx-c}x}{2d} - \frac{be^{-dx-c}}{2d^2} - \frac{ae^{dx+c}}{2x} - \frac{dae^cEi(1,-dx)}{2} + \frac{be^{dx+c}x}{2d} - \frac{be^{dx+c}}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)\*cosh(d\*x+c)/x^2,x)

[Out] -1/2\*a\*exp(-d\*x-c)/x+1/2\*d\*a\*exp(-c)\*Ei(1,d\*x)-1/2/d\*b\*exp(-d\*x-c)\*x-1/2/d^2\*b\*exp(-d\*x-c)-1/2\*a/x\*exp(d\*x+c)-1/2\*d\*a\*exp(c)\*Ei(1,-d\*x)+1/2/d\*b\*exp(d\*x+c)\*x-1/2/d^2\*b\*exp(d\*x+c)

**Maxima [A]** time = 1.15927, size = 138, normalized size = 2.51

$$-\frac{1}{4} \left( 2aEi(-dx)e^{(-c)} - 2aEi(dx)e^c + \frac{(d^2x^2e^c - 2dxe^c + 2e^c)be^{(dx)}}{d^3} + \frac{(d^2x^2 + 2dx + 2)be^{(-dx-c)}}{d^3} \right) d + \frac{1}{2} \left( bx^2 - \frac{2a}{x} \right) \cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*cosh(d\*x+c)/x^2,x, algorithm="maxima")

[Out] -1/4\*(2\*a\*Ei(-d\*x)\*e^(-c) - 2\*a\*Ei(d\*x)\*e^c + (d^2\*x^2\*e^c - 2\*d\*x\*e^c + 2\*e^c)\*b\*e^(d\*x)/d^3 + (d^2\*x^2 + 2\*d\*x + 2)\*b\*e^(-d\*x - c)/d^3)\*d + 1/2\*(b\*x^2 - 2\*a/x)\*cosh(d\*x + c)

**Fricas [A]** time = 1.82228, size = 223, normalized size = 4.05

$$\frac{2bdx^2 \sinh(dx+c) - 2(ad^2 + bx) \cosh(dx+c) + (ad^3x \operatorname{Ei}(dx) - ad^3x \operatorname{Ei}(-dx)) \cosh(c) + (ad^3x \operatorname{Ei}(dx) + ad^3x \operatorname{Ei}(-dx)) \sinh(c)}{2d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*cosh(d\*x+c)/x^2,x, algorithm="fricas")

[Out]  $\frac{1}{2}*(2*b*d*x^2*\sinh(d*x + c) - 2*(a*d^2 + b*x)*\cosh(d*x + c) + (a*d^3*x*Ei(d*x) - a*d^3*x*Ei(-d*x))*\cosh(c) + (a*d^3*x*Ei(d*x) + a*d^3*x*Ei(-d*x))*\sinh(c))/(d^2*x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^3) \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*cosh(d\*x+c)/x\*\*2,x)

[Out] Integral((a + b\*x\*\*3)\*cosh(c + d\*x)/x\*\*2, x)

**Giac [B]** time = 1.31986, size = 150, normalized size = 2.73

$$\frac{ad^3xEi(-dx)e^{(-c)} - ad^3xEi(dx)e^c - bdx^2e^{(dx+c)} + bdx^2e^{(-dx-c)} + ad^2e^{(dx+c)} + ad^2e^{(-dx-c)} + bxe^{(dx+c)} + bxe^{(-dx-c)}}{2d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*cosh(d\*x+c)/x^2,x, algorithm="giac")

[Out]  $\frac{-1/2*(a*d^3*x*Ei(-d*x)*e^{(-c)} - a*d^3*x*Ei(d*x)*e^c - b*d*x^2*e^{(d*x + c)} + b*d*x^2*e^{(-d*x - c)} + a*d^2*e^{(d*x + c)} + a*d^2*e^{(-d*x - c)} + b*x*e^{(d*x + c)} + b*x*e^{(-d*x - c)})/(d^2*x)$

$$3.85 \quad \int \frac{(a+bx^3) \cosh(c+dx)}{x^3} dx$$

**Optimal.** Leaf size=69

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + \frac{b \sinh(c+dx)}{d}$$

[Out]  $-(a*\text{Cosh}[c + d*x])/(2*x^2) + (a*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (b*\text{Sinh}[c + d*x])/d - (a*d*\text{Sinh}[c + d*x])/(2*x) + (a*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

**Rubi [A]** time = 0.138576, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {5287, 2637, 3297, 3303, 3298, 3301}

$$\frac{1}{2}ad^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2 \sinh(c)\text{Shi}(dx) - \frac{a \cosh(c+dx)}{2x^2} - \frac{ad \sinh(c+dx)}{2x} + \frac{b \sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^3)*\text{Cosh}[c + d*x])/x^3, x]$

[Out]  $-(a*\text{Cosh}[c + d*x])/(2*x^2) + (a*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (b*\text{Sinh}[c + d*x])/d - (a*d*\text{Sinh}[c + d*x])/(2*x) + (a*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

#### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_)^{(n_)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m, n\}, x \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$   $\text{FreeQ}\{c, d\}, x$

#### Rule 3297

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_)], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}*\sin[e + f*x]/(d*(m+1)), x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\cos[e + f*x], x], x] /;$   $\text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{LtQ}[m, -1]$

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3) \cosh(c + dx)}{x^3} dx &= \int \left( b \cosh(c + dx) + \frac{a \cosh(c + dx)}{x^3} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^3} dx + b \int \cosh(c + dx) dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + \frac{b \sinh(c + dx)}{d} + \frac{1}{2}(ad) \int \frac{\sinh(c + dx)}{x^2} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + \frac{b \sinh(c + dx)}{d} - \frac{ad \sinh(c + dx)}{2x} + \frac{1}{2}(ad^2) \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{2x^2} + \frac{b \sinh(c + dx)}{d} - \frac{ad \sinh(c + dx)}{2x} + \frac{1}{2}(ad^2 \cosh(c)) \int \frac{\cosh(dx)}{x} dx + \\
&= -\frac{a \cosh(c + dx)}{2x^2} + \frac{1}{2}ad^2 \cosh(c) \text{Chi}(dx) + \frac{b \sinh(c + dx)}{d} - \frac{ad \sinh(c + dx)}{2x} + \frac{1}{2}ad^2 \sinh(c) \text{Shi}(dx)
\end{aligned}$$

**Mathematica [A]** time = 0.141124, size = 86, normalized size = 1.25

$$\frac{1}{2}ad^2(\cosh(c)\text{Chi}(dx) + \sinh(c)\text{Shi}(dx)) - \frac{a \cosh(dx)(dx \sinh(c) + \cosh(c))}{2x^2} - \frac{a \sinh(dx)(dx \cosh(c) + \sinh(c))}{2x^2} + \frac{b \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^3)\*Cosh[c + d\*x])/x^3,x]

[Out] (b\*Cosh[d\*x]\*Sinh[c])/d - (a\*Cosh[d\*x]\*(Cosh[c] + d\*x\*Sinh[c]))/(2\*x^2) + (b\*Cosh[c]\*Sinh[d\*x])/d - (a\*(d\*x\*Cosh[c] + Sinh[c])\*Sinh[d\*x])/(2\*x^2) + (a\*d^2\*(Cosh[c]\*CoshIntegral[d\*x] + Sinh[c]\*SinhIntegral[d\*x]))/2

**Maple [A]** time = 0.056, size = 114, normalized size = 1.7

$$\frac{dae^{-dx-c}}{4x} - \frac{ae^{-dx-c}}{4x^2} - \frac{d^2ae^{-c}\text{Ei}(1,dx)}{4} - \frac{be^{-dx-c}}{2d} - \frac{ae^{dx+c}}{4x^2} - \frac{dae^{dx+c}}{4x} - \frac{d^2ae^c\text{Ei}(1,-dx)}{4} + \frac{be^{dx+c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)\*cosh(d\*x+c)/x^3,x)

[Out] 1/4\*d\*a\*exp(-d\*x-c)/x-1/4\*a\*exp(-d\*x-c)/x^2-1/4\*d^2\*a\*exp(-c)\*Ei(1,d\*x)-1/2\*b/d\*exp(-d\*x-c)-1/4\*a/x^2\*exp(d\*x+c)-1/4\*d\*a/x\*exp(d\*x+c)-1/4\*d^2\*a\*exp(c)\*Ei(1,-d\*x)+1/2\*b/d\*exp(d\*x+c)

**Maxima [A]** time = 1.17684, size = 117, normalized size = 1.7

$$\frac{1}{4} \left( ade^{(-c)}\Gamma(-1,dx) + ade^c\Gamma(-1,-dx) - \frac{2(dx e^c - e^c)be^{(dx)}}{d^2} - \frac{2(dx+1)be^{(-dx-c)}}{d^2} \right) d + \frac{1}{2} \left( 2bx - \frac{a}{x^2} \right) \cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*cosh(d\*x+c)/x^3,x, algorithm="maxima")

[Out] 1/4\*(a\*d\*e^(-c)\*gamma(-1, d\*x) + a\*d\*e^c\*gamma(-1, -d\*x) - 2\*(d\*x\*e^c - e^c)\*b\*e^(d\*x)/d^2 - 2\*(d\*x + 1)\*b\*e^(-d\*x - c)/d^2)\*d + 1/2\*(2\*b\*x - a/x^2)\*cosh(d\*x + c)

**Fricas [A]** time = 1.83341, size = 238, normalized size = 3.45

$$\frac{2ad \cosh(dx+c) - (ad^3x^2\text{Ei}(dx) + ad^3x^2\text{Ei}(-dx)) \cosh(c) + 2(ad^2x - 2bx^2) \sinh(dx+c) - (ad^3x^2\text{Ei}(dx) - ad^3x^2\text{Ei}(-dx)) \cosh(c)}{4dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d*cosh(d*x + c) - (a*d^3*x^2*Ei(d*x) + a*d^3*x^2*Ei(-d*x))*cosh(c)
) + 2*(a*d^2*x - 2*b*x^2)*sinh(d*x + c) - (a*d^3*x^2*Ei(d*x) - a*d^3*x^2*Ei
(-d*x))*sinh(c))/(d*x^2)
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x**3,x)
```

```
[Out] Exception raised: ValueError
```

**Giac [A]** time = 1.25569, size = 159, normalized size = 2.3

$$\frac{ad^3x^2Ei(-dx)e^{(-c)} + ad^3x^2Ei(dx)e^c - ad^2xe^{(dx+c)} + ad^2xe^{(-dx-c)} + 2bx^2e^{(dx+c)} - 2bx^2e^{(-dx-c)} - ade^{(dx+c)} - ade^{(-dx-c)}}{4dx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^3*x^2*Ei(-d*x)*e^(-c) + a*d^3*x^2*Ei(d*x)*e^c - a*d^2*x*e^(d*x + c)
) + a*d^2*x*e^(-d*x - c) + 2*b*x^2*e^(d*x + c) - 2*b*x^2*e^(-d*x - c) - a*d
*e^(d*x + c) - a*d*e^(-d*x - c))/(d*x^2)
```

$$3.86 \quad \int \frac{(a+bx^3) \cosh(c+dx)}{x^4} dx$$

**Optimal.** Leaf size=91

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{ad \sinh(c+dx)}{6x^2} - \frac{a \cosh(c+dx)}{3x^3} + b \cosh(c)\text{Chi}(d$$

[Out]  $-(a*\text{Cosh}[c + d*x])/(3*x^3) - (a*d^2*\text{Cosh}[c + d*x])/(6*x) + b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (a*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 - (a*d*\text{Sinh}[c + d*x])/(6*x^2) + (a*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6 + b*\text{Sinh}[c]*\text{SinhIntegral}[d*x]$

**Rubi [A]** time = 0.215468, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {5287, 3297, 3303, 3298, 3301}

$$\frac{1}{6}ad^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}ad^3 \cosh(c)\text{Shi}(dx) - \frac{ad^2 \cosh(c+dx)}{6x} - \frac{ad \sinh(c+dx)}{6x^2} - \frac{a \cosh(c+dx)}{3x^3} + b \cosh(c)\text{Chi}(d$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + b*x^3)*\text{Cosh}[c + d*x]}{x^4}, x]$

[Out]  $-(a*\text{Cosh}[c + d*x])/(3*x^3) - (a*d^2*\text{Cosh}[c + d*x])/(6*x) + b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (a*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 - (a*d*\text{Sinh}[c + d*x])/(6*x^2) + (a*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6 + b*\text{Sinh}[c]*\text{SinhIntegral}[d*x]$

#### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \text{ :> } \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, m, n\}, x\} \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 3297

$\text{Int}[\frac{(c_.) + (d_.)*(x_.)}{x} \sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \text{ :> } \text{Simp}[\frac{(c + d*x)^{(m + 1)}*\text{Sin}[e + f*x]}{(d*(m + 1))}, x] - \text{Dist}[f/(d*(m + 1)), \text{Int}[(c + d*x)^{(m + 1)}*\text{Cos}[e + f*x], x], x] \text{ /; } \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{LtQ}[m, -1]$

#### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3) \cosh(c + dx)}{x^4} dx &= \int \left( \frac{a \cosh(c + dx)}{x^4} + \frac{b \cosh(c + dx)}{x} \right) dx \\
&= a \int \frac{\cosh(c + dx)}{x^4} dx + b \int \frac{\cosh(c + dx)}{x} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} + \frac{1}{3}(ad) \int \frac{\sinh(c + dx)}{x^3} dx + (b \cosh(c)) \int \frac{\cosh(dx)}{x} dx + (b \sinh(c)) \int \frac{1}{x} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} + b \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{6x^2} + b \sinh(c) \text{Shi}(dx) + \frac{1}{6} (ad^2) \int \frac{1}{x} dx \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{6x} + b \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{6x^2} + b \sinh(c) \text{Shi}(dx) \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{6x} + b \cosh(c) \text{Chi}(dx) - \frac{ad \sinh(c + dx)}{6x^2} + b \sinh(c) \text{Shi}(dx) \\
&= -\frac{a \cosh(c + dx)}{3x^3} - \frac{ad^2 \cosh(c + dx)}{6x} + b \cosh(c) \text{Chi}(dx) + \frac{1}{6} ad^3 \text{Chi}(dx) \sinh(c) - \frac{ad \sinh(c + dx)}{6x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.238487, size = 73, normalized size = 0.8

$$\frac{1}{6} \left( \text{Chi}(dx) (ad^3 \sinh(c) + 6b \cosh(c)) + \text{Shi}(dx) (ad^3 \cosh(c) + 6b \sinh(c)) - \frac{a((d^2 x^2 + 2) \cosh(c + dx) + dx \sinh(c + dx))}{x^3} \right)$$

Antiderivative was successfully verified.



[In] Integrate[((a + b\*x^3)\*Cosh[c + d\*x])/x^4,x]

[Out] (CoshIntegral[d\*x]\*(6\*b\*Cosh[c] + a\*d^3\*Sinh[c]) - (a\*((2 + d^2\*x^2)\*Cosh[c + d\*x] + d\*x\*Sinh[c + d\*x]))/x^3 + (a\*d^3\*Cosh[c] + 6\*b\*Sinh[c])\*SinhIntegral[d\*x])/6

**Maple [A]** time = 0.066, size = 143, normalized size = 1.6

$$\frac{d^3 a e^{-c} \text{Ei}(1, dx)}{12} - \frac{b e^{-c} \text{Ei}(1, dx)}{2} - \frac{a d^2 e^{-dx-c}}{12 x} + \frac{d a e^{-dx-c}}{12 x^2} - \frac{a e^{-dx-c}}{6 x^3} - \frac{a e^{dx+c}}{6 x^3} - \frac{d a e^{dx+c}}{12 x^2} - \frac{a d^2 e^{dx+c}}{12 x} - \frac{d^3 a e^c \text{Ei}(1, -dx)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)\*cosh(d\*x+c)/x^4,x)

[Out] 1/12\*d^3\*a\*exp(-c)\*Ei(1,d\*x)-1/2\*b\*exp(-c)\*Ei(1,d\*x)-1/12\*d^2\*a\*exp(-d\*x-c)/x+1/12\*d\*a\*exp(-d\*x-c)/x^2-1/6\*a\*exp(-d\*x-c)/x^3-1/6\*a/x^3\*exp(d\*x+c)-1/12\*d\*a/x^2\*exp(d\*x+c)-1/12\*d^2\*a/x\*exp(d\*x+c)-1/12\*d^3\*a\*exp(c)\*Ei(1,-d\*x)-1/2\*b\*exp(c)\*Ei(1,-d\*x)

**Maxima [A]** time = 1.22964, size = 128, normalized size = 1.41

$$\frac{1}{6} \left( \left( d^2 e^{(-c)} \Gamma(-2, dx) - d^2 e^c \Gamma(-2, -dx) \right) a - \frac{2 b \cosh(dx + c) \log(x^3)}{d} + \frac{3 \left( \text{Ei}(-dx) e^{(-c)} + \text{Ei}(dx) e^c \right) b}{d} \right) d + \frac{1}{3} \left( b \log(x^3) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*cosh(d\*x+c)/x^4,x, algorithm="maxima")

[Out] 1/6\*((d^2\*e^(-c)\*gamma(-2, d\*x) - d^2\*e^c\*gamma(-2, -d\*x))\*a - 2\*b\*cosh(d\*x + c)\*log(x^3)/d + 3\*(Ei(-d\*x)\*e^(-c) + Ei(d\*x)\*e^c)\*b/d)\*d + 1/3\*(b\*log(x^3) - a/x^3)\*cosh(d\*x + c)

**Fricas [A]** time = 1.72822, size = 277, normalized size = 3.04

$$\frac{2 a dx \sinh(dx + c) + 2 (a d^2 x^2 + 2 a) \cosh(dx + c) - \left( (a d^3 + 6 b) x^3 \text{Ei}(dx) - (a d^3 - 6 b) x^3 \text{Ei}(-dx) \right) \cosh(c) - \left( (a d^3 + 6 b) x^3 \text{Ei}(dx) - (a d^3 - 6 b) x^3 \text{Ei}(-dx) \right)}{12 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

```
[Out] -1/12*(2*a*d*x*sinh(d*x + c) + 2*(a*d^2*x^2 + 2*a)*cosh(d*x + c) - ((a*d^3 + 6*b)*x^3*Ei(d*x) - (a*d^3 - 6*b)*x^3*Ei(-d*x))*cosh(c) - ((a*d^3 + 6*b)*x^3*Ei(d*x) + (a*d^3 - 6*b)*x^3*Ei(-d*x))*sinh(c))/x^3
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*cosh(d*x+c)/x**4,x)
```

```
[Out] Exception raised: ValueError
```

**Giac [A]** time = 1.27178, size = 190, normalized size = 2.09

$$\frac{ad^3x^3Ei(-dx)e^{(-c)} - ad^3x^3Ei(dx)e^c + ad^2x^2e^{(dx+c)} + ad^2x^2e^{(-dx-c)} - 6bx^3Ei(-dx)e^{(-c)} - 6bx^3Ei(dx)e^c + adxe^{(dx+c)} - 12x^3}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

```
[Out] -1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x + c) + a*d^2*x^2*e^(-d*x - c) - 6*b*x^3*Ei(-d*x)*e^(-c) - 6*b*x^3*Ei(d*x)*e^c + a*d*x*e^(d*x + c) - a*d*x*e^(-d*x - c) + 2*a*e^(d*x + c) + 2*a*e^(-d*x - c))/x^3
```

### 3.87 $\int x (a + bx^3)^2 \cosh(c + dx) dx$

**Optimal.** Leaf size=234

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} + \frac{24abx^2 \sinh(c + dx)}{d^3} - \frac{8abx^3 \cosh(c + dx)}{d^2} + \frac{48ab \sinh(c + dx)}{d^5} - \frac{48abx \cosh(c + dx)}{d^4}$$

[Out]  $(-5040*b^2*Cosh[c + d*x])/d^8 - (a^2*Cosh[c + d*x])/d^2 - (48*a*b*x*Cosh[c + d*x])/d^4 - (2520*b^2*x^2*Cosh[c + d*x])/d^6 - (8*a*b*x^3*Cosh[c + d*x])/d^2 - (210*b^2*x^4*Cosh[c + d*x])/d^4 - (7*b^2*x^6*Cosh[c + d*x])/d^2 + (48*a*b*Sinh[c + d*x])/d^5 + (5040*b^2*x*Sinh[c + d*x])/d^7 + (a^2*x*Sinh[c + d*x])/d + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (840*b^2*x^3*Sinh[c + d*x])/d^5 + (2*a*b*x^4*Sinh[c + d*x])/d + (42*b^2*x^5*Sinh[c + d*x])/d^3 + (b^2*x^7*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.39526, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {5287, 3296, 2638, 2637}

$$-\frac{a^2 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} + \frac{24abx^2 \sinh(c + dx)}{d^3} - \frac{8abx^3 \cosh(c + dx)}{d^2} + \frac{48ab \sinh(c + dx)}{d^5} - \frac{48abx \cosh(c + dx)}{d^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*x^3)^2*\text{Cosh}[c + d*x], x]$

[Out]  $(-5040*b^2*Cosh[c + d*x])/d^8 - (a^2*Cosh[c + d*x])/d^2 - (48*a*b*x*Cosh[c + d*x])/d^4 - (2520*b^2*x^2*Cosh[c + d*x])/d^6 - (8*a*b*x^3*Cosh[c + d*x])/d^2 - (210*b^2*x^4*Cosh[c + d*x])/d^4 - (7*b^2*x^6*Cosh[c + d*x])/d^2 + (48*a*b*Sinh[c + d*x])/d^5 + (5040*b^2*x*Sinh[c + d*x])/d^7 + (a^2*x*Sinh[c + d*x])/d + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (840*b^2*x^3*Sinh[c + d*x])/d^5 + (2*a*b*x^4*Sinh[c + d*x])/d + (42*b^2*x^5*Sinh[c + d*x])/d^3 + (b^2*x^7*Sinh[c + d*x])/d$

#### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m, n\}, x\} \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int x (a + bx^3)^2 \cosh(c + dx) dx &= \int (a^2 x \cosh(c + dx) + 2abx^4 \cosh(c + dx) + b^2 x^7 \cosh(c + dx)) dx \\
&= a^2 \int x \cosh(c + dx) dx + (2ab) \int x^4 \cosh(c + dx) dx + b^2 \int x^7 \cosh(c + dx) dx \\
&= \frac{a^2 x \sinh(c + dx)}{d} + \frac{2abx^4 \sinh(c + dx)}{d} + \frac{b^2 x^7 \sinh(c + dx)}{d} - \frac{a^2 \int \sinh(c + dx) dx}{d} - \frac{2ab \int x^3 \cosh(c + dx) dx}{d} - \frac{b^2 \int x^6 \cosh(c + dx) dx}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{7b^2 x^6 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} + \frac{2abx^4 \sinh(c + dx)}{d} + \frac{b^2 x^7 \sinh(c + dx)}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{7b^2 x^6 \cosh(c + dx)}{d^2} + \frac{a^2 x \sinh(c + dx)}{d} + \frac{2abx^4 \sinh(c + dx)}{d} + \frac{b^2 x^7 \sinh(c + dx)}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{210b^2 x^4 \cosh(c + dx)}{d^4} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{8abx^3 \cosh(c + dx)}{d^2} - \frac{210b^2 x^4 \cosh(c + dx)}{d^4} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2520b^2 x^2 \cosh(c + dx)}{d^6} - \frac{8abx^3 \cosh(c + dx)}{d^2} \\
&= -\frac{a^2 \cosh(c + dx)}{d^2} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2520b^2 x^2 \cosh(c + dx)}{d^6} - \frac{8abx^3 \cosh(c + dx)}{d^2} \\
&= -\frac{5040b^2 \cosh(c + dx)}{d^8} - \frac{a^2 \cosh(c + dx)}{d^2} - \frac{48abx \cosh(c + dx)}{d^4} - \frac{2520b^2 x^2 \cosh(c + dx)}{d^6}
\end{aligned}$$

**Mathematica [A]** time = 0.291489, size = 139, normalized size = 0.59

$$\frac{d (a^2 d^6 x + 2abd^2 (d^4 x^4 + 12d^2 x^2 + 24) + b^2 x (d^6 x^6 + 42d^4 x^4 + 840d^2 x^2 + 5040)) \sinh(c + dx) - (a^2 d^6 + 8abd^4 x (d^2 x^2 + 24) + b^2 x^2 (d^6 x^4 + 42d^4 x^2 + 840d^2 x + 5040)) \cosh(c + dx)}{d^8}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x^3)^2\*Cosh[c + d\*x],x]

[Out]  $(-((a^2*d^6 + 8*a*b*d^4*x*(6 + d^2*x^2) + 7*b^2*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6))*Cosh[c + d*x]) + d*(a^2*d^6*x + 2*a*b*d^2*(24 + 12*d^2*x^2 + d^4*x^4) + b^2*x*(5040 + 840*d^2*x^2 + 42*d^4*x^4 + d^6*x^6))*Sinh[c + d*x])/d^8$

**Maple [B]** time = 0.008, size = 818, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^3+a)^2\*cosh(d\*x+c),x)

[Out]  $1/d^2*(a^2*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+1/d^6*b^2*((d*x+c)^7*sinh(d*x+c)-7*(d*x+c)^6*cosh(d*x+c)+42*(d*x+c)^5*sinh(d*x+c)-210*(d*x+c)^4*cosh(d*x+c)+840*(d*x+c)^3*sinh(d*x+c)-2520*(d*x+c)^2*cosh(d*x+c)+5040*(d*x+c)*sinh(d*x+c)-5040*cosh(d*x+c))+7/d^6*b^2*c^6*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+12/d^3*b*c^2*a*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-8/d^3*b*c*a*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-21/d^6*b^2*c^5*((d*x+c)^2*sinh(d*x+c)-2*(d*x+c)*cosh(d*x+c)+2*sinh(d*x+c))-8/d^3*b*c^3*a*((d*x+c)*sinh(d*x+c)-cosh(d*x+c))+35/d^6*b^2*c^4*((d*x+c)^3*sinh(d*x+c)-3*(d*x+c)^2*cosh(d*x+c)+6*(d*x+c)*sinh(d*x+c)-6*cosh(d*x+c))-35/d^6*b^2*c^3*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))+2/d^3*b*a*((d*x+c)^4*sinh(d*x+c)-4*(d*x+c)^3*cosh(d*x+c)+12*(d*x+c)^2*sinh(d*x+c)-24*(d*x+c)*cosh(d*x+c)+24*sinh(d*x+c))-7/d^6*b^2*c*((d*x+c)^6*sinh(d*x+c)-6*(d*x+c)^5*cosh(d*x+c)+30*(d*x+c)^4*sinh(d*x+c)-120*(d*x+c)^3*cosh(d*x+c)+360*(d*x+c)^2*sinh(d*x+c)-720*(d*x+c)*cosh(d*x+c)+720*sinh(d*x+c))+21/d^6*b^2*c^2*((d*x+c)^5*sinh(d*x+c)-5*(d*x+c)^4*cosh(d*x+c)+20*(d*x+c)^3*sinh(d*x+c)-60*(d*x+c)^2*cosh(d*x+c)+120*(d*x+c)*sinh(d*x+c)-120*cosh(d*x+c))-c*a^2*sinh(d*x+c)-1/d^6*b^2*c^7*sinh(d*x+c)+2/d^3*b*c^4*a*sinh(d*x+c))$

**Maxima [A]** time = 1.07065, size = 517, normalized size = 2.21

$$-\frac{1}{80}d\left(\frac{20(d^2x^2e^c - 2dxe^c + 2e^c)a^2e^{(dx)}}{d^3} + \frac{20(d^2x^2 + 2dx + 2)a^2e^{(-dx-c)}}{d^3} + \frac{16(d^5x^5e^c - 5d^4x^4e^c + 20d^3x^3e^c - 60d^2x^2e^c - 60dxe^c + 20e^c)a^2e^{(dx)}}{d^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^3+a)^2\*cosh(d\*x+c),x, algorithm="maxima")

[Out] 
$$-1/80*d*(20*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^{(d*x)}/d^3 + 20*(d^2*x^2 + 2*d*x + 2)*a^2*e^{(-d*x - c)}/d^3 + 16*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*a*b*e^{(d*x)}/d^6 + 16*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*a*b*e^{(-d*x - c)}/d^6 + 5*(d^8*x^8*e^c - 8*d^7*x^7*e^c + 56*d^6*x^6*e^c - 336*d^5*x^5*e^c + 1680*d^4*x^4*e^c - 6720*d^3*x^3*e^c + 20160*d^2*x^2*e^c - 40320*d*x*e^c + 40320*e^c)*b^2*e^{(d*x)}/d^9 + 5*(d^8*x^8 + 8*d^7*x^7 + 56*d^6*x^6 + 336*d^5*x^5 + 1680*d^4*x^4 + 6720*d^3*x^3 + 20160*d^2*x^2 + 40320*d*x + 40320)*b^2*e^{(-d*x - c)}/d^9) + 1/40*(5*b^2*x^8 + 16*a*b*x^5 + 20*a^2*x^2)*cosh(d*x + c)$$

**Fricas [A]** time = 1.75902, size = 358, normalized size = 1.53

$$\frac{(7b^2d^6x^6 + 8abd^6x^3 + 210b^2d^4x^4 + a^2d^6 + 48abd^4x + 2520b^2d^2x^2 + 5040b^2) \cosh(dx + c) - (b^2d^7x^7 + 2abd^7x^4 + 42b^2d^5x^5 + 24a^2bd^5x^2 + 840b^2d^3x^3 + 48a^2bd^3 + (a^2d^7 + 5040b^2d)x) \sinh(dx + c)}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^3+a)^2\*cosh(d\*x+c),x, algorithm="fricas")

[Out] 
$$-((7*b^2*d^6*x^6 + 8*a*b*d^6*x^3 + 210*b^2*d^4*x^4 + a^2*d^6 + 48*a*b*d^4*x + 2520*b^2*d^2*x^2 + 5040*b^2)*\cosh(d*x + c) - (b^2*d^7*x^7 + 2*a*b*d^7*x^4 + 42*b^2*d^5*x^5 + 24*a*b*d^5*x^2 + 840*b^2*d^3*x^3 + 48*a*b*d^3 + (a^2*d^7 + 5040*b^2*d)*x)*\sinh(d*x + c))/d^8$$

**Sympy [A]** time = 15.3885, size = 284, normalized size = 1.21

$$\left\{ \frac{a^2x \sinh(c+dx)}{d} - \frac{a^2 \cosh(c+dx)}{d^2} + \frac{2abx^4 \sinh(c+dx)}{d} - \frac{8abx^3 \cosh(c+dx)}{d^2} + \frac{24abx^2 \sinh(c+dx)}{d^3} - \frac{48abx \cosh(c+dx)}{d^4} + \frac{48ab \sinh(c+dx)}{d^5} + \frac{b^2x^7 \sinh(c+dx)}{d^8} \right\} \cosh(c) + \left( \frac{a^2x^2}{2} + \frac{2abx^5}{5} + \frac{b^2x^8}{8} \right) \cosh(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*3+a)\*\*2\*cosh(d\*x+c),x)



### 3.88 $\int (a + bx^3)^2 \cosh(c + dx) dx$

**Optimal.** Leaf size=186

$$\frac{a^2 \sinh(c + dx)}{d} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{12ab \cosh(c + dx)}{d^4} + \frac{2abx^3 \sinh(c + dx)}{d} + \frac{30b^2x^4 \sinh(c + dx)}{d^3}$$

[Out]  $(-12*a*b*Cosh[c + d*x])/d^4 - (720*b^2*x*Cosh[c + d*x])/d^6 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (6*b^2*x^5*Cosh[c + d*x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (a^2*Sinh[c + d*x])/d + (12*a*b*x*Sinh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c + d*x])/d^5 + (2*a*b*x^3*Sinh[c + d*x])/d + (30*b^2*x^4*Sinh[c + d*x])/d^3 + (b^2*x^6*Sinh[c + d*x])/d$

**Rubi [A]** time = 0.287106, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5277, 2637, 3296, 2638}

$$\frac{a^2 \sinh(c + dx)}{d} - \frac{6abx^2 \cosh(c + dx)}{d^2} + \frac{12abx \sinh(c + dx)}{d^3} - \frac{12ab \cosh(c + dx)}{d^4} + \frac{2abx^3 \sinh(c + dx)}{d} + \frac{30b^2x^4 \sinh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2\*Cosh[c + d\*x], x]

[Out]  $(-12*a*b*Cosh[c + d*x])/d^4 - (720*b^2*x*Cosh[c + d*x])/d^6 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (6*b^2*x^5*Cosh[c + d*x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (a^2*Sinh[c + d*x])/d + (12*a*b*x*Sinh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c + d*x])/d^5 + (2*a*b*x^3*Sinh[c + d*x])/d + (30*b^2*x^4*Sinh[c + d*x])/d^3 + (b^2*x^6*Sinh[c + d*x])/d$

#### Rule 5277

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[Cosh[c + d\*x], (a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[p, 0]

#### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]



Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a + bx^3)^2 \cosh(c + dx) dx &= \int (a^2 \cosh(c + dx) + 2abx^3 \cosh(c + dx) + b^2x^6 \cosh(c + dx)) dx \\
&= a^2 \int \cosh(c + dx) dx + (2ab) \int x^3 \cosh(c + dx) dx + b^2 \int x^6 \cosh(c + dx) dx \\
&= \frac{a^2 \sinh(c + dx)}{d} + \frac{2abx^3 \sinh(c + dx)}{d} + \frac{b^2x^6 \sinh(c + dx)}{d} - \frac{(6ab) \int x^2 \sinh(c + dx) dx}{d} \\
&= -\frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{6b^2x^5 \cosh(c + dx)}{d^2} + \frac{a^2 \sinh(c + dx)}{d} + \frac{2abx^3 \sinh(c + dx)}{d} + \\
&= -\frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{6b^2x^5 \cosh(c + dx)}{d^2} + \frac{a^2 \sinh(c + dx)}{d} + \frac{12abx \sinh(c + dx)}{d^3} + \\
&= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c + dx)}{d^4} - \frac{6b^2x^5 \cosh(c + dx)}{d^2} \\
&= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c + dx)}{d^4} - \frac{6b^2x^5 \cosh(c + dx)}{d^2} \\
&= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{720b^2x \cosh(c + dx)}{d^6} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c + dx)}{d^4} \\
&= -\frac{12ab \cosh(c + dx)}{d^4} - \frac{720b^2x \cosh(c + dx)}{d^6} - \frac{6abx^2 \cosh(c + dx)}{d^2} - \frac{120b^2x^3 \cosh(c + dx)}{d^4}
\end{aligned}$$

**Mathematica [A]** time = 0.219807, size = 111, normalized size = 0.6

$$\frac{(a^2d^6 + 2abd^4x(d^2x^2 + 6) + b^2(d^6x^6 + 30d^4x^4 + 360d^2x^2 + 720)) \sinh(c + dx) - 6bd(ad^2(d^2x^2 + 2) + bx(d^4x^4 + 20d^2x^2 + 60d^2)) \cosh(c + dx)}{d^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*Cosh[c + d*x], x]
```

[Out]  $(-6*b*d*(a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*\text{Cosh}[c + d*x] + (a^2*d^6 + 2*a*b*d^4*x*(6 + d^2*x^2) + b^2*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6))*\text{Sinh}[c + d*x])/d^7$

**Maple [B]** time = 0.009, size = 592, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^3+a)^2*\text{cosh}(d*x+c), x)$

[Out]  $1/d*(-6/d^6*b^2*c*((d*x+c)^5*\text{sinh}(d*x+c)-5*(d*x+c)^4*\text{cosh}(d*x+c)+20*(d*x+c)^3*\text{sinh}(d*x+c)-60*(d*x+c)^2*\text{cosh}(d*x+c)+120*(d*x+c)*\text{sinh}(d*x+c)-120*\text{cosh}(d*x+c))+15/d^6*b^2*c^2*((d*x+c)^4*\text{sinh}(d*x+c)-4*(d*x+c)^3*\text{cosh}(d*x+c)+12*(d*x+c)^2*\text{sinh}(d*x+c)-24*(d*x+c)*\text{cosh}(d*x+c)+24*\text{sinh}(d*x+c))-20/d^6*b^2*c^3*((d*x+c)^3*\text{sinh}(d*x+c)-3*(d*x+c)^2*\text{cosh}(d*x+c)+6*(d*x+c)*\text{sinh}(d*x+c)-6*\text{cosh}(d*x+c))+2/d^3*b*a*((d*x+c)^3*\text{sinh}(d*x+c)-3*(d*x+c)^2*\text{cosh}(d*x+c)+6*(d*x+c)*\text{sinh}(d*x+c)-6*\text{cosh}(d*x+c))+a^2*\text{sinh}(d*x+c)+1/d^6*b^2*((d*x+c)^6*\text{sinh}(d*x+c)-6*(d*x+c)^5*\text{cosh}(d*x+c)+30*(d*x+c)^4*\text{sinh}(d*x+c)-120*(d*x+c)^3*\text{cosh}(d*x+c)+360*(d*x+c)^2*\text{sinh}(d*x+c)-720*(d*x+c)*\text{cosh}(d*x+c)+720*\text{sinh}(d*x+c))+1/d^6*b^2*c^6*\text{sinh}(d*x+c)+6/d^3*b*c^2*a*((d*x+c)*\text{sinh}(d*x+c)-\text{cosh}(d*x+c))-6/d^3*b*c*a*((d*x+c)^2*\text{sinh}(d*x+c)-2*(d*x+c)*\text{cosh}(d*x+c)+2*\text{sinh}(d*x+c))-2/d^3*b*c^3*a*\text{sinh}(d*x+c)-6/d^6*b^2*c^5*((d*x+c)*\text{sinh}(d*x+c)-\text{cosh}(d*x+c))+15/d^6*b^2*c^4*((d*x+c)^2*\text{sinh}(d*x+c)-2*(d*x+c)*\text{cosh}(d*x+c)+2*\text{sinh}(d*x+c)))$

**Maxima [A]** time = 1.04638, size = 328, normalized size = 1.76

$$\frac{a^2 e^{(dx+c)}}{2d} - \frac{a^2 e^{(-dx-c)}}{2d} + \frac{(d^3 x^3 e^c - 3d^2 x^2 e^c + 6d x e^c - 6e^c) a b e^{(dx)}}{d^4} - \frac{(d^3 x^3 + 3d^2 x^2 + 6d x + 6) a b e^{(-dx-c)}}{d^4} + \frac{(d^6 x^6 e^c - 6d^5 x^5 e^c + 30d^4 x^4 e^c - 120d^3 x^3 e^c + 360d^2 x^2 e^c - 720d x e^c + 720e^c) b^2 e^{(dx)}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x^3+a)^2*\text{cosh}(d*x+c), x, \text{algorithm}="maxima")$

[Out]  $1/2*a^2*e^{(d*x + c)}/d - 1/2*a^2*e^{(-d*x - c)}/d + (d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*b*e^{(d*x)}/d^4 - (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*b*e^{(-d*x - c)}/d^4 + 1/2*(d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*b^2*e^{(d*x)}/d^7$

$$- \frac{1}{2}(d^6x^6 + 6d^5x^5 + 30d^4x^4 + 120d^3x^3 + 360d^2x^2 + 720dx + 720)b^2e^{-(dx - c)}/d^7$$

**Fricas [A]** time = 1.7411, size = 285, normalized size = 1.53

$$\frac{6(b^2d^5x^5 + abd^5x^2 + 20b^2d^3x^3 + 2abd^3 + 120b^2dx)\cosh(dx + c) - (b^2d^6x^6 + 2abd^6x^3 + 30b^2d^4x^4 + a^2d^6 + 12abd^4)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c),x, algorithm="fricas")

[Out]  $-(6*(b^2*d^5*x^5 + a*b*d^5*x^2 + 20*b^2*d^3*x^3 + 2*a*b*d^3 + 120*b^2*d*x)*\cosh(d*x + c) - (b^2*d^6*x^6 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 + a^2*d^6 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 + 720*b^2)*\sinh(d*x + c))/d^7$

**Sympy [A]** time = 9.36333, size = 226, normalized size = 1.22

$$\left\{ \begin{array}{l} \frac{a^2 \sinh(c+dx)}{d} + \frac{2abx^3 \sinh(c+dx)}{d} - \frac{6abx^2 \cosh(c+dx)}{d^2} + \frac{12abx \sinh(c+dx)}{d^3} - \frac{12ab \cosh(c+dx)}{d^4} + \frac{b^2x^6 \sinh(c+dx)}{d} - \frac{6b^2x^5 \cosh(c+dx)}{d^2} + \frac{30b^2}{d^2} \\ \left( a^2x + \frac{abx^4}{2} + \frac{b^2x^7}{7} \right) \cosh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2\*cosh(d\*x+c),x)

[Out] Piecewise((a\*\*2\*sinh(c + d\*x)/d + 2\*a\*b\*x\*\*3\*sinh(c + d\*x)/d - 6\*a\*b\*x\*\*2\*cosh(c + d\*x)/d\*\*2 + 12\*a\*b\*x\*sinh(c + d\*x)/d\*\*3 - 12\*a\*b\*cosh(c + d\*x)/d\*\*4 + b\*\*2\*x\*\*6\*sinh(c + d\*x)/d - 6\*b\*\*2\*x\*\*5\*cosh(c + d\*x)/d\*\*2 + 30\*b\*\*2\*x\*\*4\*sinh(c + d\*x)/d\*\*3 - 120\*b\*\*2\*x\*\*3\*cosh(c + d\*x)/d\*\*4 + 360\*b\*\*2\*x\*\*2\*sinh(c + d\*x)/d\*\*5 - 720\*b\*\*2\*x\*cosh(c + d\*x)/d\*\*6 + 720\*b\*\*2\*sinh(c + d\*x)/d\*\*7, Ne(d, 0)), ((a\*\*2\*x + a\*b\*x\*\*4/2 + b\*\*2\*x\*\*7/7)\*cosh(c), True))

**Giac [A]** time = 1.38075, size = 329, normalized size = 1.77

$$\frac{(b^2d^6x^6 - 6b^2d^5x^5 + 2abd^6x^3 + 30b^2d^4x^4 - 6abd^5x^2 + a^2d^6 - 120b^2d^3x^3 + 12abd^4x + 360b^2d^2x^2 - 12abd^3 - 720b^2d)}{2d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c),x, algorithm="giac")

[Out]  $\frac{1}{2}(b^2d^6x^6 - 6b^2d^5x^5 + 2ab^2d^6x^3 + 30b^2d^4x^4 - 6ab^2d^5x^2 + a^2d^6 - 120b^2d^3x^3 + 12ab^2d^4x + 360b^2d^2x^2 - 12ab^2d^3 - 720b^2dx + 720b^2)e^{(dx+c)}/d^7 - \frac{1}{2}(b^2d^6x^6 + 6b^2d^5x^5 + 2ab^2d^6x^3 + 30b^2d^4x^4 + 6ab^2d^5x^2 + a^2d^6 + 120b^2d^3x^3 + 12ab^2d^4x + 360b^2d^2x^2 + 12ab^2d^3 + 720b^2dx + 720b^2)e^{(-dx-c)}/d^7$

$$3.89 \quad \int \frac{(a+bx^3)^2 \cosh(c+dx)}{x} dx$$

**Optimal.** Leaf size=160

$$a^2 \cosh(c)\text{Chi}(dx) + a^2 \sinh(c)\text{Shi}(dx) + \frac{4ab \sinh(c+dx)}{d^3} - \frac{4abx \cosh(c+dx)}{d^2} + \frac{2abx^2 \sinh(c+dx)}{d} + \frac{20b^2x^3 \sinh(c+dx)}{d^3}$$

[Out]  $(-120*b^2*\text{Cosh}[c + d*x])/d^6 - (4*a*b*x*\text{Cosh}[c + d*x])/d^2 - (60*b^2*x^2*\text{Cosh}[c + d*x])/d^4 - (5*b^2*x^4*\text{Cosh}[c + d*x])/d^2 + a^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (4*a*b*\text{Sinh}[c + d*x])/d^3 + (120*b^2*x*\text{Sinh}[c + d*x])/d^5 + (2*a*b*x^2*\text{Sinh}[c + d*x])/d + (20*b^2*x^3*\text{Sinh}[c + d*x])/d^3 + (b^2*x^5*\text{Sinh}[c + d*x])/d + a^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x]$

**Rubi [A]** time = 0.282343, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {5287, 3303, 3298, 3301, 3296, 2637, 2638}

$$a^2 \cosh(c)\text{Chi}(dx) + a^2 \sinh(c)\text{Shi}(dx) + \frac{4ab \sinh(c+dx)}{d^3} - \frac{4abx \cosh(c+dx)}{d^2} + \frac{2abx^2 \sinh(c+dx)}{d} + \frac{20b^2x^3 \sinh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^3)^2\*Cosh[c + d\*x])/x,x]

[Out]  $(-120*b^2*\text{Cosh}[c + d*x])/d^6 - (4*a*b*x*\text{Cosh}[c + d*x])/d^2 - (60*b^2*x^2*\text{Cosh}[c + d*x])/d^4 - (5*b^2*x^4*\text{Cosh}[c + d*x])/d^2 + a^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (4*a*b*\text{Sinh}[c + d*x])/d^3 + (120*b^2*x*\text{Sinh}[c + d*x])/d^5 + (2*a*b*x^2*\text{Sinh}[c + d*x])/d + (20*b^2*x^3*\text{Sinh}[c + d*x])/d^3 + (b^2*x^5*\text{Sinh}[c + d*x])/d + a^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x]$

### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&

NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :=> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :=> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :=> -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] :=> Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :=> -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x} dx &= \int \left( \frac{a^2 \cosh(c + dx)}{x} + 2abx^2 \cosh(c + dx) + b^2x^5 \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x} dx + (2ab) \int x^2 \cosh(c + dx) dx + b^2 \int x^5 \cosh(c + dx) dx \\
&= \frac{2abx^2 \sinh(c + dx)}{d} + \frac{b^2x^5 \sinh(c + dx)}{d} - \frac{(4ab) \int x \sinh(c + dx) dx}{d} - \frac{(5b^2) \int x^4 \sinh(c + dx) dx}{d} \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{5b^2x^4 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) + \frac{2abx^2 \sinh(c + dx)}{d} \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{5b^2x^4 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) + \frac{4ab \sinh(c + dx)}{d^3} + \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{60b^2x^2 \cosh(c + dx)}{d^4} - \frac{5b^2x^4 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) \\
&= -\frac{4abx \cosh(c + dx)}{d^2} - \frac{60b^2x^2 \cosh(c + dx)}{d^4} - \frac{5b^2x^4 \cosh(c + dx)}{d^2} + a^2 \cosh(c) \text{Chi}(dx) \\
&= -\frac{120b^2 \cosh(c + dx)}{d^6} - \frac{4abx \cosh(c + dx)}{d^2} - \frac{60b^2x^2 \cosh(c + dx)}{d^4} - \frac{5b^2x^4 \cosh(c + dx)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.468257, size = 108, normalized size = 0.68

$$a^2 \cosh(c) \text{Chi}(dx) + a^2 \sinh(c) \text{Shi}(dx) + \frac{b(2ad^2(d^2x^2 + 2) + bx(d^4x^4 + 20d^2x^2 + 120)) \sinh(c + dx)}{d^5} - \frac{b(4ad^4x + 5b^2x^4) \cosh(c + dx)}{d^5}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^3)^2\*Cosh[c + d\*x])/x,x]

[Out] -((b\*(4\*a\*d^4\*x + 5\*b\*(24 + 12\*d^2\*x^2 + d^4\*x^4))\*Cosh[c + d\*x])/d^6) + a^2\*Cosh[c]\*CoshIntegral[d\*x] + (b\*(2\*a\*d^2\*(2 + d^2\*x^2) + b\*x\*(120 + 20\*d^2\*x^2 + d^4\*x^4))\*Sinh[c + d\*x])/d^5 + a^2\*Sinh[c]\*SinhIntegral[d\*x]

**Maple [B]** time = 0.086, size = 335, normalized size = 2.1

$$-\frac{b^2e^{-dx-c}x^5}{2d} - \frac{5b^2e^{-dx-c}x^4}{2d^2} - 10\frac{b^2e^{-dx-c}x^3}{d^3} - 30\frac{b^2e^{-dx-c}x^2}{d^4} - 60\frac{b^2e^{-dx-c}x}{d^5} - \frac{a^2e^{-c}\text{Ei}(1, dx)}{2} - \frac{a^2e^c\text{Ei}(1, -dx)}{2} - 60\frac{e^c}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2\*cosh(d\*x+c)/x,x)

[Out]  $-1/2/d*b^2*\exp(-d*x-c)*x^5-5/2/d^2*b^2*\exp(-d*x-c)*x^4-10/d^3*b^2*\exp(-d*x-c)*x^3-30/d^4*b^2*\exp(-d*x-c)*x^2-60/d^5*b^2*\exp(-d*x-c)*x-1/2*a^2*\exp(-c)*\text{Ei}(1,d*x)-1/2*a^2*\exp(c)*\text{Ei}(1,-d*x)-60/d^6*b^2*\exp(d*x+c)-2/d^3*a*b*\exp(-d*x-c)+1/d*a*b*\exp(d*x+c)*x^2-2/d^2*a*b*\exp(d*x+c)*x+2/d^3*a*b*\exp(d*x+c)-60/d^6*b^2*\exp(-d*x-c)-30/d^4*b^2*\exp(d*x+c)*x^2+60/d^5*b^2*\exp(d*x+c)*x+1/2/d*b^2*\exp(d*x+c)*x^5-5/2/d^2*b^2*\exp(d*x+c)*x^4+10/d^3*b^2*\exp(d*x+c)*x^3-1/d*a*b*\exp(-d*x-c)*x^2-2/d^2*a*b*\exp(-d*x-c)*x$

**Maxima [A]** time = 1.23268, size = 390, normalized size = 2.44

$$-\frac{1}{12} \left( 4ab \left( \frac{(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)e^{(dx)}}{d^4} + \frac{(d^3x^3 + 3d^2x^2 + 6dx + 6)e^{(-dx-c)}}{d^4} \right) + b^2 \left( \frac{(d^6x^6e^c - 6d^5x^5e^c + 30d^4x^4e^c - 120d^3x^3e^c + 360d^2x^2e^c - 720dxe^c + 720e^c)e^{(dx)}}{d^7} + \frac{(d^6x^6 + 6d^5x^5 + 30d^4x^4 + 120d^3x^3 + 360d^2x^2 + 720dx + 720)e^{(-dx-c)}}{d^7} + 4a^2*\cosh(d*x + c)*\log(x^3)/d - 6*(\text{Ei}(-d*x)*e^{(-c)} + \text{Ei}(d*x)*e^c)*a^2/d)*d + 1/6*(b^2*x^6 + 4*a*b*x^3 + 2*a^2*\log(x^3))*\cosh(d*x + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c)/x,x, algorithm="maxima")

[Out]  $-1/12*(4*a*b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^{(d*x)}/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^{(-d*x - c)}/d^4) + b^2*((d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*e^{(d*x)}/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*e^{(-d*x - c)}/d^7) + 4*a^2*\cosh(d*x + c)*\log(x^3)/d - 6*(\text{Ei}(-d*x)*e^{(-c)} + \text{Ei}(d*x)*e^c)*a^2/d)*d + 1/6*(b^2*x^6 + 4*a*b*x^3 + 2*a^2*\log(x^3))*\cosh(d*x + c)$

**Fricas [A]** time = 1.78628, size = 365, normalized size = 2.28

$$\frac{2(5b^2d^4x^4 + 4abd^4x + 60b^2d^2x^2 + 120b^2)\cosh(dx + c) - (a^2d^6\text{Ei}(dx) + a^2d^6\text{Ei}(-dx))\cosh(c) - 2(b^2d^5x^5 + 2abd^5x^4 + 20b^2d^3x^3 + 4a*b*d^3 + 120b^2*d*x)*\sinh(d*x + c) - (a^2*d^6*\log(x^3) + 6*a*d^6*\log(x^3))*\cosh(d*x + c)}{2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c)/x,x, algorithm="fricas")

[Out]  $-1/2*(2*(5*b^2*d^4*x^4 + 4*a*b*d^4*x + 60*b^2*d^2*x^2 + 120*b^2)*\cosh(d*x + c) - (a^2*d^6*\text{Ei}(d*x) + a^2*d^6*\text{Ei}(-d*x))*\cosh(c) - 2*(b^2*d^5*x^5 + 2*a*b*d^5*x^4 + 20*b^2*d^3*x^3 + 4*a*b*d^3 + 120*b^2*d*x)*\sinh(d*x + c) - (a^2*d^6*\log(x^3) + 6*a*d^6*\log(x^3))*\cosh(d*x + c)$



$$d^6 \operatorname{Ei}(dx) - a^2 d^6 \operatorname{Ei}(-dx) \sinh(c) / d^6$$

**Sympy [A]** time = 11.71, size = 168, normalized size = 1.05

$$a^2 \sinh(c) \operatorname{Shi}(dx) + a^2 \cosh(c) \operatorname{Chi}(dx) + 2ab \left( \begin{cases} \frac{x^2 \sinh(c+dx)}{d} - \frac{2x \cosh(c+dx)}{d^2} + \frac{2 \sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \frac{x^3 \cosh(c)}{3} & \text{otherwise} \end{cases} \right) + b^2 \left( \begin{cases} \frac{x^5 \sinh(c)}{d} - \frac{5x^4 \cosh(c)}{d^2} + \frac{20x^3 \sinh(c)}{d^3} - \frac{60x^2 \cosh(c)}{d^4} + \frac{120x \sinh(c)}{d^5} - \frac{120 \cosh(c)}{d^6} & \text{for } d \neq 0 \\ \frac{x^6 \cosh(c)}{6} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2\*cosh(d\*x+c)/x,x)

[Out] a\*\*2\*sinh(c)\*Shi(d\*x) + a\*\*2\*cosh(c)\*Chi(d\*x) + 2\*a\*b\*Piecewise((x\*\*2\*sinh(c + d\*x)/d - 2\*x\*cosh(c + d\*x)/d\*\*2 + 2\*sinh(c + d\*x)/d\*\*3, Ne(d, 0)), (x\*\*3\*cosh(c)/3, True)) + b\*\*2\*Piecewise((x\*\*5\*sinh(c + d\*x)/d - 5\*x\*\*4\*cosh(c + d\*x)/d\*\*2 + 20\*x\*\*3\*sinh(c + d\*x)/d\*\*3 - 60\*x\*\*2\*cosh(c + d\*x)/d\*\*4 + 120\*x\*sinh(c + d\*x)/d\*\*5 - 120\*cosh(c + d\*x)/d\*\*6, Ne(d, 0)), (x\*\*6\*cosh(c)/6, True))

**Giac [B]** time = 1.21348, size = 447, normalized size = 2.79

$$b^2 d^5 x^5 e^{(dx+c)} - b^2 d^5 x^5 e^{(-dx-c)} - 5 b^2 d^4 x^4 e^{(dx+c)} - 5 b^2 d^4 x^4 e^{(-dx-c)} + 2 a b d^5 x^2 e^{(dx+c)} - 2 a b d^5 x^2 e^{(-dx-c)} + a^2 d^6 \operatorname{Ei}(-dx) e^{(-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c)/x,x, algorithm="giac")

[Out] 1/2\*(b^2\*d^5\*x^5\*e^(d\*x + c) - b^2\*d^5\*x^5\*e^(-d\*x - c) - 5\*b^2\*d^4\*x^4\*e^(d\*x + c) - 5\*b^2\*d^4\*x^4\*e^(-d\*x - c) + 2\*a\*b\*d^5\*x^2\*e^(d\*x + c) - 2\*a\*b\*d^5\*x^2\*e^(-d\*x - c) + a^2\*d^6\*Ei(-d\*x)\*e^(-c) + a^2\*d^6\*Ei(d\*x)\*e^c + 20\*b^2\*d^3\*x^3\*e^(d\*x + c) - 20\*b^2\*d^3\*x^3\*e^(-d\*x - c) - 4\*a\*b\*d^4\*x\*e^(d\*x + c) - 4\*a\*b\*d^4\*x\*e^(-d\*x - c) - 60\*b^2\*d^2\*x^2\*e^(d\*x + c) - 60\*b^2\*d^2\*x^2\*e^(-d\*x - c) + 4\*a\*b\*d^3\*e^(d\*x + c) - 4\*a\*b\*d^3\*e^(-d\*x - c) + 120\*b^2\*d\*x\*e^(d\*x + c) - 120\*b^2\*d\*x\*e^(-d\*x - c) - 120\*b^2\*e^(d\*x + c) - 120\*b^2\*e^(-d\*x - c))/d^6

$$3.90 \quad \int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx$$

**Optimal.** Leaf size=143

$$a^2 d \sinh(c) \text{Chi}(dx) + a^2 d \cosh(c) \text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} - \frac{2ab \cosh(c+dx)}{d^2} + \frac{2abx \sinh(c+dx)}{d} + \frac{12b^2 x^2 \sinh(c+dx)}{d^3}$$

[Out] (-2\*a\*b\*Cosh[c + d\*x])/d^2 - (a^2\*Cosh[c + d\*x])/x - (24\*b^2\*x\*Cosh[c + d\*x])/d^4 - (4\*b^2\*x^3\*Cosh[c + d\*x])/d^2 + a^2\*d\*CoshIntegral[d\*x]\*Sinh[c] + (24\*b^2\*Sinh[c + d\*x])/d^5 + (2\*a\*b\*x\*Sinh[c + d\*x])/d + (12\*b^2\*x^2\*Sinh[c + d\*x])/d^3 + (b^2\*x^4\*Sinh[c + d\*x])/d + a^2\*d\*Cosh[c]\*SinhIntegral[d\*x]

**Rubi [A]** time = 0.251996, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638, 2637}

$$a^2 d \sinh(c) \text{Chi}(dx) + a^2 d \cosh(c) \text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} - \frac{2ab \cosh(c+dx)}{d^2} + \frac{2abx \sinh(c+dx)}{d} + \frac{12b^2 x^2 \sinh(c+dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^3)^2\*Cosh[c + d\*x])/x^2,x]

[Out] (-2\*a\*b\*Cosh[c + d\*x])/d^2 - (a^2\*Cosh[c + d\*x])/x - (24\*b^2\*x\*Cosh[c + d\*x])/d^4 - (4\*b^2\*x^3\*Cosh[c + d\*x])/d^2 + a^2\*d\*CoshIntegral[d\*x]\*Sinh[c] + (24\*b^2\*Sinh[c + d\*x])/d^5 + (2\*a\*b\*x\*Sinh[c + d\*x])/d + (12\*b^2\*x^2\*Sinh[c + d\*x])/d^3 + (b^2\*x^4\*Sinh[c + d\*x])/d + a^2\*d\*Cosh[c]\*SinhIntegral[d\*x]

### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^2} dx &= \int \left( \frac{a^2 \cosh(c + dx)}{x^2} + 2abx \cosh(c + dx) + b^2 x^4 \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^2} dx + (2ab) \int x \cosh(c + dx) dx + b^2 \int x^4 \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{x} + \frac{2abx \sinh(c + dx)}{d} + \frac{b^2 x^4 \sinh(c + dx)}{d} - \frac{(2ab) \int \sinh(c + dx) dx}{d} \\
&= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{x} - \frac{4b^2 x^3 \cosh(c + dx)}{d^2} + \frac{2abx \sinh(c + dx)}{d} + \frac{b^2 x^4 \sinh(c + dx)}{d} \\
&= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{x} - \frac{4b^2 x^3 \cosh(c + dx)}{d^2} + a^2 d \operatorname{Chi}(dx) \sinh(c) + \frac{2abx \sinh(c + dx)}{d} \\
&= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{x} - \frac{24b^2 x \cosh(c + dx)}{d^4} - \frac{4b^2 x^3 \cosh(c + dx)}{d^2} + a^2 d \operatorname{Chi}(dx) \sinh(c) \\
&= -\frac{2ab \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{x} - \frac{24b^2 x \cosh(c + dx)}{d^4} - \frac{4b^2 x^3 \cosh(c + dx)}{d^2} + a^2 d \operatorname{Chi}(dx) \sinh(c)
\end{aligned}$$

**Mathematica [A]** time = 0.347878, size = 143, normalized size = 1.

$$a^2 d \sinh(c) \operatorname{Chi}(dx) + a^2 d \cosh(c) \operatorname{Shi}(dx) - \frac{a^2 \cosh(c + dx)}{x} - \frac{2ab \cosh(c + dx)}{d^2} + \frac{2abx \sinh(c + dx)}{d} + \frac{12b^2 x^2 \sinh(c + dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^3)^2\*Cosh[c + d\*x])/x^2,x]

[Out] (-2\*a\*b\*Cosh[c + d\*x])/d^2 - (a^2\*Cosh[c + d\*x])/x - (24\*b^2\*x\*Cosh[c + d\*x])/d^4 - (4\*b^2\*x^3\*Cosh[c + d\*x])/d^2 + a^2\*d\*CoshIntegral[d\*x]\*Sinh[c] + (24\*b^2\*Sinh[c + d\*x])/d^5 + (2\*a\*b\*x\*Sinh[c + d\*x])/d + (12\*b^2\*x^2\*Sinh[c + d\*x])/d^3 + (b^2\*x^4\*Sinh[c + d\*x])/d + a^2\*d\*Cosh[c]\*SinhIntegral[d\*x]

**Maple [B]** time = 0.102, size = 296, normalized size = 2.1

$$-\frac{b^2 e^{-dx-c} x^4}{2d} - 2 \frac{b^2 e^{-dx-c} x^3}{d^2} - 6 \frac{b^2 e^{-dx-c} x^2}{d^3} - 12 \frac{b^2 e^{-dx-c} x}{d^4} + \frac{da^2 e^{-c} \operatorname{Ei}(1, dx)}{2} - \frac{abe^{-dx-c}}{d^2} - \frac{a^2 e^{-dx-c}}{2x} - 12 \frac{b^2 e^{-dx-c}}{d^5} - \frac{abe^{-dx-c}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2\*cosh(d\*x+c)/x^2,x)

[Out]  $-1/2/d*b^2*\exp(-d*x-c)*x^4-2/d^2*b^2*\exp(-d*x-c)*x^3-6/d^3*b^2*\exp(-d*x-c)*x^2-12/d^4*b^2*\exp(-d*x-c)*x+1/2*d*a^2*\exp(-c)*\text{Ei}(1,d*x)-a/d^2*b*\exp(-d*x-c)-1/2*a^2*\exp(-d*x-c)/x-12/d^5*b^2*\exp(-d*x-c)-a/d*b*\exp(-d*x-c)*x-1/2*a^2/x*\exp(d*x+c)-1/2*d*a^2*\exp(c)*\text{Ei}(1,-d*x)+12/d^5*b^2*\exp(d*x+c)+a/d*b*\exp(d*x+c)*x-a/d^2*b*\exp(d*x+c)+6/d^3*b^2*\exp(d*x+c)*x^2-12/d^4*b^2*\exp(d*x+c)*x+1/2/d*b^2*\exp(d*x+c)*x^4-2/d^2*b^2*\exp(d*x+c)*x^3$

**Maxima [A]** time = 1.19747, size = 317, normalized size = 2.22

$$-\frac{1}{10} \left( 5 a^2 \text{Ei}(-dx) e^{(-c)} - 5 a^2 \text{Ei}(dx) e^c + \frac{5 (d^2 x^2 e^c - 2 dx e^c + 2 e^c) a b e^{(dx)}}{d^3} + \frac{5 (d^2 x^2 + 2 dx + 2) a b e^{(-dx-c)}}{d^3} + \frac{(d^5 x^5 e^c - 5 d^4 x^4 e^c + 20 d^3 x^3 e^c - 60 d^2 x^2 e^c + 120 d x e^c - 120 e^c) b^2 e^{(d x)}}{d^6} + \frac{(d^5 x^5 + 5 d^4 x^4 + 20 d^3 x^3 + 60 d^2 x^2 + 120 d x + 120) b^2 e^{(-d x - c)}}{d^6} \right) d + \frac{1}{5} (b^2 x^5 + 5 a b x^2 - 5 a^2/x) \cosh(d x + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")`

[Out]  $-1/10*(5*a^2*\text{Ei}(-d*x)*e^{(-c)} - 5*a^2*\text{Ei}(d*x)*e^c + 5*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*b*e^{(d*x)}/d^3 + 5*(d^2*x^2 + 2*d*x + 2)*a*b*e^{(-d*x - c)}/d^3 + (d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b^2*e^{(d*x)}/d^6 + (d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b^2*e^{(-d*x - c)}/d^6)*d + 1/5*(b^2*x^5 + 5*a*b*x^2 - 5*a^2/x)*\cosh(d*x + c)$

**Fricas [A]** time = 1.77474, size = 358, normalized size = 2.5

$$\frac{2 (4 b^2 d^3 x^4 + a^2 d^5 + 2 a b d^3 x + 24 b^2 d x^2) \cosh(dx + c) - (a^2 d^6 x \text{Ei}(dx) - a^2 d^6 x \text{Ei}(-dx)) \cosh(c) - 2 (b^2 d^4 x^5 + 2 a b d^4 x^2 + 12 b^2 d^2 x^3 + 24 b^2 d x) \sinh(dx + c) - (a^2 d^6 x \text{Ei}(d x) + a^2 d^6 x \text{Ei}(-d x)) \sinh(c)}{2 d^5 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")`

[Out]  $-1/2*(2*(4*b^2*d^3*x^4 + a^2*d^5 + 2*a*b*d^3*x + 24*b^2*d*x^2)*\cosh(d*x + c) - (a^2*d^6*x*\text{Ei}(d*x) - a^2*d^6*x*\text{Ei}(-d*x))*\cosh(c) - 2*(b^2*d^4*x^5 + 2*a*b*d^4*x^2 + 12*b^2*d^2*x^3 + 24*b^2*d*x)*\sinh(d*x + c) - (a^2*d^6*x*\text{Ei}(d*x) + a^2*d^6*x*\text{Ei}(-d*x))*\sinh(c))/(d^5*x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2\*cosh(d\*x+c)/x\*\*2,x)

[Out] Integral((a + b\*x\*\*3)\*\*2\*cosh(c + d\*x)/x\*\*2, x)

**Giac [B]** time = 1.26092, size = 416, normalized size = 2.91

$$\frac{b^2 d^4 x^5 e^{(dx+c)} - b^2 d^4 x^5 e^{(-dx-c)} - a^2 d^6 x \operatorname{Ei}(-dx) e^{(-c)} + a^2 d^6 x \operatorname{Ei}(dx) e^c - 4 b^2 d^3 x^4 e^{(dx+c)} - 4 b^2 d^3 x^4 e^{(-dx-c)} + 2 a b d^4 x^2 e^{(dx+c)}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c)/x^2,x, algorithm="giac")

[Out]  $\frac{1}{2} * (b^2 * d^4 * x^5 * e^{(d * x + c)} - b^2 * d^4 * x^5 * e^{(-d * x - c)} - a^2 * d^6 * x * \operatorname{Ei}(-d * x) * e^{(-c)} + a^2 * d^6 * x * \operatorname{Ei}(d * x) * e^c - 4 * b^2 * d^3 * x^4 * e^{(d * x + c)} - 4 * b^2 * d^3 * x^4 * e^{(-d * x - c)} + 2 * a * b * d^4 * x^2 * e^{(d * x + c)} - 2 * a * b * d^4 * x^2 * e^{(-d * x - c)} - a^2 * d^5 * e^{(d * x + c)} + 12 * b^2 * d^2 * x^3 * e^{(d * x + c)} - a^2 * d^5 * e^{(-d * x - c)} - 12 * b^2 * d^2 * x^3 * e^{(-d * x - c)} - 2 * a * b * d^3 * x * e^{(d * x + c)} - 2 * a * b * d^3 * x * e^{(-d * x - c)} - 24 * b^2 * d * x^2 * e^{(d * x + c)} - 24 * b^2 * d * x^2 * e^{(-d * x - c)} + 24 * b^2 * x * e^{(d * x + c)} - 24 * b^2 * x * e^{(-d * x - c)}) / (d^5 * x)$

$$3.91 \quad \int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3} dx$$

**Optimal.** Leaf size=141

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + \frac{2ab \sinh(c+dx)}{d} - \frac{3b^2x^2 \cosh(c+dx)}{d^2}$$

[Out]  $(-6*b^2*\text{Cosh}[c + d*x])/d^4 - (a^2*\text{Cosh}[c + d*x])/(2*x^2) - (3*b^2*x^2*\text{Cosh}[c + d*x])/d^2 + (a^2*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (2*a*b*\text{Sinh}[c + d*x])/d - (a^2*d*\text{Sinh}[c + d*x])/(2*x) + (6*b^2*x*\text{Sinh}[c + d*x])/d^3 + (b^2*x^3*\text{Sinh}[c + d*x])/d + (a^2*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

**Rubi [A]** time = 0.232806, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {5287, 2637, 3297, 3303, 3298, 3301, 3296, 2638}

$$\frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{2x^2} - \frac{a^2d \sinh(c+dx)}{2x} + \frac{2ab \sinh(c+dx)}{d} - \frac{3b^2x^2 \cosh(c+dx)}{d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(a + b*x^3)^2*\text{Cosh}[c + d*x]}{x^3}, x]$

[Out]  $(-6*b^2*\text{Cosh}[c + d*x])/d^4 - (a^2*\text{Cosh}[c + d*x])/(2*x^2) - (3*b^2*x^2*\text{Cosh}[c + d*x])/d^2 + (a^2*d^2*\text{Cosh}[c]*\text{CoshIntegral}[d*x])/2 + (2*a*b*\text{Sinh}[c + d*x])/d - (a^2*d*\text{Sinh}[c + d*x])/(2*x) + (6*b^2*x*\text{Sinh}[c + d*x])/d^3 + (b^2*x^3*\text{Sinh}[c + d*x])/d + (a^2*d^2*\text{Sinh}[c]*\text{SinhIntegral}[d*x])/2$

### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_))^{(n_.)}]^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m, n\}, x\} \ \&\& \ \text{IGtQ}[p, 0]$

### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$   $\text{FreeQ}\{c, d\}, x\}$

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^3} dx &= \int \left( 2ab \cosh(c + dx) + \frac{a^2 \cosh(c + dx)}{x^3} + b^2 x^3 \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^3} dx + (2ab) \int \cosh(c + dx) dx + b^2 \int x^3 \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} + \frac{2ab \sinh(c + dx)}{d} + \frac{b^2 x^3 \sinh(c + dx)}{d} - \frac{(3b^2) \int x^2 \sinh(c + dx) dx}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} + \frac{2ab \sinh(c + dx)}{d} - \frac{a^2 d \sinh(c + dx)}{2x} + \frac{b^2 x^3 \cosh(c + dx)}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{2x^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} + \frac{2ab \sinh(c + dx)}{d} - \frac{a^2 d \sinh(c + dx)}{2x} + \frac{6b^2 x^3 \cosh(c + dx)}{d} \\
&= -\frac{6b^2 \cosh(c + dx)}{d^4} - \frac{a^2 \cosh(c + dx)}{2x^2} - \frac{3b^2 x^2 \cosh(c + dx)}{d^2} + \frac{1}{2} a^2 d^2 \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{2x} + \frac{2ab \sinh(c + dx)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.347951, size = 136, normalized size = 0.96

$$\frac{1}{2} \left( a^2 d^2 \cosh(c) \text{Chi}(dx) + a^2 d^2 \sinh(c) \text{Shi}(dx) - \frac{a^2 \cosh(c + dx)}{x^2} - \frac{a^2 d \sinh(c + dx)}{x} + \frac{4ab \sinh(c + dx)}{d} - \frac{6b^2 x^2 \cosh(c + dx)}{d^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^3)^2\*Cosh[c + d\*x])/x^3,x]

[Out] ((-12\*b^2\*Cosh[c + d\*x])/d^4 - (a^2\*Cosh[c + d\*x])/x^2 - (6\*b^2\*x^2\*Cosh[c + d\*x])/d^2 + a^2\*d^2\*Cosh[c]\*CoshIntegral[d\*x] + (4\*a\*b\*Sinh[c + d\*x])/d - (a^2\*d\*Sinh[c + d\*x])/x + (12\*b^2\*x\*Sinh[c + d\*x])/d^3 + (2\*b^2\*x^3\*Sinh[c + d\*x])/d + a^2\*d^2\*Sinh[c]\*SinhIntegral[d\*x])/2

**Maple [A]** time = 0.115, size = 265, normalized size = 1.9

$$-\frac{b^2 e^{-dx-c} x^3}{2d} - \frac{3b^2 e^{-dx-c} x^2}{2d^2} - 3 \frac{b^2 e^{-dx-c} x}{d^3} - \frac{d^2 a^2 e^{-c} \text{Ei}(1, dx)}{4} - \frac{a b e^{-dx-c}}{d} + \frac{d a^2 e^{-dx-c}}{4x} - \frac{a^2 e^{-dx-c}}{4x^2} - 3 \frac{b^2 e^{-dx-c}}{d^4} - \frac{d^2 a^2 e^{-c} \text{Chi}(dx)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2\*cosh(d\*x+c)/x^3,x)

[Out] -1/2/d\*b^2\*exp(-d\*x-c)\*x^3-3/2/d^2\*b^2\*exp(-d\*x-c)\*x^2-3/d^3\*b^2\*exp(-d\*x-c)\*x-1/4\*d^2\*a^2\*exp(-c)\*Ei(1,d\*x)-a\*b/d\*exp(-d\*x-c)+1/4\*d\*a^2\*exp(-d\*x-c)/x

$$-1/4*a^2*\exp(-d*x-c)/x^2-3/d^4*b^2*\exp(-d*x-c)-1/4*d^2*a^2*\exp(c)*\text{Ei}(1,-d*x)-3/d^4*b^2*\exp(d*x+c)+a*b/d*\exp(d*x+c)-3/2/d^2*b^2*\exp(d*x+c)*x^2+3/d^3*b^2*\exp(d*x+c)*x-1/4*a^2/x^2*\exp(d*x+c)-1/4*d*a^2/x*\exp(d*x+c)+1/2/d*b^2*\exp(d*x+c)*x^3$$

**Maxima [A]** time = 1.20039, size = 274, normalized size = 1.94

$$\frac{1}{8} \left( 2a^2de^{(-c)}\Gamma(-1, dx) + 2a^2de^c\Gamma(-1, -dx) - \frac{8(dx e^c - e^c)abe^{(dx)}}{d^2} - \frac{8(dx + 1)abe^{(-dx-c)}}{d^2} - \frac{(d^4x^4e^c - 4d^3x^3e^c + 12d^2x^2e^c - 4d^4x^4e^{-c} + 4d^3x^3e^{-c} + 12d^2x^2e^{-c} + 24dx + 24)b^2e^{(dx)}}{d^5} + \frac{(d^4x^4 + 4d^3x^3 + 12d^2x^2 + 24dx + 24)b^2e^{(-dx-c)}}{d^5} * d + 1/4*(b^2*x^4 + 8*a*b*x - 2*a^2/x^2)*\cosh(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c)/x^3,x, algorithm="maxima")

[Out] 1/8\*(2\*a^2\*d\*e^(-c)\*gamma(-1, d\*x) + 2\*a^2\*d\*e^c\*gamma(-1, -d\*x) - 8\*(d\*x\*e^c - e^c)\*a\*b\*e^(d\*x)/d^2 - 8\*(d\*x + 1)\*a\*b\*e^(-d\*x - c)/d^2 - (d^4\*x^4\*e^c - 4\*d^3\*x^3\*e^c + 12\*d^2\*x^2\*e^c - 24\*d\*x\*e^c + 24\*e^c)\*b^2\*e^(d\*x)/d^5 - (d^4\*x^4 + 4\*d^3\*x^3 + 12\*d^2\*x^2 + 24\*d\*x + 24)\*b^2\*e^(-d\*x - c)/d^5)\*d + 1/4\*(b^2\*x^4 + 8\*a\*b\*x - 2\*a^2/x^2)\*cosh(d\*x + c)

**Fricas [A]** time = 1.75178, size = 351, normalized size = 2.49

$$\frac{2(6b^2d^2x^4 + a^2d^4 + 12b^2x^2) \cosh(dx + c) - (a^2d^6x^2\text{Ei}(dx) + a^2d^6x^2\text{Ei}(-dx)) \cosh(c) - 2(2b^2d^3x^5 - a^2d^5x + 4abd^3)}{4d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c)/x^3,x, algorithm="fricas")

[Out] -1/4\*(2\*(6\*b^2\*d^2\*x^4 + a^2\*d^4 + 12\*b^2\*x^2)\*cosh(d\*x + c) - (a^2\*d^6\*x^2\*Ei(d\*x) + a^2\*d^6\*x^2\*Ei(-d\*x))\*cosh(c) - 2\*(2\*b^2\*d^3\*x^5 - a^2\*d^5\*x + 4\*a\*b\*d^3\*x^2 + 12\*b^2\*d\*x^3)\*sinh(d\*x + c) - (a^2\*d^6\*x^2\*Ei(d\*x) - a^2\*d^6\*x^2\*Ei(-d\*x))\*sinh(c))/(d^4\*x^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2\*cosh(d\*x+c)/x\*\*3,x)

[Out] Integral((a + b\*x\*\*3)\*\*2\*cosh(c + d\*x)/x\*\*3, x)

**Giac [B]** time = 1.33646, size = 378, normalized size = 2.68

$a^2 d^6 x^2 \operatorname{Ei}(-dx) e^{(-c)} + a^2 d^6 x^2 \operatorname{Ei}(dx) e^c + 2 b^2 d^3 x^5 e^{(dx+c)} - 2 b^2 d^3 x^5 e^{(-dx-c)} - a^2 d^5 x e^{(dx+c)} - 6 b^2 d^2 x^4 e^{(dx+c)} + a^2 d^5 x e^{(-dx-c)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c)/x^3,x, algorithm="giac")

[Out]  $\frac{1}{4} * (a^2 * d^6 * x^2 * \operatorname{Ei}(-d * x) * e^{(-c)} + a^2 * d^6 * x^2 * \operatorname{Ei}(d * x) * e^c + 2 * b^2 * d^3 * x^5 * e^{(d * x + c)} - 2 * b^2 * d^3 * x^5 * e^{(-d * x - c)} - a^2 * d^5 * x * e^{(d * x + c)} - 6 * b^2 * d^2 * x^4 * e^{(d * x + c)} + a^2 * d^5 * x * e^{(-d * x - c)} - 6 * b^2 * d^2 * x^4 * e^{(-d * x - c)} + 4 * a * b * d^3 * x^2 * e^{(d * x + c)} - 4 * a * b * d^3 * x^2 * e^{(-d * x - c)} - a^2 * d^4 * e^{(d * x + c)} + 12 * b^2 * d * x^3 * e^{(d * x + c)} - a^2 * d^4 * e^{(-d * x - c)} - 12 * b^2 * d * x^3 * e^{(-d * x - c)} - 12 * b^2 * x^2 * e^{(d * x + c)} - 12 * b^2 * x^2 * e^{(-d * x - c)}) / (d^4 * x^2)$

$$3.92 \quad \int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4} dx$$

**Optimal.** Leaf size=150

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2d \sinh(c+dx)}{6x^2} - \frac{a^2 \cosh(c+dx)}{3x^3} + 2ab \cosh(c)$$

[Out]  $-(a^2 \text{Cosh}[c + d*x])/(3*x^3) - (a^2*d^2*\text{Cosh}[c + d*x])/(6*x) - (2*b^2*x*\text{Cosh}[c + d*x])/d^2 + 2*a*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (a^2*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 + (2*b^2*\text{Sinh}[c + d*x])/d^3 - (a^2*d*\text{Sinh}[c + d*x])/(6*x^2) + (b^2*x^2*\text{Sinh}[c + d*x])/d + (a^2*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6 + 2*a*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x]$

**Rubi [A]** time = 0.278846, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2637}

$$\frac{1}{6}a^2d^3 \sinh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3 \cosh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{6x} - \frac{a^2d \sinh(c+dx)}{6x^2} - \frac{a^2 \cosh(c+dx)}{3x^3} + 2ab \cosh(c)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^3)^2*\text{Cosh}[c + d*x])/x^4, x]$

[Out]  $-(a^2*\text{Cosh}[c + d*x])/(3*x^3) - (a^2*d^2*\text{Cosh}[c + d*x])/(6*x) - (2*b^2*x*\text{Cosh}[c + d*x])/d^2 + 2*a*b*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + (a^2*d^3*\text{CoshIntegral}[d*x]*\text{Sinh}[c])/6 + (2*b^2*\text{Sinh}[c + d*x])/d^3 - (a^2*d*\text{Sinh}[c + d*x])/(6*x^2) + (b^2*x^2*\text{Sinh}[c + d*x])/d + (a^2*d^3*\text{Cosh}[c]*\text{SinhIntegral}[d*x])/6 + 2*a*b*\text{Sinh}[c]*\text{SinhIntegral}[d*x]$

### Rule 5287

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Cosh}[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x \ \&\& \ \text{IGtQ}[p, 0]$

### Rule 3297

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(m+1)}*\text{Sin}[e + f*x]/(d*(m+1)), x] - \text{Dist}[f/(d*(m+1)), \text{Int}[(c + d*x)^{(m+1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{LtQ}[m, -1]$

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^4} dx &= \int \left( \frac{a^2 \cosh(c + dx)}{x^4} + \frac{2ab \cosh(c + dx)}{x} + b^2 x^2 \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^4} dx + (2ab) \int \frac{\cosh(c + dx)}{x} dx + b^2 \int x^2 \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} + \frac{b^2 x^2 \sinh(c + dx)}{d} - \frac{(2b^2) \int x \sinh(c + dx) dx}{d} + \frac{1}{3} (a^2 d) \int \frac{\sinh(c + dx)}{x^3} dx \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{2b^2 x \cosh(c + dx)}{d^2} + 2ab \cosh(c) \text{Chi}(dx) - \frac{a^2 d \sinh(c + dx)}{6x^2} + \frac{b^2 x^2 \sinh(c + dx)}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{a^2 d^2 \cosh(c + dx)}{6x} - \frac{2b^2 x \cosh(c + dx)}{d^2} + 2ab \cosh(c) \text{Chi}(dx) + \frac{b^2 x^2 \sinh(c + dx)}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{a^2 d^2 \cosh(c + dx)}{6x} - \frac{2b^2 x \cosh(c + dx)}{d^2} + 2ab \cosh(c) \text{Chi}(dx) + \frac{b^2 x^2 \sinh(c + dx)}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{3x^3} - \frac{a^2 d^2 \cosh(c + dx)}{6x} - \frac{2b^2 x \cosh(c + dx)}{d^2} + 2ab \cosh(c) \text{Chi}(dx) + \frac{1}{6} a^2 d^2 \sinh(c + dx)
\end{aligned}$$

**Mathematica [A]** time = 0.569738, size = 135, normalized size = 0.9

$$\frac{1}{6} \left( -\frac{a^2 d^2 \cosh(c + dx)}{x} - \frac{a^2 d \sinh(c + dx)}{x^2} - \frac{2a^2 \cosh(c + dx)}{x^3} + a \text{Chi}(dx) (ad^3 \sinh(c) + 12b \cosh(c)) + a \text{Shi}(dx) (ad^3 \cosh(c) + 12b \sinh(c)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^3)^2\*Cosh[c + d\*x])/x^4,x]

[Out] ((-2\*a^2\*Cosh[c + d\*x])/x^3 - (a^2\*d^2\*Cosh[c + d\*x])/x - (12\*b^2\*x\*Cosh[c + d\*x])/d^2 + a\*CoshIntegral[d\*x]\*(12\*b\*Cosh[c] + a\*d^3\*Sinh[c]) + (12\*b^2\*Sinh[c + d\*x])/d^3 - (a^2\*d\*Sinh[c + d\*x])/x^2 + (6\*b^2\*x^2\*Sinh[c + d\*x])/d + a\*(a\*d^3\*Cosh[c] + 12\*b\*Sinh[c])\*SinhIntegral[d\*x])/6

**Maple [A]** time = 0.128, size = 261, normalized size = 1.7

$$-\frac{b^2 e^{-dx-c} x^2}{2d} - \frac{b^2 e^{-dx-c} x}{d^2} + \frac{d^3 a^2 e^{-c} \text{Ei}(1, dx)}{12} - \frac{a^2 d^2 e^{-dx-c}}{12x} + \frac{da^2 e^{-dx-c}}{12x^2} - \frac{a^2 e^{-dx-c}}{6x^3} - abe^{-c} \text{Ei}(1, dx) - \frac{b^2 e^{-dx-c}}{d^3} - \frac{d^3 a^2 e^{-c}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2\*cosh(d\*x+c)/x^4,x)

[Out]  $-1/2/d*b^2*\exp(-d*x-c)*x^2-1/d^2*b^2*\exp(-d*x-c)*x+1/12*d^3*a^2*\exp(-c)*Ei(1,d*x)-1/12*d^2*a^2*\exp(-d*x-c)/x+1/12*d*a^2*\exp(-d*x-c)/x^2-1/6*a^2*\exp(-d*x-c)/x^3-a*b*\exp(-c)*Ei(1,d*x)-1/d^3*b^2*\exp(-d*x-c)-1/12*d^3*a^2*\exp(c)*Ei(1,-d*x)+1/d^3*b^2*\exp(d*x+c)-1/6*a^2/x^3*\exp(d*x+c)-1/12*d*a^2/x^2*\exp(d*x+c)-1/12*d^2*a^2/x*\exp(d*x+c)-a*b*\exp(c)*Ei(1,-d*x)+1/2/d*b^2*\exp(d*x+c)*x^2-1/d^2*b^2*\exp(d*x+c)*x$

**Maxima [A]** time = 1.25149, size = 254, normalized size = 1.69

$$\frac{1}{6} \left( (d^2 e^{(-c)} \Gamma(-2, dx) - d^2 e^c \Gamma(-2, -dx)) a^2 - b^2 \left( \frac{(d^3 x^3 e^c - 3 d^2 x^2 e^c + 6 dx e^c - 6 e^c) e^{(dx)}}{d^4} + \frac{(d^3 x^3 + 3 d^2 x^2 + 6 dx + 6) e^{(-dx)}}{d^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c)/x^4,x, algorithm="maxima")

[Out]  $1/6*((d^2*e^{(-c)}*\gamma(-2, d*x) - d^2*e^c*\gamma(-2, -d*x))*a^2 - b^2*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^{(d*x)}/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^{(-d*x - c)}/d^4) - 4*a*b*cosh(d*x + c)*\log(x^3)/d + 6*(Ei(-d*x)*e^{(-c)} + Ei(d*x)*e^c)*a*b/d)*d + 1/3*(b^2*x^3 + 2*a*b*\log(x^3) - a^2/x^3)*cosh(d*x + c)$

**Fricas [A]** time = 1.75958, size = 412, normalized size = 2.75

$$\frac{2(a^2 d^5 x^2 + 12 b^2 d x^4 + 2 a^2 d^3) \cosh(dx + c) - ((a^2 d^6 + 12 a b d^3) x^3 Ei(dx) - (a^2 d^6 - 12 a b d^3) x^3 Ei(-dx)) \cosh(c) - 2}{12 d^3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c)/x^4,x, algorithm="fricas")

[Out]  $-1/12*(2*(a^2*d^5*x^2 + 12*b^2*d*x^4 + 2*a^2*d^3)*cosh(d*x + c) - ((a^2*d^6 + 12*a*b*d^3)*x^3*Ei(d*x) - (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*cosh(c) - 2*(6*b^2*d^2*x^5 - a^2*d^4*x + 12*b^2*x^3)*sinh(d*x + c) - ((a^2*d^6 + 12*a*b*d^3)*x^3*Ei(d*x) + (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*sinh(c))/(d^3*x^3)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2\*cosh(d\*x+c)/x\*\*4,x)

[Out] Integral((a + b\*x\*\*3)\*\*2\*cosh(c + d\*x)/x\*\*4, x)

**Giac [A]** time = 1.33523, size = 377, normalized size = 2.51

$$\frac{a^2 d^6 x^3 \operatorname{Ei}(-dx) e^{(-c)} - a^2 d^6 x^3 \operatorname{Ei}(dx) e^c + a^2 d^5 x^2 e^{(dx+c)} - 6 b^2 d^2 x^5 e^{(dx+c)} + a^2 d^5 x^2 e^{(-dx-c)} + 6 b^2 d^2 x^5 e^{(-dx-c)} - 12 a b d^3 x^3 \operatorname{Ei}(-dx) e^{(-c)} - 12 a b d^3 x^3 \operatorname{Ei}(dx) e^c}{(d^3 x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c)/x^4,x, algorithm="giac")

[Out] -1/12\*(a^2\*d^6\*x^3\*Ei(-d\*x)\*e^(-c) - a^2\*d^6\*x^3\*Ei(d\*x)\*e^c + a^2\*d^5\*x^2\*e^(d\*x + c) - 6\*b^2\*d^2\*x^5\*e^(d\*x + c) + a^2\*d^5\*x^2\*e^(-d\*x - c) + 6\*b^2\*d^2\*x^5\*e^(-d\*x - c) - 12\*a\*b\*d^3\*x^3\*Ei(-d\*x)\*e^(-c) - 12\*a\*b\*d^3\*x^3\*Ei(d\*x)\*e^c + a^2\*d^4\*x\*e^(d\*x + c) + 12\*b^2\*d\*x^4\*e^(d\*x + c) - a^2\*d^4\*x\*e^(-d\*x - c) + 12\*b^2\*d\*x^4\*e^(-d\*x - c) + 2\*a^2\*d^3\*e^(d\*x + c) - 12\*b^2\*x^3\*e^(d\*x + c) + 2\*a^2\*d^3\*e^(-d\*x - c) + 12\*b^2\*x^3\*e^(-d\*x - c))/(d^3\*x^3)



$$3.93 \quad \int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} dx$$

**Optimal.** Leaf size=167

$$\frac{1}{24}a^2d^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4 \sinh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{24x^2} - \frac{a^2d^3 \sinh(c+dx)}{24x} - \frac{a^2d \sinh(c+dx)}{12x^3} - \frac{a^2 \cos$$

[Out]  $-\left(\frac{b^2 \text{Cosh}[c+dx]}{d^2}\right) - \frac{a^2 \text{Cosh}[c+dx]}{4x^4} - \frac{a^2 d^2 \text{Cosh}[c+dx]}{24x^2} - \frac{2ab \text{Cosh}[c+dx]}{x} + \frac{a^2 d^4 \text{Cosh}[c] \text{CoshIntegral}[dx]}{24} + 2abd \text{CoshIntegral}[dx] \text{Sinh}[c] - \frac{a^2 d \text{Sinh}[c+dx]}{12x^3} - \frac{a^2 d^3 \text{Sinh}[c+dx]}{24x} + \frac{b^2 x \text{Sinh}[c+dx]}{d} + 2abd \text{Cosh}[c] \text{SinhIntegral}[dx] + \frac{a^2 d^4 \text{Sinh}[c] \text{SinhIntegral}[dx]}{24}$

**Rubi [A]** time = 0.30798, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {5287, 3297, 3303, 3298, 3301, 3296, 2638}

$$\frac{1}{24}a^2d^4 \cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4 \sinh(c)\text{Shi}(dx) - \frac{a^2d^2 \cosh(c+dx)}{24x^2} - \frac{a^2d^3 \sinh(c+dx)}{24x} - \frac{a^2d \sinh(c+dx)}{12x^3} - \frac{a^2 \cos$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x^3)^2\*Cosh[c + d\*x])/x^5,x]

[Out]  $-\left(\frac{b^2 \text{Cosh}[c+dx]}{d^2}\right) - \frac{a^2 \text{Cosh}[c+dx]}{4x^4} - \frac{a^2 d^2 \text{Cosh}[c+dx]}{24x^2} - \frac{2ab \text{Cosh}[c+dx]}{x} + \frac{a^2 d^4 \text{Cosh}[c] \text{CoshIntegral}[dx]}{24} + 2abd \text{CoshIntegral}[dx] \text{Sinh}[c] - \frac{a^2 d \text{Sinh}[c+dx]}{12x^3} - \frac{a^2 d^3 \text{Sinh}[c+dx]}{24x} + \frac{b^2 x \text{Sinh}[c+dx]}{d} + 2abd \text{Cosh}[c] \text{SinhIntegral}[dx] + \frac{a^2 d^4 \text{Sinh}[c] \text{SinhIntegral}[dx]}{24}$

#### Rule 5287

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], (e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

#### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x])/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^5} dx &= \int \left( \frac{a^2 \cosh(c + dx)}{x^5} + \frac{2ab \cosh(c + dx)}{x^2} + b^2 x \cosh(c + dx) \right) dx \\
&= a^2 \int \frac{\cosh(c + dx)}{x^5} dx + (2ab) \int \frac{\cosh(c + dx)}{x^2} dx + b^2 \int x \cosh(c + dx) dx \\
&= -\frac{a^2 \cosh(c + dx)}{4x^4} - \frac{2ab \cosh(c + dx)}{x} + \frac{b^2 x \sinh(c + dx)}{d} - \frac{b^2 \int \sinh(c + dx) dx}{d} + \frac{1}{4} \left( \frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{4x^4} - \frac{2ab \cosh(c + dx)}{x} - \frac{a^2 d \sinh(c + dx)}{12x^3} + \frac{b^2 x \sinh(c + dx)}{d} \right) \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{4x^4} - \frac{2ab \cosh(c + dx)}{x} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{2ab \cosh(c + dx)}{x} + 2abd \operatorname{Chi}(dx) \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{4x^4} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{2ab \cosh(c + dx)}{x} + 2abd \operatorname{Chi}(dx) \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{4x^4} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{2ab \cosh(c + dx)}{x} + 2abd \operatorname{Chi}(dx) \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{4x^4} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{2ab \cosh(c + dx)}{x} + 2abd \operatorname{Chi}(dx) \\
&= -\frac{b^2 \cosh(c + dx)}{d^2} - \frac{a^2 \cosh(c + dx)}{4x^4} - \frac{a^2 d^2 \cosh(c + dx)}{24x^2} - \frac{2ab \cosh(c + dx)}{x} + \frac{1}{24} a^2 d^3 \operatorname{Chi}(dx)
\end{aligned}$$

**Mathematica [A]** time = 0.539467, size = 150, normalized size = 0.9

$$\frac{1}{24} \left( -\frac{a^2 d^2 \cosh(c + dx)}{x^2} - \frac{a^2 d^3 \sinh(c + dx)}{x} - \frac{2a^2 d \sinh(c + dx)}{x^3} - \frac{6a^2 \cosh(c + dx)}{x^4} + ad \operatorname{Chi}(dx) \right) (ad^3 \cosh(c) + 48bd)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x^3)^2\*Cosh[c + d\*x])/x^5,x]

[Out] ((-24\*b^2\*Cosh[c + d\*x])/d^2 - (6\*a^2\*Cosh[c + d\*x])/x^4 - (a^2\*d^2\*Cosh[c + d\*x])/x^2 - (48\*a\*b\*Cosh[c + d\*x])/x + a\*d\*CoshIntegral[d\*x]\*(a\*d^3\*Cosh[c] + 48\*b\*Sinh[c]) - (2\*a^2\*d\*Sinh[c + d\*x])/x^3 - (a^2\*d^3\*Sinh[c + d\*x])/x + (24\*b^2\*x\*Sinh[c + d\*x])/d + a\*d\*(48\*b\*Cosh[c] + a\*d^3\*Sinh[c])\*SinhIntegral[d\*x])/24

**Maple [A]** time = 0.174, size = 292, normalized size = 1.8

$$-\frac{b^2 e^{-dx-c} x}{2d} - \frac{d^4 a^2 e^{-c} \operatorname{Ei}(1, dx)}{48} + \frac{d^3 a^2 e^{-dx-c}}{48x} - \frac{b^2 e^{-dx-c}}{2d^2} - \frac{a^2 d^2 e^{-dx-c}}{48x^2} + \frac{da^2 e^{-dx-c}}{24x^3} - \frac{a^2 e^{-dx-c}}{8x^4} - \frac{abe^{-dx-c}}{x} + dabe^{-c} \operatorname{Ei}(1, dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^2*cosh(d*x+c)/x^5,x)`

[Out] 
$$-1/2*b^2/d*\exp(-d*x-c)*x-1/48*d^4*a^2*\exp(-c)*\text{Ei}(1,d*x)+1/48*d^3*a^2*\exp(-d*x-c)/x-1/2*b^2/d^2*\exp(-d*x-c)-1/48*d^2*a^2*\exp(-d*x-c)/x^2+1/24*d*a^2*\exp(-d*x-c)/x^3-1/8*a^2*\exp(-d*x-c)/x^4-a*b*\exp(-d*x-c)/x+d*a*b*\exp(-c)*\text{Ei}(1,d*x)-1/48*d^4*a^2*\exp(c)*\text{Ei}(1,-d*x)-1/2*b^2/d^2*\exp(d*x+c)-1/48*d^2*a^2/x^2*\exp(d*x+c)-1/48*d^3*a^2/x*\exp(d*x+c)-1/8*a^2/x^4*\exp(d*x+c)-1/24*d*a^2/x^3*\exp(d*x+c)-d*a*b*\exp(c)*\text{Ei}(1,-d*x)+1/2*b^2/d*\exp(d*x+c)*x-a*b/x*\exp(d*x+c)$$

**Maxima [A]** time = 1.24745, size = 208, normalized size = 1.25

$$\frac{1}{8} \left( a^2 d^3 e^{(-c)} \Gamma(-3, dx) + a^2 d^3 e^c \Gamma(-3, -dx) - 8 ab \text{Ei}(-dx) e^{(-c)} + 8 ab \text{Ei}(dx) e^c - \frac{2(d^2 x^2 e^c - 2 dx e^c + 2 e^c) b^2 e^{(dx)}}{d^3} - \frac{2(d^2 x^2 e^c - 2 dx e^c + 2 e^c) b^2 e^{(dx)}}{d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")`

[Out] 
$$\frac{1}{8} * (a^2 * d^3 * e^{(-c)} * \text{gamma}(-3, d*x) + a^2 * d^3 * e^c * \text{gamma}(-3, -d*x) - 8 * a * b * \text{Ei}(-d*x) * e^{(-c)} + 8 * a * b * \text{Ei}(d*x) * e^c - 2 * (d^2 * x^2 * e^c - 2 * d * x * e^c + 2 * e^c) * b^2 * e^{(d*x)} / d^3 - 2 * (d^2 * x^2 * e^c - 2 * d * x * e^c + 2 * e^c) * b^2 * e^{(d*x)} / d^3 * d + 1/4 * (2 * b^2 * x^2 - (8 * a * b * x^3 + a^2) / x^4) * \cosh(d*x + c))$$

**Fricas [A]** time = 1.80379, size = 435, normalized size = 2.6

$$\frac{2(a^2 d^4 x^2 + 48 a b d^2 x^3 + 24 b^2 x^4 + 6 a^2 d^2) \cosh(dx + c) - ((a^2 d^6 + 48 a b d^3) x^4 \text{Ei}(dx) + (a^2 d^6 - 48 a b d^3) x^4 \text{Ei}(-dx)) \cosh(c) + 2(a^2 d^5 x^3 - 24 b^2 d^2 x^5 + 2 a^2 d^3 x) \sinh(dx + c) - ((a^2 d^6 + 48 a b d^3) x^4 \text{Ei}(dx) - (a^2 d^6 - 48 a b d^3) x^4 \text{Ei}(-dx)) \sinh(c)}{d^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")`

[Out] 
$$-1/48 * (2 * (a^2 * d^4 * x^2 + 48 * a * b * d^2 * x^3 + 24 * b^2 * x^4 + 6 * a^2 * d^2) * \cosh(d*x + c) - ((a^2 * d^6 + 48 * a * b * d^3) * x^4 * \text{Ei}(d*x) + (a^2 * d^6 - 48 * a * b * d^3) * x^4 * \text{Ei}(-d*x)) * \cosh(c) + 2 * (a^2 * d^5 * x^3 - 24 * b^2 * d^2 * x^5 + 2 * a^2 * d^3 * x) * \sinh(d*x + c) - ((a^2 * d^6 + 48 * a * b * d^3) * x^4 * \text{Ei}(d*x) - (a^2 * d^6 - 48 * a * b * d^3) * x^4 * \text{Ei}(-d*x)) * \sinh(c)) / (d^2 * x^4)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^3)^2 \cosh(c + dx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2\*cosh(d\*x+c)/x\*\*5,x)

[Out] Integral((a + b\*x\*\*3)\*\*2\*cosh(c + d\*x)/x\*\*5, x)

**Giac [B]** time = 1.33292, size = 425, normalized size = 2.54

$$a^2 d^6 x^4 \operatorname{Ei}(-dx) e^{(-c)} + a^2 d^6 x^4 \operatorname{Ei}(dx) e^c - a^2 d^5 x^3 e^{(dx+c)} + a^2 d^5 x^3 e^{(-dx-c)} - 48 a b d^3 x^4 \operatorname{Ei}(-dx) e^{(-c)} + 48 a b d^3 x^4 \operatorname{Ei}(dx) e^c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*cosh(d\*x+c)/x^5,x, algorithm="giac")

[Out] 1/48\*(a^2\*d^6\*x^4\*Ei(-d\*x)\*e^(-c) + a^2\*d^6\*x^4\*Ei(d\*x)\*e^c - a^2\*d^5\*x^3\*e^(d\*x + c) + a^2\*d^5\*x^3\*e^(-d\*x - c) - 48\*a\*b\*d^3\*x^4\*Ei(-d\*x)\*e^(-c) + 48\*a\*b\*d^3\*x^4\*Ei(d\*x)\*e^c - a^2\*d^4\*x^2\*e^(d\*x + c) + 24\*b^2\*d\*x^5\*e^(d\*x + c) - a^2\*d^4\*x^2\*e^(-d\*x - c) - 24\*b^2\*d\*x^5\*e^(-d\*x - c) - 48\*a\*b\*d^2\*x^3\*e^(d\*x + c) - 48\*a\*b\*d^2\*x^3\*e^(-d\*x - c) - 2\*a^2\*d^3\*x\*e^(d\*x + c) - 24\*b^2\*x^4\*e^(d\*x + c) + 2\*a^2\*d^3\*x\*e^(-d\*x - c) - 24\*b^2\*x^4\*e^(-d\*x - c) - 6\*a^2\*d^2\*e^(d\*x + c) - 6\*a^2\*d^2\*e^(-d\*x - c))/(d^2\*x^4)

### 3.94 $\int \frac{x^4 \cosh(c+dx)}{a+bx^3} dx$

**Optimal.** Leaf size=373

$$\frac{(-1)^{2/3} a^{2/3} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1} a^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{5/3}} + \frac{a^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{5/3}}$$

[Out]  $-(\text{Cosh}[c + d*x]/(b*d^2)) + ((-1)^{(2/3)}*a^{(2/3)}*\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}])*(\text{CoshIntegral}[( (-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(3*b^{(5/3)}) - ((-1)^{(1/3)}*a^{(2/3)}*\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}])*(\text{CoshIntegral}[-(( (-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}) - d*x])/(3*b^{(5/3)}) + (a^{(2/3)}*\text{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}])*(\text{CoshIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(5/3)}) + (x*\text{Sinh}[c + d*x])/(b*d) - ((-1)^{(2/3)}*a^{(2/3)}*\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}])*(\text{SinhIntegral}[( (-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(3*b^{(5/3)}) + (a^{(2/3)}*\text{Sinh}[c - (a^{(1/3)}*d)/b^{(1/3)}])*(\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(5/3)}) - ((-1)^{(1/3)}*a^{(2/3)}*\text{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}])*(\text{SinhIntegral}[( (-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(5/3)})$

**Rubi [A]** time = 0.937188, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5293, 3296, 2638, 3303, 3298, 3301}

$$\frac{(-1)^{2/3} a^{2/3} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1} a^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{5/3}} + \frac{a^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{5/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^4*\text{Cosh}[c + d*x])/(a + b*x^3), x]$

[Out]  $-(\text{Cosh}[c + d*x]/(b*d^2)) + ((-1)^{(2/3)}*a^{(2/3)}*\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}])*(\text{CoshIntegral}[( (-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(3*b^{(5/3)}) - ((-1)^{(1/3)}*a^{(2/3)}*\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}])*(\text{CoshIntegral}[-(( (-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}) - d*x])/(3*b^{(5/3)}) + (a^{(2/3)}*\text{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}])*(\text{CoshIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(5/3)}) + (x*\text{Sinh}[c + d*x])/(b*d) - ((-1)^{(2/3)}*a^{(2/3)}*\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}])*(\text{SinhIntegral}[( (-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(3*b^{(5/3)}) + (a^{(2/3)}*\text{Sinh}[c - (a^{(1/3)}*d)/b^{(1/3)}])*(\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(5/3)}) - ((-1)^{(1/3)}*a^{(2/3)}*\text{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}])*(\text{SinhIntegral}[( (-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(5/3)})$

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol]
:> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol]
:> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 \cosh(c+dx)}{a+bx^3} dx &= \int \left( \frac{x \cosh(c+dx)}{b} - \frac{ax \cosh(c+dx)}{b(a+bx^3)} \right) dx \\
&= \frac{\int x \cosh(c+dx) dx}{b} - \frac{a \int \frac{x \cosh(c+dx)}{a+bx^3} dx}{b} \\
&= \frac{x \sinh(c+dx)}{bd} - \frac{a \int \left( -\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a}+\sqrt[3]{bx})} - \frac{(-1)^{2/3} \cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{bx})} + \frac{\sqrt[3]{-1} \cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{bx})} \right) dx}{b} - \int \sin \\
&= -\frac{\cosh(c+dx)}{bd^2} + \frac{x \sinh(c+dx)}{bd} + \frac{a^{2/3} \int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{3b^{4/3}} - \frac{(\sqrt[3]{-1}a^{2/3}) \int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{bx}} dx}{3b^{4/3}} + \frac{((-1)^{2/3}) \int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{bx}} dx}{3b^{4/3}} \\
&= -\frac{\cosh(c+dx)}{bd^2} + \frac{x \sinh(c+dx)}{bd} + \frac{\left( a^{2/3} \cosh\left( c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \right) \int \frac{\cosh\left( \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx \right)}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{3b^{4/3}} - \frac{\left( \sqrt[3]{-1}a^{2/3} \cosh\left( c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \right) \int \frac{\cosh\left( \frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx \right)}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{3b^{4/3}} \\
&= -\frac{\cosh(c+dx)}{bd^2} + \frac{(-1)^{2/3}a^{2/3} \cosh\left( c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \text{Chi}\left( \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx \right)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3} \cosh\left( c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}} \right) \text{Chi}\left( \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx \right)}{3b^{5/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.462271, size = 213, normalized size = 0.57

$$ad^2 \text{RootSum}\left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \text{Shi}(d(x - \#1))}{\#1} \& \right] + a a$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*Cosh[c + d\*x])/(a + b\*x^3),x]

[Out] -(a\*d^2\*RootSum[a + b\*#1^3 &, (Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] - CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] - Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)])/#1 &] + a\*d^2\*RootSum[a + b\*#1^3 &, (Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] + CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] + Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)])/#1 &] + 6\*b\*(Cosh[c + d\*x] - d\*x\*Sinh[c + d\*x]))/(6\*b^2\*d^2)

**Maple [C]** time = 0.087, size = 925, normalized size = 2.5

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^4 \cosh(dx+c)/(b*x^3+a), x)$

[Out] 
$$-1/2/d*\exp(-d*x-c)/b*x-1/2/d^2*\exp(-d*x-c)/b-1/6/d^2/b^2*\text{sum}((6*_R1^2*b*c^2 -_R1*a*d^3-8*_R1*b*c^3-3*a*c*d^3+3*b*c^4)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2*c^4/b*\text{sum}(1/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c^3/b*\text{sum}(_R1/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/d^2*c^2/b*\text{sum}(_R1^2/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c/b^2*\text{sum}((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d/b*\exp(d*x+c)*x-1/2/d^2/b*\exp(d*x+c)-1/6/d^2/b^2*\text{sum}((6*_R1^2*b*c^2-_R1*a*d^3-8*_R1*b*c^3-3*a*c*d^3+3*b*c^4)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2*c^4/b*\text{sum}(1/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c^3/b*\text{sum}(_R1/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/d^2*c^2/b*\text{sum}(_R1^2/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2*c/b^2*\text{sum}((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^4 \cosh(dx+c)/(b*x^3+a), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.13585, size = 2627, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/12*((a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh
(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c
)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(
-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*
(sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(d*x + 1
/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1)
- c) - (-a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*si
nh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*
d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - si
nh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(
a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) + (a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c
)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*((s
qrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/
2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
- c) + (a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*si
nh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x +
c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqr
t(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(-a*d^3/b)^(2
/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c
+ (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)
*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) + 12*d*x*sinh(d*x + c
) - 12*cosh(d*x + c))/(b*d^2*cosh(d*x + c)^2 - b*d^2*sinh(d*x + c)^2)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*cosh(d*x+c)/(b*x**3+a),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(dx + c)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*cosh(d\*x+c)/(b\*x^3+a),x, algorithm="giac")

[Out] integrate(x^4\*cosh(d\*x + c)/(b\*x^3 + a), x)

$$3.95 \quad \int \frac{x^3 \cosh(c+dx)}{a+bx^3} dx$$

**Optimal.** Leaf size=358

$$\frac{\sqrt[3]{-1}\sqrt[3]{a} \cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{4/3}} - \frac{\sqrt[3]{a} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{4/3}}$$

[Out]  $((-1)^{(1/3)}*a^{(1/3)}*\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[( (-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(3*b^{(4/3)}) - ((-1)^{(2/3)}*a^{(1/3)}*\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[-(((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}) - d*x])/(3*b^{(4/3)}) - (a^{(1/3)}*\text{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(4/3)}) + \text{Sinh}[c + d*x]/(b*d) - ((-1)^{(1/3)}*a^{(1/3)}*\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(3*b^{(4/3)}) - (a^{(1/3)}*\text{Sinh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(4/3)}) - ((-1)^{(2/3)}*a^{(1/3)}*\text{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(4/3)})$

**Rubi [A]** time = 0.648427, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5293, 2637, 5281, 3303, 3298, 3301}

$$\frac{\sqrt[3]{-1}\sqrt[3]{a} \cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{4/3}} - \frac{\sqrt[3]{a} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*Cosh[c + d\*x])/(a + b\*x^3), x]

[Out]  $((-1)^{(1/3)}*a^{(1/3)}*\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[( (-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(3*b^{(4/3)}) - ((-1)^{(2/3)}*a^{(1/3)}*\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[-(((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}) - d*x])/(3*b^{(4/3)}) - (a^{(1/3)}*\text{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(4/3)}) + \text{Sinh}[c + d*x]/(b*d) - ((-1)^{(1/3)}*a^{(1/3)}*\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(3*b^{(4/3)}) - (a^{(1/3)}*\text{Sinh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(4/3)}) - ((-1)^{(2/3)}*a^{(1/3)}*\text{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d*x])/(3*b^{(4/3)})$

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol]
:> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \cosh(c + dx)}{a + bx^3} dx &= \int \left( \frac{\cosh(c + dx)}{b} - \frac{a \cosh(c + dx)}{b(a + bx^3)} \right) dx \\
&= \frac{\int \cosh(c + dx) dx}{b} - \frac{a \int \frac{\cosh(c + dx)}{a + bx^3} dx}{b} \\
&= \frac{\sinh(c + dx)}{bd} - \frac{a \int \left( -\frac{\cosh(c + dx)}{3a^{2/3}(-\sqrt[3]{a} - \sqrt[3]{bx})} - \frac{\cosh(c + dx)}{3a^{2/3}(-\sqrt[3]{a} + \sqrt[3]{-1}\sqrt[3]{bx})} - \frac{\cosh(c + dx)}{3a^{2/3}(-\sqrt[3]{a} - (-1)^{2/3}\sqrt[3]{bx})} \right) dx}{b} \\
&= \frac{\sinh(c + dx)}{bd} + \frac{\sqrt[3]{a} \int \frac{\cosh(c + dx)}{-\sqrt[3]{a} - \sqrt[3]{bx}} dx}{3b} + \frac{\sqrt[3]{a} \int \frac{\cosh(c + dx)}{-\sqrt[3]{a} + \sqrt[3]{-1}\sqrt[3]{bx}} dx}{3b} + \frac{\sqrt[3]{a} \int \frac{\cosh(c + dx)}{-\sqrt[3]{a} - (-1)^{2/3}\sqrt[3]{bx}} dx}{3b} \\
&= \frac{\sinh(c + dx)}{bd} + \frac{\left( \sqrt[3]{a} \cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{-\sqrt[3]{a} - \sqrt[3]{bx}} dx + \left( \sqrt[3]{a} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{a}}{\sqrt[3]{b}}\right)}{-\sqrt[3]{a} - (-1)^{2/3}\sqrt[3]{bx}} dx \right)}{3b} \right)}{3b} \\
&= \frac{\sqrt[3]{-1}\sqrt[3]{a} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right) - (-1)^{2/3}\sqrt[3]{a} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b^{4/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.259845, size = 198, normalized size = 0.55

$$adRootSum\left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \text{Shi}(d(x - \#1))}{\#1^2} \&\right] + ad$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*Cosh[c + d\*x])/(a + b\*x^3), x]

[Out] -(a\*d\*RootSum[a + b\*#1^3 &, (Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] - CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] - Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)])/#1^2 &] + a\*d\*RootSum[a + b\*#1^3 &, (Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] + CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] + Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)])/#1^2 &] - 6\*b\*Sinh[c + d\*x]/(6\*b^2\*d)

**Maple [C]** time = 0.036, size = 671, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*cosh(d*x+c)/(b*x^3+a),x)`

[Out] 
$$-1/2/d*\exp(-d*x-c)/b-1/6/d/b^2*\sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^3/b*\sum(1/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d*c^2/b*\sum(_R1/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d*c/b*\sum(_R1^2/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d/b*\exp(d*x+c)-1/6/d/b^2*\sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^3/b*\sum(1/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d*c^2/b*\sum(_R1/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d*c/b*\sum(_R1^2/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.20258, size = 2591, normalized size = 7.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")`

[Out] 
$$1/12*((a*d^3/b)^{(1/3)}*((\sqrt{-3} + 1)*\cosh(d*x + c)^2 - (\sqrt{-3} + 1)*\sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\cosh(1/2*(a*d^3/b$$

$$\begin{aligned} &)^{(1/3)} * (\sqrt{-3} + 1) + c) - (-a*d^3/b)^{(1/3)} * ((\sqrt{-3} + 1) * \cosh(d*x + c) \\ &)^2 - (\sqrt{-3} + 1) * \sinh(d*x + c)^2 * \text{Ei}(-d*x - 1/2 * (-a*d^3/b)^{(1/3)} * (\sqrt{-3} \\ &- 1)) * \cosh(1/2 * (-a*d^3/b)^{(1/3)} * (\sqrt{-3} + 1) - c) - (a*d^3/b)^{(1/3)} * \\ &(\sqrt{-3} - 1) * \cosh(d*x + c)^2 - (\sqrt{-3} - 1) * \sinh(d*x + c)^2 * \text{Ei}(d*x + 1 \\ &/2 * (a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1)) * \cosh(1/2 * (a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1) \\ &- c) + (-a*d^3/b)^{(1/3)} * ((\sqrt{-3} - 1) * \cosh(d*x + c)^2 - (\sqrt{-3} - 1) * \sinh(d*x + c)^2) * \text{Ei}(-d*x + 1/2 * (-a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1)) * \cosh(1/2 * (-a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1) + c) + 2 * (-a*d^3/b)^{(1/3)} * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(-d*x + (-a*d^3/b)^{(1/3)}) * \cosh(c + (-a*d^3/b)^{(1/3)}) - 2 * (a*d^3/b)^{(1/3)} * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(d*x + (a*d^3/b)^{(1/3)}) * \cosh(-c + (a*d^3/b)^{(1/3)}) + (a*d^3/b)^{(1/3)} * ((\sqrt{-3} + 1) * \cosh(d*x + c)^2 - (\sqrt{-3} + 1) * \sinh(d*x + c)^2) * \text{Ei}(d*x - 1/2 * (a*d^3/b)^{(1/3)} * (\sqrt{-3} + 1)) * \sinh(1/2 * (a*d^3/b)^{(1/3)} * (\sqrt{-3} + 1) + c) - (-a*d^3/b)^{(1/3)} * ((\sqrt{-3} + 1) * \cosh(d*x + c)^2 - (\sqrt{-3} + 1) * \sinh(d*x + c)^2) * \text{Ei}(-d*x - 1/2 * (-a*d^3/b)^{(1/3)} * (\sqrt{-3} + 1)) * \sinh(1/2 * (-a*d^3/b)^{(1/3)} * (\sqrt{-3} + 1) - c) + (a*d^3/b)^{(1/3)} * ((\sqrt{-3} - 1) * \cosh(d*x + c)^2 - (\sqrt{-3} - 1) * \sinh(d*x + c)^2) * \text{Ei}(d*x + 1/2 * (a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1)) * \sinh(1/2 * (a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1) - c) - (-a*d^3/b)^{(1/3)} * ((\sqrt{-3} - 1) * \cosh(d*x + c)^2 - (\sqrt{-3} - 1) * \sinh(d*x + c)^2) * \text{Ei}(-d*x + 1/2 * (-a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1)) * \sinh(1/2 * (-a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1) + c) - 2 * (-a*d^3/b)^{(1/3)} * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(-d*x + (-a*d^3/b)^{(1/3)}) * \sinh(c + (-a*d^3/b)^{(1/3)}) + 2 * (a*d^3/b)^{(1/3)} * (\cosh(d*x + c)^2 - \sinh(d*x + c)^2) * \text{Ei}(d*x + (a*d^3/b)^{(1/3)}) * \sinh(-c + (a*d^3/b)^{(1/3)}) + 12 * \sinh(d*x + c) / (b*d * \cosh(d*x + c)^2 - b*d * \sinh(d*x + c)^2) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(c + dx)}{a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*cosh(d\*x+c)/(b\*x\*\*3+a), x)

[Out] Integral(x\*\*3\*cosh(c + d\*x)/(a + b\*x\*\*3), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(dx + c)}{bx^3 + a} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a), x)
```

### 3.96 $\int \frac{x^2 \cosh(c+dx)}{a+bx^3} dx$

**Optimal.** Leaf size=283

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b} + \frac{\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b} + \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b}$$

[Out] (Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*b) + (Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)) - d\*x])/(3\*b) + (Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*b) - (Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*b) + (Sinh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*b) + (Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*b)

**Rubi [A]** time = 0.455137, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {5293, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b} + \frac{\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b} + \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*Cosh[c + d\*x])/(a + b\*x^3), x]

[Out] (Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*b) + (Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)) - d\*x])/(3\*b) + (Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*b) - (Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*b) + (Sinh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*b) + (Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*b)

#### Rule 5293

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Int[ExpandIntegrand[Cosh[c + d\*x], x^m\*(a + b\*x^n)^p, x], x] /; Fr

eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned} \int \frac{x^2 \cosh(c + dx)}{a + bx^3} dx &= \int \left( \frac{\cosh(c + dx)}{3b^{2/3} (\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{\cosh(c + dx)}{3b^{2/3} (-\sqrt[3]{-1} \sqrt[3]{a} + \sqrt[3]{bx})} + \frac{\cosh(c + dx)}{3b^{2/3} ((-1)^{2/3} \sqrt[3]{a} + \sqrt[3]{bx})} \right) dx \\ &= \frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3b^{2/3}} + \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{-1} \sqrt[3]{a} + \sqrt[3]{bx}} dx}{3b^{2/3}} + \frac{\int \frac{\cosh(c+dx)}{(-1)^{2/3} \sqrt[3]{a} + \sqrt[3]{bx}} dx}{3b^{2/3}} \\ &= \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3b^{2/3}} + \frac{\cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cos\left(\frac{(-1)^{5/6} \sqrt[3]{ad}}{\sqrt[3]{b}} - idx\right)}{-\sqrt[3]{-1} \sqrt[3]{a} + \sqrt[3]{bx}} dx}{3b^{2/3}} + \frac{\cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3b^{2/3}} \\ &= \frac{\cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b} + \frac{\cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-\frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3b} + \frac{\cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{3b} \end{aligned}$$

**Mathematica [C]** time = 0.225213, size = 170, normalized size = 0.6

RootSum[#1^3 b + a &, -sinh(#1 d + c) Chi(d(x - #1)) + cosh(#1 d + c) Chi(d(x - #1)) + sinh(#1 d + c) Shi(d(x - #1))]

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Cosh[c + d\*x])/(a + b\*x^3),x]

[Out] (RootSum[a + b\*#1^3 & , Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] - CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] - Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] & ] + RootSum[a + b\*#1^3 & , Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] + CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] + Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] & ])/(6\*b)

**Maple [C]** time = 0.032, size = 423, normalized size = 1.5

$$\frac{1}{6b} \sum_{_R1=\text{RootOf}(b_Z^3-3_Z^2bc+3_Zbc^2+ad^3-bc^3)} \frac{-_R1^2 e^{-_R1} \text{Ei}(1, dx - _R1 + c)}{-_R1^2 - 2\_R1 c + c^2} - \frac{c^2}{6b} \sum_{_R1=\text{RootOf}(b_Z^3-3_Z^2bc+3_Zbc^2+ad^3-bc^3)} \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*cosh(d\*x+c)/(b\*x^3+a),x)

[Out] -1/6/b\*sum(\_R1^2/(\_R1^2-2\*\_R1\*c+c^2)\*exp(-\_R1)\*Ei(1,d\*x-\_R1+c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))-1/6\*c^2/b\*sum(1/(\_R1^2-2\*\_R1\*c+c^2)\*exp(-\_R1)\*Ei(1,d\*x-\_R1+c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))+1/3\*c/b\*sum(\_R1/(\_R1^2-2\*\_R1\*c+c^2)\*exp(-\_R1)\*Ei(1,d\*x-\_R1+c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))-1/6/b\*sum(\_R1^2/(\_R1^2-2\*\_R1\*c+c^2)\*exp(\_R1)\*Ei(1,-d\*x+\_R1-c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))-1/6\*c^2/b\*sum(1/(\_R1^2-2\*\_R1\*c+c^2)\*exp(\_R1)\*Ei(1,-d\*x+\_R1-c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))+1/3\*c/b\*sum(\_R1/(\_R1^2-2\*\_R1\*c+c^2)\*exp(\_R1)\*Ei(1,-d\*x+\_R1-c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(dx^2 e^{2c} + x e^{2c}) e^{dx} - (dx^2 - x) e^{-dx}}{2(bd^2 x^3 e^c + ad^2 e^c)} + \frac{1}{2} \int \frac{(2bx^3 e^c - 3adx e^c - ae^c) e^{dx}}{b^2 d^2 x^6 + 2abd^2 x^3 + a^2 d^2} dx + \frac{1}{2} \int \frac{(2bx^3 + 3adx - a) e^{-dx}}{b^2 d^2 x^6 e^c + 2abd^2 x^3 e^c + a^2 d^2 e^c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x^2*e^(2*c) + x*e^(2*c))*e^(d*x) - (d*x^2 - x)*e^(-d*x))/(b*d^2*x^3
*e^c + a*d^2*e^c) + 1/2*integrate((2*b*x^3*e^c - 3*a*d*x*e^c - a*e^c)*e^(d*
x)/(b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2), x) + 1/2*integrate((2*b*x^3 + 3
*a*d*x - a)*e^(-d*x)/(b^2*d^2*x^6*e^c + 2*a*b*d^2*x^3*e^c + a^2*d^2*e^c), x
)
```

**Fricas [B]** time = 1.95249, size = 1322, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqr
t(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + Ei(-d*x + 1/2*(-
a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c
) + Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + Ei(d*x + (a*d^
3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt
(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a
*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c)
- Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(s
qrt(-3) - 1) - c) - Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c
+ (-a*d^3/b)^(1/3)) - Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3))
)/b
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(c + dx)}{a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(d*x+c)/(b*x**3+a),x)
```

[Out] Integral(x\*\*2\*cosh(c + d\*x)/(a + b\*x\*\*3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(dx + c)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x^3+a),x, algorithm="giac")

[Out] integrate(x^2\*cosh(d\*x + c)/(b\*x^3 + a), x)

$$3.97 \quad \int \frac{x \cosh(c+dx)}{a+bx^3} dx$$

**Optimal.** Leaf size=345

$$\frac{(-1)^{2/3} \cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3\sqrt[3]{ab^{2/3}}} + \frac{\sqrt[3]{-1} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3\sqrt[3]{ab^{2/3}}} - \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3\sqrt[3]{ab^{2/3}}}$$

[Out]  $-\left((-1)^{2/3}\right) \text{Cosh}\left[c + \left((-1)^{1/3}\right) a^{1/3} d / b^{1/3}\right] \text{CoshIntegral}\left[\left((-1)^{1/3}\right) a^{1/3} d / b^{1/3} - d x\right] / \left(3 a^{1/3} b^{2/3}\right) + \left((-1)^{1/3}\right) \text{Cosh}\left[c - \left((-1)^{2/3}\right) a^{1/3} d / b^{1/3}\right] \text{CoshIntegral}\left[-\left(\left((-1)^{2/3}\right) a^{1/3} d / b^{1/3}\right) - d x\right] / \left(3 a^{1/3} b^{2/3}\right) - \left(\text{Cosh}\left[c - \left(a^{1/3} d / b^{1/3}\right)\right] \text{CoshIntegral}\left[\left(a^{1/3} d / b^{1/3} + d x\right) / \left(3 a^{1/3} b^{2/3}\right) + \left((-1)^{2/3}\right) \text{Sinh}\left[c + \left((-1)^{1/3}\right) a^{1/3} d / b^{1/3}\right] \text{SinhIntegral}\left[\left((-1)^{1/3}\right) a^{1/3} d / b^{1/3} - d x\right] / \left(3 a^{1/3} b^{2/3}\right) - \left(\text{Sinh}\left[c - \left(a^{1/3} d / b^{1/3}\right)\right] \text{SinhIntegral}\left[\left(a^{1/3} d / b^{1/3} + d x\right) / \left(3 a^{1/3} b^{2/3}\right) + \left((-1)^{1/3}\right) \text{Sinh}\left[c - \left((-1)^{2/3}\right) a^{1/3} d / b^{1/3}\right] \text{SinhIntegral}\left[\left((-1)^{2/3}\right) a^{1/3} d / b^{1/3} + d x\right] / \left(3 a^{1/3} b^{2/3}\right)\right)$

**Rubi [A]** time = 0.411566, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {5293, 3303, 3298, 3301}

$$\frac{(-1)^{2/3} \cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3\sqrt[3]{ab^{2/3}}} + \frac{\sqrt[3]{-1} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3\sqrt[3]{ab^{2/3}}} - \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3\sqrt[3]{ab^{2/3}}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\frac{x \text{Cosh}\left[c + d x\right]}{a + b x^3}, x\right]$

[Out]  $-\left((-1)^{2/3}\right) \text{Cosh}\left[c + \left((-1)^{1/3}\right) a^{1/3} d / b^{1/3}\right] \text{CoshIntegral}\left[\left((-1)^{1/3}\right) a^{1/3} d / b^{1/3} - d x\right] / \left(3 a^{1/3} b^{2/3}\right) + \left((-1)^{1/3}\right) \text{Cosh}\left[c - \left((-1)^{2/3}\right) a^{1/3} d / b^{1/3}\right] \text{CoshIntegral}\left[-\left(\left((-1)^{2/3}\right) a^{1/3} d / b^{1/3}\right) - d x\right] / \left(3 a^{1/3} b^{2/3}\right) - \left(\text{Cosh}\left[c - \left(a^{1/3} d / b^{1/3}\right)\right] \text{CoshIntegral}\left[\left(a^{1/3} d / b^{1/3} + d x\right) / \left(3 a^{1/3} b^{2/3}\right) + \left((-1)^{2/3}\right) \text{Sinh}\left[c + \left((-1)^{1/3}\right) a^{1/3} d / b^{1/3}\right] \text{SinhIntegral}\left[\left((-1)^{1/3}\right) a^{1/3} d / b^{1/3} - d x\right] / \left(3 a^{1/3} b^{2/3}\right) - \left(\text{Sinh}\left[c - \left(a^{1/3} d / b^{1/3}\right)\right] \text{SinhIntegral}\left[\left(a^{1/3} d / b^{1/3} + d x\right) / \left(3 a^{1/3} b^{2/3}\right) + \left((-1)^{1/3}\right) \text{Sinh}\left[c - \left((-1)^{2/3}\right) a^{1/3} d / b^{1/3}\right] \text{SinhIntegral}\left[\left((-1)^{2/3}\right) a^{1/3} d / b^{1/3} + d x\right] / \left(3 a^{1/3} b^{2/3}\right)\right)$

Rule 5293

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[Cosh[c + d\*x], x^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x \cosh(c + dx)}{a + bx^3} dx &= \int \left( -\frac{\cosh(c + dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{(-1)^{2/3} \cosh(c + dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a} - \sqrt[3]{-1}\sqrt[3]{bx})} + \frac{\sqrt[3]{-1} \cosh(c + dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a} + (-1)^{2/3}\sqrt[3]{bx})} \right) dx \\
 &= -\frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1} \int \frac{\cosh(c+dx)}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{bx}} dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3} \int \frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{bx}} dx}{3\sqrt[3]{a}\sqrt[3]{b}} \\
 &= -\frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\left(\sqrt[3]{-1} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\right) \int \frac{\cosh\left(\frac{(-1)^{5/6}\sqrt[3]{ad} - idx}{\sqrt[3]{b}}\right)}{\sqrt[3]{a} + (-1)^{2/3}\sqrt[3]{bx}} dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{\sqrt[3]{a} - \sqrt[3]{-1}\sqrt[3]{bx}} dx}{3\sqrt[3]{a}\sqrt[3]{b}} \\
 &= -\frac{(-1)^{2/3} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3\sqrt[3]{ab}^{2/3}} + \frac{\sqrt[3]{-1} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3\sqrt[3]{ab}^{2/3}}
 \end{aligned}$$





```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.13832, size = 1773, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*((a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(sq
rt(-3) - 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x + 1/2
*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) -
c) - (-a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + (a*d^3/b)^(2/3)*(s
qrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(-d*x - 1/2
*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1)
- c) + (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3)
) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(2/3)*(sq
rt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3
/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/
3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(2/3)*Ei(d*x + (a*d^3/b)^(1/3)
)*cosh(-c + (a*d^3/b)^(1/3)) - 2*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3)
))*sinh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(2/3)*Ei(d*x + (a*d^3/b)^(1/3)
)*sinh(-c + (a*d^3/b)^(1/3)))/(a*d^2)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(c + dx)}{a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x**3+a),x)
```

[Out] Integral(x\*cosh(c + d\*x)/(a + b\*x\*\*3), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(dx + c)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x^3+a),x, algorithm="giac")

[Out] integrate(x\*cosh(d\*x + c)/(b\*x^3 + a), x)

### 3.98 $\int \frac{\cosh(c+dx)}{a+bx^3} dx$

**Optimal.** Leaf size=345

$$\frac{\sqrt[3]{-1} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{2/3} \sqrt[3]{b}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{2/3} \sqrt[3]{b}} + \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{2/3} \sqrt[3]{b}}$$

[Out]  $-((-1)^{(1/3)} * \text{Cosh}[c + ((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{CoshIntegral} [((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)} - d * x]) / (3 * a^{(2/3)} * b^{(1/3)}) + ((-1)^{(2/3)} * \text{Cosh}[c - ((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{CoshIntegral} [-(((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)}) - d * x]) / (3 * a^{(2/3)} * b^{(1/3)}) + (\text{Cosh}[c - (a^{(1/3)} * d) / b^{(1/3)}] * \text{CoshIntegral} [(a^{(1/3)} * d) / b^{(1/3)} + d * x]) / (3 * a^{(2/3)} * b^{(1/3)}) + ((-1)^{(1/3)} * \text{Sinh}[c + ((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{SinhIntegral} [((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)} - d * x]) / (3 * a^{(2/3)} * b^{(1/3)}) + (\text{Sinh}[c - (a^{(1/3)} * d) / b^{(1/3)}] * \text{SinhIntegral} [(a^{(1/3)} * d) / b^{(1/3)} + d * x]) / (3 * a^{(2/3)} * b^{(1/3)}) + ((-1)^{(2/3)} * \text{Sinh}[c - ((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{SinhIntegral} [((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)} + d * x]) / (3 * a^{(2/3)} * b^{(1/3)})$

**Rubi [A]** time = 0.401321, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5281, 3303, 3298, 3301}

$$\frac{\sqrt[3]{-1} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{2/3} \sqrt[3]{b}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{2/3} \sqrt[3]{b}} + \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{2/3} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(a + b\*x^3), x]

[Out]  $-((-1)^{(1/3)} * \text{Cosh}[c + ((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{CoshIntegral} [((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)} - d * x]) / (3 * a^{(2/3)} * b^{(1/3)}) + ((-1)^{(2/3)} * \text{Cosh}[c - ((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{CoshIntegral} [-(((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)}) - d * x]) / (3 * a^{(2/3)} * b^{(1/3)}) + (\text{Cosh}[c - (a^{(1/3)} * d) / b^{(1/3)}] * \text{CoshIntegral} [(a^{(1/3)} * d) / b^{(1/3)} + d * x]) / (3 * a^{(2/3)} * b^{(1/3)}) + ((-1)^{(1/3)} * \text{Sinh}[c + ((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{SinhIntegral} [((-1)^{(1/3)} * a^{(1/3)} * d) / b^{(1/3)} - d * x]) / (3 * a^{(2/3)} * b^{(1/3)}) + (\text{Sinh}[c - (a^{(1/3)} * d) / b^{(1/3)}] * \text{SinhIntegral} [(a^{(1/3)} * d) / b^{(1/3)} + d * x]) / (3 * a^{(2/3)} * b^{(1/3)}) + ((-1)^{(2/3)} * \text{Sinh}[c - ((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)}] * \text{SinhIntegral} [((-1)^{(2/3)} * a^{(1/3)} * d) / b^{(1/3)} + d * x]) / (3 * a^{(2/3)} * b^{(1/3)})$

Rule 5281

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[Cosh[c + d\*x], (a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(c+dx)}{a+bx^3} dx &= \int \left( -\frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{bx})} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx})} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx})} \right) dx \\
 &= -\frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx}{3a^{2/3}} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx}} dx}{3a^{2/3}} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}} dx}{3a^{2/3}} \\
 &= -\frac{\cosh\left(c-\frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx}{3a^{2/3}} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cos\left(\frac{(-1)^{5/6}\sqrt[3]{ad}-idx}{\sqrt[3]{b}}\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}} dx}{3a^{2/3}} - \frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}} dx}{3a^{2/3}} \\
 &= -\frac{\sqrt[3]{-1} \cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3} \cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}
 \end{aligned}$$

**Mathematica [C]** time = 0.135678, size = 180, normalized size = 0.52

$$\text{RootSum}\left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \text{Shi}(d(x - \#1))}{\#1^2} \&\right] + \text{RootS}$$


---

6b

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(a + b\*x^3), x]

[Out] (RootSum[a + b\*#1^3 &, (Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] - CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] - Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)])/#1^2 &] + RootSum[a + b\*#1^3 &, (Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] + CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] + Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)])/#1^2 & ])/(6\*b)

**Maple [C]** time = 0.022, size = 143, normalized size = 0.4

$$\frac{d^2}{6b} \sum_{_R1=\text{RootOf}(b_Z^3-3_Z^2bc+3_Zbc^2+ad^3-bc^3)} \frac{e^{-R1} \text{Ei}(1, dx - _R1 + c)}{-R1^2 - 2_R1 c + c^2} - \frac{d^2}{6b} \sum_{_R1=\text{RootOf}(b_Z^3-3_Z^2bc+3_Zbc^2+ad^3-bc^3)} \frac{e^{-R1} \text{Ei}(1, -dx + _R1 - c)}{-R1^2 - 2_R1 c + c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/(b\*x^3+a), x)

[Out] -1/6\*d^2/b\*sum(1/(\_R1^2-2\*\_R1\*c+c^2)\*exp(-\_R1)\*Ei(1, d\*x-\_R1+c), \_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))-1/6\*d^2/b\*sum(1/(\_R1^2-2\*\_R1\*c+c^2)\*exp(\_R1)\*Ei(1, -d\*x+\_R1-c), \_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x^3+a), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.04197, size = 1770, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & -1/12*((a*d^3/b)^{1/3}*(\sqrt{-3} + 1)*Ei(d*x - 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1)) * \cosh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1) + c) - (-a*d^3/b)^{1/3}*(\sqrt{-3} + 1)*Ei(-d*x - 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1)) * \cosh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1) - c) - (a*d^3/b)^{1/3}*(\sqrt{-3} - 1)*Ei(d*x + 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1)) * \cosh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1) - c) + (-a*d^3/b)^{1/3}*(\sqrt{-3} - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1)) * \cosh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1) + c) + (a*d^3/b)^{1/3}*(\sqrt{-3} + 1)*Ei(d*x - 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1)) * \sinh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} + 1) + c) - (-a*d^3/b)^{1/3}*(\sqrt{-3} + 1)*Ei(-d*x - 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1)) * \sinh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} + 1) - c) + (a*d^3/b)^{1/3}*(\sqrt{-3} - 1)*Ei(d*x + 1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1)) * \sinh(1/2*(a*d^3/b)^{1/3}*(\sqrt{-3} - 1) - c) - (-a*d^3/b)^{1/3}*(\sqrt{-3} - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1)) * \sinh(1/2*(-a*d^3/b)^{1/3}*(\sqrt{-3} - 1) + c) + 2*(-a*d^3/b)^{1/3}*Ei(-d*x + (-a*d^3/b)^{1/3}) * \cosh(c + (-a*d^3/b)^{1/3}) - 2*(a*d^3/b)^{1/3}*Ei(d*x + (a*d^3/b)^{1/3}) * \cosh(-c + (a*d^3/b)^{1/3}) - 2*(-a*d^3/b)^{1/3}*Ei(-d*x + (-a*d^3/b)^{1/3}) * \sinh(c + (-a*d^3/b)^{1/3}) + 2*(a*d^3/b)^{1/3}*Ei(d*x + (a*d^3/b)^{1/3}) * \sinh(-c + (a*d^3/b)^{1/3}))/ (a*d) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx)}{a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(b*x**3+a),x)`

[Out] `Integral(cosh(c + d*x)/(a + b*x**3), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*x^3 + a), x)
```



$$3.99 \quad \int \frac{\cosh(c+dx)}{x(a+bx^3)} dx$$

**Optimal.** Leaf size=303

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a} - \frac{\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a} - \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a}$$

[Out] (Cosh[c]\*CoshIntegral[d\*x])/a - (Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*a) - (Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*a) - (Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a) + (Sinh[c]\*SinhIntegral[d\*x])/a + (Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*a) - (Sinh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a) - (Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a)

**Rubi [A]** time = 0.523232, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {5293, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a} - \frac{\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a} - \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x\*(a + b\*x^3)),x]

[Out] (Cosh[c]\*CoshIntegral[d\*x])/a - (Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*a) - (Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*a) - (Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a) + (Sinh[c]\*SinhIntegral[d\*x])/a + (Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*a) - (Sinh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a) - (Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a)

**Rule 5293**

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx^3)} dx &= \int \left( \frac{\cosh(c+dx)}{ax} - \frac{bx^2 \cosh(c+dx)}{a(a+bx^3)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x} dx}{a} - \frac{b \int \frac{x^2 \cosh(c+dx)}{a+bx^3} dx}{a} \\
&= -\frac{b \int \left( \frac{\cosh(c+dx)}{3b^{2/3}(\sqrt[3]{a}+\sqrt[3]{bx})} + \frac{\cosh(c+dx)}{3b^{2/3}(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{bx})} + \frac{\cosh(c+dx)}{3b^{2/3}((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{bx})} \right) dx}{a} + \frac{\cosh(c) \int \frac{\cosh(dx)}{x} dx}{a} + \frac{\sinh(c)}{a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\sqrt[3]{b} \int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{3a} - \frac{\sqrt[3]{b} \int \frac{\cosh(c+dx)}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{bx}} dx}{3a} - \frac{\sqrt[3]{b} \int \frac{\cosh(c+dx)}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{bx}} dx}{3a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\left(\sqrt[3]{b} \cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{3a} - \frac{\left(\sqrt[3]{b} \cosh\left(c + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}}+dx\right)}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{3a} \\
&= \frac{\cosh(c)\text{Chi}(dx)}{a} - \frac{\cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a} - \frac{\cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a}
\end{aligned}$$

**Mathematica [C]** time = 0.267678, size = 186, normalized size = 0.61

$$\text{RootSum}\left[\#1^3b+a\&, -\sinh(\#1d+c)\text{Chi}(d(x-\#1)) + \cosh(\#1d+c)\text{Chi}(d(x-\#1)) + \sinh(\#1d+c)\text{Shi}(d(x-\#1))\right]$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x\*(a + b\*x^3)), x]

[Out]  $-( -6*\text{Cosh}[c]*\text{CoshIntegral}[d*x] + \text{RootSum}[a + b*\#1^3 \&, \text{Cosh}[c + d*\#1]*\text{CoshIntegral}[d*(x - \#1)] - \text{CoshIntegral}[d*(x - \#1)]*\text{Sinh}[c + d*\#1] - \text{Cosh}[c + d*\#1]*\text{SinhIntegral}[d*(x - \#1)] + \text{Sinh}[c + d*\#1]*\text{SinhIntegral}[d*(x - \#1)] \& ] + \text{RootSum}[a + b*\#1^3 \&, \text{Cosh}[c + d*\#1]*\text{CoshIntegral}[d*(x - \#1)] + \text{CoshIntegral}[d*(x - \#1)]*\text{Sinh}[c + d*\#1] + \text{Cosh}[c + d*\#1]*\text{SinhIntegral}[d*(x - \#1)] + \text{Sinh}[c + d*\#1]*\text{SinhIntegral}[d*(x - \#1)] \& ] - 6*\text{Sinh}[c]*\text{SinhIntegral}[d*x] )/(6*a)$

**Maple [C]** time = 0.036, size = 138, normalized size = 0.5

$$\frac{e^{-c}\text{Ei}(1, dx)}{2a} + \frac{\sum_{R1=\text{RootOf}(bZ^3-3Z^2bc+3Zbc^2+ad^3-bc^3)} e^{-R1}\text{Ei}(1, dx - R1 + c)}{6a} - \frac{e^c\text{Ei}(1, -dx)}{2a} + \frac{\sum_{R1=\text{RootOf}(bZ^3-3Z^2bc+3Zbc^2+ad^3-bc^3)} e^{R1}\text{Ei}(1, -dx - R1 - c)}{6a}$$



$$\frac{1}{b^{1/3}}(\sqrt{-3} - 1) \sinh\left(\frac{1}{2}(-a d^3/b)^{1/3}(\sqrt{-3} - 1) + c\right) - \operatorname{Ei}(-d x + (-a d^3/b)^{1/3}) \sinh(c + (-a d^3/b)^{1/3}) - 3(\operatorname{Ei}(d x) - \operatorname{Ei}(-d x)) \sinh(c) - \operatorname{Ei}(d x + (a d^3/b)^{1/3}) \sinh(-c + (a d^3/b)^{1/3}) / a$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x\*\*3+a), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^3+a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x^3+a), x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((b\*x^3 + a)\*x), x)

$$3.100 \quad \int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx$$

**Optimal.** Leaf size=381

$$\frac{(-1)^{2/3} \sqrt[3]{b} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1} \sqrt[3]{b} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{4/3}} + \frac{\sqrt[3]{b} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{4/3}}$$

[Out] -(Cosh[c + d\*x]/(a\*x)) + ((-1)^(2/3)\*b^(1/3)\*Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(3\*a^(4/3)) - ((-1)^(1/3)\*b^(1/3)\*Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(3\*a^(4/3)) + (b^(1/3)\*Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x]/(3\*a^(4/3)) + (d\*CoshIntegral[d\*x]\*Sinh[c])/a + (d\*Cosh[c]\*SinhIntegral[d\*x])/a - ((-1)^(2/3)\*b^(1/3)\*Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(3\*a^(4/3)) + (b^(1/3)\*Sinh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x]/(3\*a^(4/3)) - ((-1)^(1/3)\*b^(1/3)\*Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x]/(3\*a^(4/3))

**Rubi [A]** time = 0.600099, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {5293, 3297, 3303, 3298, 3301}

$$\frac{(-1)^{2/3} \sqrt[3]{b} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \operatorname{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1} \sqrt[3]{b} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{4/3}} + \frac{\sqrt[3]{b} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x^2\*(a + b\*x^3)),x]

[Out] -(Cosh[c + d\*x]/(a\*x)) + ((-1)^(2/3)\*b^(1/3)\*Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(3\*a^(4/3)) - ((-1)^(1/3)\*b^(1/3)\*Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(3\*a^(4/3)) + (b^(1/3)\*Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x]/(3\*a^(4/3)) + (d\*CoshIntegral[d\*x]\*Sinh[c])/a + (d\*Cosh[c]\*SinhIntegral[d\*x])/a - ((-1)^(2/3)\*b^(1/3)\*Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(3\*a^(4/3)) + (b^(1/3)\*Sinh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x]/(3\*a^(4/3)) - ((-1)^(1/3)\*b^(1/3)\*Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x]/(3\*a^(4/3))

) $\cdot b^{1/3} \cdot \sinh\left[c - \frac{(-1)^{2/3} a^{1/3} d}{b^{1/3}}\right] \cdot \operatorname{SinhIntegral}\left[\frac{(-1)^{2/3} a^{1/3} d}{b^{1/3} + d x}\right] / (3 a^{4/3})$

### Rule 5293

$\operatorname{Int}[\operatorname{Cosh}[c + d x] (x)^m (a + b x^n)^p, x]$   $\rightarrow$   $\operatorname{Int}[\operatorname{ExpandIntegrand}[\operatorname{Cosh}[c + d x], x^m (a + b x^n)^p, x], x]$  /;  $\operatorname{FreeQ}\{a, b, c, d, x\}$  &&  $\operatorname{ILtQ}[p, 0]$  &&  $\operatorname{IntegerQ}[m]$  &&  $\operatorname{IGtQ}[n, 0]$  &&  $(\operatorname{EqQ}[n, 2] \mid \mid \operatorname{EqQ}[p, -1])$

### Rule 3297

$\operatorname{Int}[(c + d x)^m \sin[e + f x], x]$   $\rightarrow$   $\operatorname{Simp}[(c + d x)^{m+1} \sin[e + f x] / (d(m+1)), x] - \operatorname{Dist}[f / (d(m+1)), \operatorname{Int}[(c + d x)^{m+1} \cos[e + f x], x], x]$  /;  $\operatorname{FreeQ}\{c, d, e, f, x\}$  &&  $\operatorname{LtQ}[m, -1]$

### Rule 3303

$\operatorname{Int}[\sin[e + f x] / (c + d x), x]$   $\rightarrow$   $\operatorname{Dist}[\cos[(d e - c f) / d], \operatorname{Int}[\sin[(c f) / d + f x] / (c + d x), x], x] + \operatorname{Dist}[\sin[(d e - c f) / d], \operatorname{Int}[\cos[(c f) / d + f x] / (c + d x), x], x]$  /;  $\operatorname{FreeQ}\{c, d, e, f, x\}$  &&  $\operatorname{NeQ}[d e - c f, 0]$

### Rule 3298

$\operatorname{Int}[\sin[e + (c + d x) f z] / (c + d x), x]$   $\rightarrow$   $\operatorname{Simp}[\operatorname{I} \operatorname{SinhIntegral}[(c f f z) / d + f f z x] / d, x]$  /;  $\operatorname{FreeQ}\{c, d, e, f, f z, x\}$  &&  $\operatorname{EqQ}[d e - c f f z \operatorname{I}, 0]$

### Rule 3301

$\operatorname{Int}[\sin[e + (c + d x) f z] / (c + d x), x]$   $\rightarrow$   $\operatorname{Simp}[\operatorname{CoshIntegral}[(c f f z) / d + f f z x] / d, x]$  /;  $\operatorname{FreeQ}\{c, d, e, f, f z, x\}$  &&  $\operatorname{EqQ}[d(e - \operatorname{Pi}/2) - c f f z \operatorname{I}, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx &= \int \left( \frac{\cosh(c+dx)}{ax^2} - \frac{bx \cosh(c+dx)}{a(a+bx^3)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{a} - \frac{b \int \frac{x \cosh(c+dx)}{a+bx^3} dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} - \frac{b \int \left( -\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a}+\sqrt[3]{bx})} - \frac{(-1)^{2/3} \cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{bx})} + \frac{\sqrt[3]{-1} \cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a+(-1)^{2/3}\sqrt[3]{bx})} \right) dx}{a} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{a} \\
&= -\frac{\cosh(c+dx)}{ax} + \frac{b^{2/3} \int \frac{\cosh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{3a^{4/3}} - \frac{(\sqrt[3]{-1}b^{2/3}) \int \frac{\cosh(c+dx)}{\sqrt[3]{a+(-1)^{2/3}\sqrt[3]{bx}}} dx}{3a^{4/3}} + \frac{((-1)^{2/3}b^{2/3}) \int \frac{\cosh(c+dx)}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{bx}}} dx}{3a^{4/3}} \\
&= -\frac{\cosh(c+dx)}{ax} + \frac{d \operatorname{Chi}(dx) \sinh(c)}{a} + \frac{d \cosh(c) \operatorname{Shi}(dx)}{a} + \frac{\left( b^{2/3} \cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{3a^{4/3}} \\
&= -\frac{\cosh(c+dx)}{ax} + \frac{(-1)^{2/3} \sqrt[3]{b} \cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \operatorname{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{4/3}} - \frac{\sqrt[3]{-1} \sqrt[3]{b} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{4/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.406599, size = 215, normalized size = 0.56

$$x \operatorname{RootSum}\left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \operatorname{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \operatorname{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \operatorname{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \operatorname{Shi}(d(x - \#1))}{\#1}\right] + x \operatorname{RootSum}\left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \operatorname{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \operatorname{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \operatorname{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \operatorname{Shi}(d(x - \#1))}{\#1}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x^2\*(a + b\*x^3)),x]

[Out]  $-(6 * \operatorname{Cosh}[c + d * x] + x * \operatorname{RootSum}[a + b * \#1^3 \&, (\operatorname{Cosh}[c + d * \#1] * \operatorname{CoshIntegral}[d * (x - \#1)] - \operatorname{CoshIntegral}[d * (x - \#1)] * \operatorname{Sinh}[c + d * \#1] - \operatorname{Cosh}[c + d * \#1] * \operatorname{SinhIntegral}[d * (x - \#1)] + \operatorname{Sinh}[c + d * \#1] * \operatorname{SinhIntegral}[d * (x - \#1)]) / \#1 \&] + x * \operatorname{RootSum}[a + b * \#1^3 \&, (\operatorname{Cosh}[c + d * \#1] * \operatorname{CoshIntegral}[d * (x - \#1)] + \operatorname{CoshIntegral}[d * (x - \#1)] * \operatorname{Sinh}[c + d * \#1] + \operatorname{Cosh}[c + d * \#1] * \operatorname{SinhIntegral}[d * (x - \#1)] + \operatorname{Sinh}[c + d * \#1] * \operatorname{SinhIntegral}[d * (x - \#1)]) / \#1 \&] - 6 * d * x * \operatorname{CoshIntegral}[d * x] * \operatorname{Sinh}[c] - 6 * d * x * \operatorname{Cosh}[c] * \operatorname{SinhIntegral}[d * x]) / (6 * a * x)$

**Maple [C]** time = 0.047, size = 187, normalized size = 0.5

$$-\frac{e^{-dx-c}}{2ax} + \frac{de^{-c} \operatorname{Ei}(1, dx)}{2a} + \frac{d}{6a} \sum_{\substack{R1 = \operatorname{RootOf}(bZ^3 - 3Z^2bc + 3Zbc^2 + ad^3 - bc^3) \\ R1 = -c}} \frac{e^{-R1} \operatorname{Ei}(1, dx - R1 + c)}{R1 - c} - \frac{e^{dx+c}}{2ax} - \frac{de^c \operatorname{Ei}(1, -dx)}{2a}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/x^2/(b*x^3+a),x)`

[Out] 
$$-1/2*\exp(-d*x-c)/a/x+1/2*d/a*\exp(-c)*Ei(1,d*x)+1/6*d/a*\sum(1/(_R1-c)*\exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/a/x*\exp(d*x+c)-1/2*d/a*\exp(c)*Ei(1,-d*x)+1/6*d/a*\sum(1/(_R1-c)*\exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.18902, size = 2921, normalized size = 7.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")`

[Out] 
$$-1/12*(12*a*d^2*\cosh(d*x + c) - (a*d^3/b)^{(2/3)}*((\sqrt{-3}*b*x - b*x)*\cosh(d*x + c)^2 - (\sqrt{-3}*b*x - b*x)*\sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) + c) - (a*d^3/b)^{(2/3)}*((\sqrt{-3}*b*x - b*x)*\cosh(d*x + c)^2 - (\sqrt{-3}*b*x - b*x)*\sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\cosh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) - c) + (a*d^3/b)^{(2/3)}*((\sqrt{-3}*b*x + b*x)*\cosh(d*x + c)^2 - (\sqrt{-3}*b*x + b*x)*\sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) - c) + (-a*d^3/b)^{(2/3)}*((\sqrt{-3}*b*x + b*x)*\cosh(d*x + c)^2 - (\sqrt{-3}*b*x + b*x)*\sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\cosh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) + c) - 2*(b*x*\cosh(d*x + c)^2 - b*x*\sinh(d*x + c)^2)*(-a*d^3/b)^{(2/3)}*Ei(-d*x + (-a*d^3/b)^{(1/3)})*\cosh(c + (-a*d^3/b)^{(1/3)})$$

) - 2\*(b\*x\*cosh(d\*x + c)^2 - b\*x\*sinh(d\*x + c)^2)\*(a\*d^3/b)^(2/3)\*Ei(d\*x + (a\*d^3/b)^(1/3))\*cosh(-c + (a\*d^3/b)^(1/3)) - (a\*d^3/b)^(2/3)\*((sqrt(-3)\*b\*x - b\*x)\*cosh(d\*x + c)^2 - (sqrt(-3)\*b\*x - b\*x)\*sinh(d\*x + c)^2)\*Ei(d\*x - 1/2\*(a\*d^3/b)^(1/3)\*(sqrt(-3) + 1))\*sinh(1/2\*(a\*d^3/b)^(1/3)\*(sqrt(-3) + 1) + c) - (-a\*d^3/b)^(2/3)\*((sqrt(-3)\*b\*x - b\*x)\*cosh(d\*x + c)^2 - (sqrt(-3)\*b\*x - b\*x)\*sinh(d\*x + c)^2)\*Ei(-d\*x - 1/2\*(-a\*d^3/b)^(1/3)\*(sqrt(-3) + 1))\*sinh(1/2\*(-a\*d^3/b)^(1/3)\*(sqrt(-3) + 1) - c) - (a\*d^3/b)^(2/3)\*((sqrt(-3)\*b\*x + b\*x)\*cosh(d\*x + c)^2 - (sqrt(-3)\*b\*x + b\*x)\*sinh(d\*x + c)^2)\*Ei(d\*x + 1/2\*(a\*d^3/b)^(1/3)\*(sqrt(-3) - 1))\*sinh(1/2\*(a\*d^3/b)^(1/3)\*(sqrt(-3) - 1) - c) - (-a\*d^3/b)^(2/3)\*((sqrt(-3)\*b\*x + b\*x)\*cosh(d\*x + c)^2 - (sqrt(-3)\*b\*x + b\*x)\*sinh(d\*x + c)^2)\*Ei(-d\*x + 1/2\*(-a\*d^3/b)^(1/3)\*(sqrt(-3) - 1))\*sinh(1/2\*(-a\*d^3/b)^(1/3)\*(sqrt(-3) - 1) + c) + 2\*(b\*x\*cosh(d\*x + c)^2 - b\*x\*sinh(d\*x + c)^2)\*(-a\*d^3/b)^(2/3)\*Ei(-d\*x + (-a\*d^3/b)^(1/3))\*sinh(c + (-a\*d^3/b)^(1/3)) + 2\*(b\*x\*cosh(d\*x + c)^2 - b\*x\*sinh(d\*x + c)^2)\*(a\*d^3/b)^(2/3)\*Ei(d\*x + (a\*d^3/b)^(1/3))\*sinh(-c + (a\*d^3/b)^(1/3)) - 6\*(a\*d^3\*x\*Ei(d\*x) - a\*d^3\*x\*Ei(-d\*x))\*cosh(c) - 6\*(a\*d^3\*x\*Ei(d\*x) + a\*d^3\*x\*Ei(-d\*x))\*sinh(c))/(a^2\*d^2\*x\*cosh(d\*x + c)^2 - a^2\*d^2\*x\*sinh(d\*x + c)^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x\*\*2/(b\*x\*\*3+a),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^3 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^2/(b\*x^3+a),x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((b\*x^3 + a)\*x^2), x)

$$3.101 \quad \int \frac{\cosh(c+dx)}{x^3(a+bx^3)} dx$$

**Optimal.** Leaf size=410

$$\frac{\sqrt[3]{-1}b^{2/3} \cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{5/3}} - \frac{(-1)^{2/3}b^{2/3} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{5/3}} - \frac{b^{2/3} \cosh(c)}{3a^{5/3}}$$

[Out] -Cosh[c + d\*x]/(2\*a\*x^2) + (d^2\*Cosh[c]\*CoshIntegral[d\*x])/(2\*a) + ((-1)^(1/3)\*b^(2/3)\*Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(3\*a^(5/3)) - ((-1)^(2/3)\*b^(2/3)\*Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)) - d\*x]/(3\*a^(5/3)) - (b^(2/3)\*Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a^(5/3)) - (d\*Sinh[c + d\*x])/(2\*a\*x) + (d^2\*Sinh[c]\*SinhIntegral[d\*x])/(2\*a) - ((-1)^(1/3)\*b^(2/3)\*Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*a^(5/3)) - (b^(2/3)\*Sinh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a^(5/3)) - ((-1)^(2/3)\*b^(2/3)\*Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a^(5/3))

**Rubi [A]** time = 0.639085, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5293, 3297, 3303, 3298, 3301, 5281}

$$\frac{\sqrt[3]{-1}b^{2/3} \cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{5/3}} - \frac{(-1)^{2/3}b^{2/3} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{3a^{5/3}} - \frac{b^{2/3} \cosh(c)}{3a^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(x^3\*(a + b\*x^3)),x]

[Out] -Cosh[c + d\*x]/(2\*a\*x^2) + (d^2\*Cosh[c]\*CoshIntegral[d\*x])/(2\*a) + ((-1)^(1/3)\*b^(2/3)\*Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(3\*a^(5/3)) - ((-1)^(2/3)\*b^(2/3)\*Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)) - d\*x]/(3\*a^(5/3)) - (b^(2/3)\*Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a^(5/3)) - (d\*Sinh[c + d\*x])/(2\*a\*x) + (d^2\*Sinh[c]\*SinhIntegral[d\*x])/(2\*a) - ((-1)^(1/3)\*b^(2/3)\*Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(3\*a^(5/3)) - (b^(2/3)\*Sinh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a^(5/3)) - ((-1)^(2/3)\*b^(2/3)\*Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x])/(3\*a^(5/3))

$$3a^{(5/3)} - (b^{(2/3)} \operatorname{Sinh}[c - (a^{(1/3)}d)/b^{(1/3)}] \operatorname{SinhIntegral}[(a^{(1/3)}d)/b^{(1/3)} + d*x]) / (3a^{(5/3)}) - ((-1)^{(2/3)} b^{(2/3)} \operatorname{Sinh}[c - ((-1)^{(2/3)} a^{(1/3)}d)/b^{(1/3)}] \operatorname{SinhIntegral}[((-1)^{(2/3)} a^{(1/3)}d)/b^{(1/3)} + d*x]) / (3a^{(5/3)})$$
Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^3(a+bx^3)} dx &= \int \left( \frac{\cosh(c+dx)}{ax^3} - \frac{b \cosh(c+dx)}{a(a+bx^3)} \right) dx \\
&= \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{a} - \frac{b \int \frac{\cosh(c+dx)}{a+bx^3} dx}{a} \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{b \int \left( -\frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{bx})} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx})} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx})} \right) dx}{a} + \frac{d \int \frac{\sinh(c)}{x^2} dx}{2a} \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{d \sinh(c+dx)}{2ax} + \frac{b \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx}{3a^{5/3}} + \frac{b \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx}} dx}{3a^{5/3}} + \frac{b \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}} dx}{3a^{5/3}} \\
&= -\frac{\cosh(c+dx)}{2ax^2} - \frac{d \sinh(c+dx)}{2ax} + \frac{(d^2 \cosh(c)) \int \frac{\cosh(dx)}{x} dx}{2a} + \frac{\left( b \cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx \right)}{3a^{5/3}} \\
&= -\frac{\cosh(c+dx)}{2ax^2} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{2a} + \frac{\sqrt[3]{-1} b^{2/3} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^{5/3}} - \frac{(-1)^{2/3}}{3a^{5/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.37738, size = 237, normalized size = 0.58

$$x^2 \text{RootSum}\left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \text{Shi}(d(x - \#1))}{\#1^2} \& \right] + x^2$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(x^3\*(a + b\*x^3)), x]

[Out]  $-(3 \text{Cosh}[c + d*x] - 3 d^2 x^2 \text{Cosh}[c] \text{CoshIntegral}[d*x] + x^2 \text{RootSum}[a + b * \#1^3 \&, (\text{Cosh}[c + d*\#1] \text{CoshIntegral}[d*(x - \#1)] - \text{CoshIntegral}[d*(x - \#1)]) * \text{Sinh}[c + d*\#1] - \text{Cosh}[c + d*\#1] \text{SinhIntegral}[d*(x - \#1)] + \text{Sinh}[c + d*\#1] * \text{SinhIntegral}[d*(x - \#1)]) / \#1^2 \& ] + x^2 \text{RootSum}[a + b * \#1^3 \&, (\text{Cosh}[c + d*\#1] \text{CoshIntegral}[d*(x - \#1)] + \text{CoshIntegral}[d*(x - \#1)] * \text{Sinh}[c + d*\#1] + \text{Cosh}[c + d*\#1] * \text{SinhIntegral}[d*(x - \#1)] + \text{Sinh}[c + d*\#1] * \text{SinhIntegral}[d*(x - \#1)]) / \#1^2 \& ] + 3 d * x * \text{Sinh}[c + d*x] - 3 d^2 x^2 \text{Sinh}[c] \text{SinhIntegral}[d*x] / (6 * a * x^2)$

**Maple [C]** time = 0.054, size = 240, normalized size = 0.6

$$\frac{de^{-dx-c}}{4ax} - \frac{e^{-dx-c}}{4ax^2} - \frac{d^2e^{-c}\text{Ei}(1,dx)}{4a} + \frac{d^2}{6a} \sum_{_R1=\text{RootOf}(b_Z^3-3_Z^2bc+3_Zbc^2+ad^3-bc^3)} \frac{e^{-_R1}\text{Ei}(1,dx-_R1+c)}{-_R1^2-2\_R1c+c^2} - \frac{de^{dx+c}}{4ax} - \frac{e^{dx+c}}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/x^3/(b\*x^3+a),x)

[Out] 1/4\*d\*exp(-d\*x-c)/a/x-1/4\*exp(-d\*x-c)/a/x^2-1/4\*d^2/a\*exp(-c)\*Ei(1,d\*x)+1/6\*d^2/a\*sum(1/(\_R1^2-2\*\_R1\*c+c^2)\*exp(-\_R1)\*Ei(1,d\*x-\_R1+c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))-1/4\*d/a/x\*exp(d\*x+c)-1/4/a/x^2\*exp(d\*x+c)-1/4\*d^2/a\*exp(c)\*Ei(1,-d\*x)+1/6\*d^2/a\*sum(1/(\_R1^2-2\*\_R1\*c+c^2)\*exp(\_R1)\*Ei(1,-d\*x+\_R1-c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^3/(b\*x^3+a),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.13039, size = 3071, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x^3/(b\*x^3+a),x, algorithm="fricas")

[Out] -1/12\*(6\*a\*d^2\*x\*sinh(d\*x + c) - (a\*d^3/b)^(1/3)\*((sqrt(-3)\*b\*x^2 + b\*x^2)\*cosh(d\*x + c)^2 - (sqrt(-3)\*b\*x^2 + b\*x^2)\*sinh(d\*x + c)^2)\*Ei(d\*x - 1/2\*(a\*d^3/b)^(1/3)\*(sqrt(-3) + 1))\*cosh(1/2\*(a\*d^3/b)^(1/3)\*(sqrt(-3) + 1) + c) + (-a\*d^3/b)^(1/3)\*((sqrt(-3)\*b\*x^2 + b\*x^2)\*cosh(d\*x + c)^2 - (sqrt(-3)\*b\*x^2 + b\*x^2)\*sinh(d\*x + c)^2)\*Ei(-d\*x - 1/2\*(-a\*d^3/b)^(1/3)\*(sqrt(-3) + 1))

```

)*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(1/3)*((sqrt(-3)
)*b*x^2 - b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2
)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sq
rt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh(d*x + c)
^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/
3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(b*x^2
*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^
3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(b*x^2*cosh(d*x + c)^2 - b*x^2*si
nh(d*x + c)^2)*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/
b)^(1/3)) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sq
rt(-3)*b*x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-
3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(1/3)*((
sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 + b*x^2)*sinh(d*x
+ c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)
^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh
(d*x + c)^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-
a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2
- b*x^2)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*si
nh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(b*x^2*cosh(d*x + c)^2 - b*
x^2*sinh(d*x + c)^2)*(-a*d^3/b)^(1/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c +
(-a*d^3/b)^(1/3)) - 2*(b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*(a*d^
3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)) + 6*a*d*cos
h(d*x + c) - 3*(a*d^3*x^2*Ei(d*x) + a*d^3*x^2*Ei(-d*x))*cosh(c) - 3*(a*d^3*
x^2*Ei(d*x) - a*d^3*x^2*Ei(-d*x))*sinh(c))/(a^2*d*x^2*cosh(d*x + c)^2 - a^2
*d*x^2*sinh(d*x + c)^2)

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x\*\*3/(b\*x\*\*3+a), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^3+a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/((b*x^3 + a)*x^3), x)
```



$$3.102 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=718

result too large to display

```
[Out] -(x*Cosh[c + d*x])/(3*b*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d]/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) + ((-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(9*a^(2/3)*b^(4/3)) + (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) - ((-1)^(2/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) + ((-1)^(1/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) + ((-1)^(1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
```

**Rubi [A]** time = 1.08119, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {5291, 5281, 3303, 3298, 3301, 5292}

$$-\frac{\sqrt[3]{-1} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} + \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*Cosh[c + d\*x])/(a + b\*x^3)^2,x]

```
[Out] -(x*Cosh[c + d*x])/(3*b*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d]/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) + ((-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(9*a^(2/3)*b^(4/3)) + (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) - ((-1)^(2/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) + ((-1)^(1/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(9*a^(2/3)*b^(4/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3)) + ((-1)^(1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(1/3)*b^(5/3)) + ((-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(9*a^(2/3)*b^(4/3))
```

$$\begin{aligned}
& 2/3)*b^{(4/3)} + ((-1)^{(2/3)}*\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[-(((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(9*a^{(2/3)}*b^{(4/3)}) + (\text{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{CoshIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(9*a^{(2/3)}*b^{(4/3)}) - (d*\text{CoshIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x]*\text{Sinh}[c - (a^{(1/3)}*d)/b^{(1/3)}])/(9*a^{(1/3)}*b^{(5/3)}) - ((-1)^{(2/3)}*d*\text{CoshIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x]*\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}])/(9*a^{(1/3)}*b^{(5/3)}) + ((-1)^{(1/3)}*d*\text{CoshIntegral}[-(((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x]*\text{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}])/(9*a^{(1/3)}*b^{(5/3)}) + ((-1)^{(2/3)}*d*\text{Cosh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(9*a^{(1/3)}*b^{(5/3)}) + ((-1)^{(1/3)}*\text{Sinh}[c + ((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(1/3)}*a^{(1/3)}*d)/b^{(1/3)} - d*x])/(9*a^{(2/3)}*b^{(4/3)}) - (d*\text{Cosh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(9*a^{(1/3)}*b^{(5/3)}) + (\text{Sinh}[c - (a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[(a^{(1/3)}*d)/b^{(1/3)} + d*x])/(9*a^{(2/3)}*b^{(4/3)}) + ((-1)^{(1/3)}*d*\text{Cosh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d*x])/(9*a^{(1/3)}*b^{(5/3)}) + ((-1)^{(2/3)}*\text{Sinh}[c - ((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)}]*\text{SinhIntegral}[((-1)^{(2/3)}*a^{(1/3)}*d)/b^{(1/3)} + d*x])/(9*a^{(2/3)}*b^{(4/3)})
\end{aligned}$$

### Rule 5291

```

Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:= Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x]
+ (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x]
- Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x])
/; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])

```

### Rule 5281

```

Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:= Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x]
/; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

```

### Rule 3303

```

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x]
+ Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x]
/; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

```

### Rule 3298

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x]
/; FreeQ[{c, d, e, f}, x]

```

, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 5292

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^p)\*Sinh[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Int[ExpandIntegrand[Sinh[c + d\*x], x^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \cosh(c + dx)}{(a + bx^3)^2} dx &= -\frac{x \cosh(c + dx)}{3b(a + bx^3)} + \frac{\int \frac{\cosh(c+dx)}{a+bx^3} dx}{3b} + \frac{d \int \frac{x \sinh(c+dx)}{a+bx^3} dx}{3b} \\
 &= -\frac{x \cosh(c + dx)}{3b(a + bx^3)} + \frac{\int \left( -\frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{bx})} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx})} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx})} \right) dx}{3b} + \frac{d \int \left( -\frac{\sinh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{bx}} \right) dx}{9\sqrt[3]{ab^4/3}} \\
 &= -\frac{x \cosh(c + dx)}{3b(a + bx^3)} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx}{9a^{2/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx}} dx}{9a^{2/3}b} - \frac{\int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}} dx}{9a^{2/3}b} - \frac{d \int \frac{\sinh(c+dx)}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{9\sqrt[3]{ab^4/3}} \\
 &= -\frac{x \cosh(c + dx)}{3b(a + bx^3)} - \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx}{9a^{2/3}b} - \frac{\left(d \cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\right) \int \frac{\sinh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{\sqrt[3]{a}+\sqrt[3]{bx}} dx}{9\sqrt[3]{ab^4/3}} \\
 &= -\frac{x \cosh(c + dx)}{3b(a + bx^3)} - \frac{\sqrt[3]{-1} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{2/3}b^4/3} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{2/3}b^4/3}
 \end{aligned}$$

**Mathematica [C]** time = 0.335862, size = 363, normalized size = 0.51

$$\frac{-\text{RootSum}\left[\#1^3 b + a \&, \frac{\sinh(\#1 d + c) \text{Chi}(d(x - \#1)) - \#1 d \sinh(\#1 d + c) \text{Chi}(d(x - \#1)) - \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \#1 d \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) - \sinh(\#1 d + c) \text{Chi}(d(x - \#1))}{\#1^2}\right]}{\#1^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

```
[Out] ((-6*b*x*Cosh[c + d*x])/(a + b*x^3) - RootSum[a + b*#1^3 & , (-Cosh[c + d*
#1]*CoshIntegral[d*(x - #1)]) + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + C
osh[c + d*#1]*SinhIntegral[d*(x - #1)] - Sinh[c + d*#1]*SinhIntegral[d*(x -
#1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x
- #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d
*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] + RootSum[a + b*#1^3
& , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sin
h[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*Sinh
Integral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*Cos
hIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(
x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ])/(18*b
^2)
```

**Maple [C]** time = 0.15, size = 877, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] -1/6*d^3*exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)*x-1/18/d/a/b^2*sum((3*_R1^2*b*c^2-
_R1*a*d^3-5*_R1*b*c^3-2*a*c*d^3+2*b*c^4+3*_R1*b*c^2+a*d^3-b*c^3)/(_R1^2-2*_
R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))+1/18/d*c^3/a/b*sum((_R1-c+2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*E
i(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d*
c^2/a/b*sum((_R1^2-_R1*c+_R1+c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+
c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c/a/b^2*sum(
(2*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3+2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R
1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/
6*d^3*exp(d*x+c)/b/(b*d^3*x^3+a*d^3)*x+1/18/d/a/b^2*sum((3*_R1^2*b*c^2-_R1*
a*d^3-5*_R1*b*c^3-2*a*c*d^3+2*b*c^4-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c
+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))-1/18/d*c^3/a/b*sum((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-
d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c^2/
a/b*sum((_R1^2-_R1*c-_R1-c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),
_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d*c/a/b^2*sum((2*_
R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3-2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei
(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.40792, size = 5017, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x^3+a)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/36*(12*a*d*x*cosh(d*x + c) - ((a*d^3/b)^{(2/3)}*((b*x^3 - \sqrt{-3})*(b*x^3 \\ & + a) + a)*cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3})*(b*x^3 + a) + a)*sinh(d*x + c \\ & )^2) - (a*d^3/b)^{(1/3)}*((b*x^3 + \sqrt{-3})*(b*x^3 + a) + a)*cosh(d*x + c)^2 \\ & - (b*x^3 + \sqrt{-3})*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/ \\ & b)^{(1/3)}*(\sqrt{-3} + 1))*cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) + c) + ((- \\ & a*d^3/b)^{(2/3)}*((b*x^3 - \sqrt{-3})*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 \\ & - \sqrt{-3})*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^{(1/3)}*((b*x^3 + \\ & \sqrt{-3})*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3})*(b*x^3 + a) + \\ & a)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*cosh(1 \\ & /2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) - c) - ((a*d^3/b)^{(2/3)}*((b*x^3 + \sqrt{-3} \\ & )*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + \sqrt{-3})*(b*x^3 + a) + a)*si \\ & nh(d*x + c)^2) - (a*d^3/b)^{(1/3)}*((b*x^3 - \sqrt{-3})*(b*x^3 + a) + a)*cosh(d \\ & *x + c)^2 - (b*x^3 - \sqrt{-3})*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x + 1 \\ & /2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) \\ & - c) + ((-a*d^3/b)^{(2/3)}*((b*x^3 + \sqrt{-3})*(b*x^3 + a) + a)*cosh(d*x + c)^2 \\ & - (b*x^3 + \sqrt{-3})*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^{(1/3)}* \\ & ((b*x^3 - \sqrt{-3})*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3})*(b \\ & *x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - \\ & 1))*cosh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) + c) - 2*((-a*d^3/b)^{(2/3)}*((b \\ & *x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2) - (-a*d^3/b)^{(1/3)} \\ & *((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2))*Ei(-d*x + (-a \\ & *d^3/b)^{(1/3))*cosh(c + (-a*d^3/b)^{(1/3)}) + 2*((a*d^3/b)^{(2/3)}*((b*x^3 + a) \end{aligned}$$

$$\begin{aligned}
& * \cosh(dx + c)^2 - (bx^3 + a) \sinh(dx + c)^2 - (ad^3/b)^{1/3} ((bx^3 + a) \cosh(dx + c)^2 - (bx^3 + a) \sinh(dx + c)^2) \operatorname{Ei}(dx + (ad^3/b)^{1/3}) \\
& ) * \cosh(-c + (ad^3/b)^{1/3}) - ((ad^3/b)^{2/3} ((bx^3 - \sqrt{-3}(bx^3 + a) + a) \cosh(dx + c)^2 - (bx^3 - \sqrt{-3}(bx^3 + a) + a) \sinh(dx + c)^2) \\
& - (ad^3/b)^{1/3} ((bx^3 + \sqrt{-3}(bx^3 + a) + a) \cosh(dx + c)^2 - (bx^3 + \sqrt{-3}(bx^3 + a) + a) \sinh(dx + c)^2)) \operatorname{Ei}(dx - 1/2(ad^3/b)^{1/3} \\
& (\sqrt{-3} + 1)) \sinh(1/2(ad^3/b)^{1/3} (\sqrt{-3} + 1) + c) + ((-ad^3/b)^{2/3} ((bx^3 - \sqrt{-3}(bx^3 + a) + a) \cosh(dx + c)^2 - (bx^3 - \sqrt{-3}(bx^3 + a) + a) \sinh(dx + c)^2) \\
& - (-ad^3/b)^{1/3} ((bx^3 + \sqrt{-3}(bx^3 + a) + a) \cosh(dx + c)^2 - (bx^3 + \sqrt{-3}(bx^3 + a) + a) \sinh(dx + c)^2)) \operatorname{Ei}(-dx - 1/2(-ad^3/b)^{1/3} (\sqrt{-3} + 1)) \sinh(1/2(-ad^3/b)^{1/3} \\
& (\sqrt{-3} + 1) - c) + ((ad^3/b)^{2/3} ((bx^3 + \sqrt{-3}(bx^3 + a) + a) \cosh(dx + c)^2 - (bx^3 + \sqrt{-3}(bx^3 + a) + a) \sinh(dx + c)^2) - (ad^3/b)^{1/3} ((bx^3 - \sqrt{-3}(bx^3 + a) + a) \cosh(dx + c)^2 - (bx^3 - \sqrt{-3}(bx^3 + a) + a) \sinh(dx + c)^2)) \operatorname{Ei}(dx + 1/2(ad^3/b)^{1/3} (\sqrt{-3} - 1)) \sinh(1/2(ad^3/b)^{1/3} (\sqrt{-3} - 1) - c) - ((-ad^3/b)^{2/3} ((bx^3 + \sqrt{-3}(bx^3 + a) + a) \cosh(dx + c)^2 - (bx^3 + \sqrt{-3}(bx^3 + a) + a) \sinh(dx + c)^2) - (-ad^3/b)^{1/3} ((bx^3 - \sqrt{-3}(bx^3 + a) + a) \cosh(dx + c)^2 - (bx^3 - \sqrt{-3}(bx^3 + a) + a) \sinh(dx + c)^2)) \operatorname{Ei}(-dx + 1/2(-ad^3/b)^{1/3} (\sqrt{-3} - 1)) \sinh(1/2(-ad^3/b)^{1/3} (\sqrt{-3} - 1) + c) + 2((-ad^3/b)^{2/3} ((bx^3 + a) \cosh(dx + c)^2 - (bx^3 + a) \sinh(dx + c)^2) - (-ad^3/b)^{1/3} ((bx^3 + a) \cosh(dx + c)^2 - (bx^3 + a) \sinh(dx + c)^2)) \operatorname{Ei}(-dx + (-ad^3/b)^{1/3}) \sinh(c + (-ad^3/b)^{1/3}) - 2((ad^3/b)^{2/3} ((bx^3 + a) \cosh(dx + c)^2 - (bx^3 + a) \sinh(dx + c)^2) - (ad^3/b)^{1/3} ((bx^3 + a) \cosh(dx + c)^2 - (bx^3 + a) \sinh(dx + c)^2)) \operatorname{Ei}(dx + (ad^3/b)^{1/3}) \sinh(-c + (ad^3/b)^{1/3}) / ((ab^2dx^3 + a^2bd) \cosh(dx + c)^2 - (ab^2dx^3 + a^2bd) \sinh(dx + c)^2)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*cosh(dx+c)/(b\*x\*\*3+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(dx + c)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

$$3.103 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=373

$$\frac{d \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} - \frac{\sqrt[3]{-1}d \sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3}d \sinh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-x\right)}{9a^{2/3}b^{4/3}}$$

[Out]  $-\text{Cosh}[c + d*x]/(3*b*(a + b*x^3)) + (d*\text{CoshIntegral}[(a^{1/3}*d)/b^{1/3} + d*x]*\text{Sinh}[c - (a^{1/3}*d)/b^{1/3}])/(9*a^{2/3}*b^{4/3}) - ((-1)^{1/3}*d*\text{CoshIntegral}[( (-1)^{1/3}*a^{1/3}*d)/b^{1/3} - d*x]*\text{Sinh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}])/(9*a^{2/3}*b^{4/3}) + ((-1)^{2/3}*d*\text{CoshIntegral}[-(((-1)^{2/3})*a^{1/3}*d)/b^{1/3}) - d*x]*\text{Sinh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}])/(9*a^{2/3}*b^{4/3}) + ((-1)^{1/3}*d*\text{Cosh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[( (-1)^{1/3}*a^{1/3}*d)/b^{1/3} - d*x])/(9*a^{2/3}*b^{4/3}) + (d*\text{Cosh}[c - (a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(9*a^{2/3}*b^{4/3}) + ((-1)^{2/3}*d*\text{Cosh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[( (-1)^{2/3}*a^{1/3}*d)/b^{1/3} + d*x])/(9*a^{2/3}*b^{4/3})$

**Rubi [A]** time = 0.601165, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {5289, 5280, 3303, 3298, 3301}

$$\frac{d \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} - \frac{\sqrt[3]{-1}d \sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{2/3}b^{4/3}} + \frac{(-1)^{2/3}d \sinh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-x\right)}{9a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*\text{Cosh}[c + d*x])/(a + b*x^3)^2, x]$

[Out]  $-\text{Cosh}[c + d*x]/(3*b*(a + b*x^3)) + (d*\text{CoshIntegral}[(a^{1/3}*d)/b^{1/3} + d*x]*\text{Sinh}[c - (a^{1/3}*d)/b^{1/3}])/(9*a^{2/3}*b^{4/3}) - ((-1)^{1/3}*d*\text{CoshIntegral}[( (-1)^{1/3}*a^{1/3}*d)/b^{1/3} - d*x]*\text{Sinh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}])/(9*a^{2/3}*b^{4/3}) + ((-1)^{2/3}*d*\text{CoshIntegral}[-(((-1)^{2/3})*a^{1/3}*d)/b^{1/3}) - d*x]*\text{Sinh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}])/(9*a^{2/3}*b^{4/3}) + ((-1)^{1/3}*d*\text{Cosh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[( (-1)^{1/3}*a^{1/3}*d)/b^{1/3} - d*x])/(9*a^{2/3}*b^{4/3}) + (d*\text{Cosh}[c - (a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(9*a^{2/3}*b^{4/3}) + ((-1)^{2/3}*d*\text{Cosh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[( (-1)^{2/3}*a^{1/3}*d)/b^{1/3} + d*x])/(9*a^{2/3}*b^{4/3})$



$\text{hIntegral}[( (-1)^{2/3} a^{1/3} d / b^{1/3} + d x ] / (9 a^{2/3} b^{4/3})$

### Rule 5289

$\text{Int}[\text{Cosh}[(c_.) + (d_.) (x_)] * ((e_.) (x_))^{(m_.)} * ((a_.) + (b_.) (x_))^{(n_.)} (p_.) , x\_Symbol] \rightarrow \text{Simp}[(e^m (a + b x^n)^{(p+1)} \text{Cosh}[c + d x]) / (b n (p+1)) , x] - \text{Dist}[(d e^m) / (b n (p+1)), \text{Int}[(a + b x^n)^{(p+1)} \text{Sinh}[c + d x], x], x] /;$  FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])

### Rule 5280

$\text{Int}[( (a_.) + (b_.) (x_))^{(n_.)} (p_.) \text{Sinh}[(c_.) + (d_.) (x_)] , x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Sinh}[c + d x], (a + b x^n)^p, x], x] /;$  FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

### Rule 3303

$\text{Int}[\sin[(e_.) + (f_.) (x_)] / ((c_.) + (d_.) (x_)) , x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d e - c f) / d], \text{Int}[\text{Sin}[(c f) / d + f x] / (c + d x), x], x] + \text{Dist}[\text{Sin}[(d e - c f) / d], \text{Int}[\text{Cos}[(c f) / d + f x] / (c + d x), x], x] /;$  FreeQ[{c, d, e, f}, x] && NeQ[d e - c f, 0]

### Rule 3298

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_]) (f_.) (x_)] / ((c_.) + (d_.) (x_)) , x\_Symbol] \rightarrow \text{Simp}[(I \text{SinhIntegral}[(c f fz) / d + f fz x]) / d, x] /;$  FreeQ[{c, d, e, f, fz}, x] && EqQ[d e - c f fz I, 0]

### Rule 3301

$\text{Int}[\sin[(e_.) + (\text{Complex}[0, fz_]) (f_.) (x_)] / ((c_.) + (d_.) (x_)) , x\_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[(c f fz) / d + f fz x] / d, x] /;$  FreeQ[{c, d, e, f, fz}, x] && EqQ[d (e - Pi/2) - c f fz I, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} dx &= -\frac{\cosh(c+dx)}{3b(a+bx^3)} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3b(a+bx^3)} + \frac{d \int \left( -\frac{\sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{bx})} - \frac{\sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx})} - \frac{\sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx})} \right) dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3b(a+bx^3)} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx}{9a^{2/3}b} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx}} dx}{9a^{2/3}b} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}} dx}{9a^{2/3}b} \\
&= -\frac{\cosh(c+dx)}{3b(a+bx^3)} - \frac{\left( d \cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \right) \int \frac{\sinh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}}+dx\right)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx}{9a^{2/3}b} - \frac{\left( id \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \right) \int \frac{\sin\left(\frac{(-1)^{5/6}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}} dx}{9a^{2/3}b} \\
&= -\frac{\cosh(c+dx)}{3b(a+bx^3)} + \frac{d \operatorname{Chi}\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}}+dx\right) \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}} - \frac{\sqrt[3]{-1}d \operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}-dx\right) \sinh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.159526, size = 203, normalized size = 0.54

$$\frac{-d \operatorname{RootSum}\left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \operatorname{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \operatorname{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \operatorname{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \operatorname{Shi}(d(x - \#1))}{\#1^2}\right] \& + d \operatorname{RootSum}\left[\#1^3 b + a \&, \frac{-\sinh(\#1 d + c) \operatorname{Chi}(d(x - \#1)) + \cosh(\#1 d + c) \operatorname{Chi}(d(x - \#1)) + \sinh(\#1 d + c) \operatorname{Shi}(d(x - \#1)) - \cosh(\#1 d + c) \operatorname{Shi}(d(x - \#1))}{\#1^2}\right] \&}{18}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Cosh[c + d\*x])/(a + b\*x^3)^2,x]

[Out] ((-6\*b\*Cosh[c + d\*x])/(a + b\*x^3) - d\*RootSum[a + b\*#1^3 & , (Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] - CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] - Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]))/#1^2 & ] + d\*RootSum[a + b\*#1^3 & , (Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] + CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] + Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]))/#1^2 & ])/(18\*b^2)

**Maple [C]** time = 0.044, size = 594, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*cosh(d*x+c)/(b*x^3+a)^2,x)`

[Out] 
$$-1/6*d^3*exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)-1/18/a/b^2*sum((2*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3+2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),$$

$$_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*c^2/a/b*sum((_R1-c+2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),$$

$$_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*c/a/b*sum((_R1^2-_R1*c+_R1+c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),$$

$$_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d^3*exp(d*x+c)/b/(b*d^3*x^3+a*d^3)+1/18/a/b^2*sum((2*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3-2*_R1*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,$$

$$-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*c^2/a/b*sum((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),$$

$$_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9*c/a/b*sum((_R1^2-_R1*c-_R1-c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),$$

$$_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.22021, size = 3200, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")`

[Out] 
$$-1/36*((a*d^3/b)^{(1/3)}*((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2$$

$$- (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)$$

$$)^{(1/3)}*(\sqrt{-3} + 1))*\cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) + c) + (-a*$$

$$\begin{aligned}
& d^3/b)^{1/3} * ((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a) * \cosh(d*x + c)^2 - (b*x^3 + \\
& \sqrt{-3}*(b*x^3 + a) + a) * \sinh(d*x + c)^2) * \text{Ei}(-d*x - 1/2*(-a*d^3/b)^{1/3} * \\
& (\sqrt{-3} + 1)) * \cosh(1/2*(-a*d^3/b)^{1/3} * (\sqrt{-3} + 1) - c) + (a*d^3/b)^{1/3} * \\
& ((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a) * \cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3} * \\
& (b*x^3 + a) + a) * \sinh(d*x + c)^2) * \text{Ei}(d*x + 1/2*(a*d^3/b)^{1/3} * (\sqrt{-3} \\
& - 1)) * \cosh(1/2*(a*d^3/b)^{1/3} * (\sqrt{-3} - 1) - c) + (-a*d^3/b)^{1/3} * ((b*x \\
& ^3 - \sqrt{-3}*(b*x^3 + a) + a) * \cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + \\
& a) + a) * \sinh(d*x + c)^2) * \text{Ei}(-d*x + 1/2*(-a*d^3/b)^{1/3} * (\sqrt{-3} - 1)) * \text{co} \\
& \text{sh}(1/2*(-a*d^3/b)^{1/3} * (\sqrt{-3} - 1) + c) - 2*(-a*d^3/b)^{1/3} * ((b*x^3 + \\
& a) * \cosh(d*x + c)^2 - (b*x^3 + a) * \sinh(d*x + c)^2) * \text{Ei}(-d*x + (-a*d^3/b)^{1/3} \\
& )) * \cosh(c + (-a*d^3/b)^{1/3}) - 2*(a*d^3/b)^{1/3} * ((b*x^3 + a) * \cosh(d*x + c \\
& )^2 - (b*x^3 + a) * \sinh(d*x + c)^2) * \text{Ei}(d*x + (a*d^3/b)^{1/3}) * \cosh(-c + (a*d \\
& ^3/b)^{1/3}) + (a*d^3/b)^{1/3} * ((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a) * \cosh(d*x \\
& + c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a) * \sinh(d*x + c)^2) * \text{Ei}(d*x - 1/2 * \\
& (a*d^3/b)^{1/3} * (\sqrt{-3} + 1)) * \sinh(1/2*(a*d^3/b)^{1/3} * (\sqrt{-3} + 1) + c) \\
& + (-a*d^3/b)^{1/3} * ((b*x^3 + \sqrt{-3}*(b*x^3 + a) + a) * \cosh(d*x + c)^2 - \\
& (b*x^3 + \sqrt{-3}*(b*x^3 + a) + a) * \sinh(d*x + c)^2) * \text{Ei}(-d*x - 1/2*(-a*d^3/b \\
& )^{1/3} * (\sqrt{-3} + 1)) * \sinh(1/2*(-a*d^3/b)^{1/3} * (\sqrt{-3} + 1) - c) - (a * \\
& d^3/b)^{1/3} * ((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a) * \cosh(d*x + c)^2 - (b*x^3 - \\
& \sqrt{-3}*(b*x^3 + a) + a) * \sinh(d*x + c)^2) * \text{Ei}(d*x + 1/2*(a*d^3/b)^{1/3} * (s \\
& \text{qrt}(-3) - 1)) * \sinh(1/2*(a*d^3/b)^{1/3} * (\sqrt{-3} - 1) - c) - (-a*d^3/b)^{1/3} * \\
& ((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a) * \cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3} * \\
& (b*x^3 + a) + a) * \sinh(d*x + c)^2) * \text{Ei}(-d*x + 1/2*(-a*d^3/b)^{1/3} * (\sqrt{-3} \\
& - 1)) * \sinh(1/2*(-a*d^3/b)^{1/3} * (\sqrt{-3} - 1) + c) + 2*(-a*d^3/b)^{1/3} * (( \\
& b*x^3 + a) * \cosh(d*x + c)^2 - (b*x^3 + a) * \sinh(d*x + c)^2) * \text{Ei}(-d*x + (-a*d^3 \\
& /b)^{1/3}) * \sinh(c + (-a*d^3/b)^{1/3}) + 2*(a*d^3/b)^{1/3} * ((b*x^3 + a) * \cosh \\
& (d*x + c)^2 - (b*x^3 + a) * \sinh(d*x + c)^2) * \text{Ei}(d*x + (a*d^3/b)^{1/3}) * \sinh(- \\
& c + (a*d^3/b)^{1/3}) + 12*a * \cosh(d*x + c) / ((a*b^2*x^3 + a^2*b) * \cosh(d*x + \\
& c)^2 - (a*b^2*x^3 + a^2*b) * \sinh(d*x + c)^2)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cosh(d\*x+c)/(b\*x\*\*3+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(dx + c)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

$$3.104 \quad \int \frac{x \cosh(c+dx)}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=695

$$\frac{(-1)^{2/3} \cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} - \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$

[Out] Cosh[c + d\*x]/(3\*a\*b\*x) - Cosh[c + d\*x]/(3\*b\*x\*(a + b\*x^3)) - ((-1)^(2/3)\*Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(9\*a^(4/3)\*b^(2/3)) + ((-1)^(1/3)\*Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)) - d\*x])/(9\*a^(4/3)\*b^(2/3)) - (Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(9\*a^(4/3)\*b^(2/3)) - (d\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x]\*Sinh[c - (a^(1/3)\*d)/b^(1/3)])/(9\*a\*b) - (d\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]\*Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)])/(9\*a\*b) - (d\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)) - d\*x]\*Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)])/(9\*a\*b) + (d\*Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(9\*a\*b) + ((-1)^(2/3)\*Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x])/(9\*a^(4/3)\*b^(2/3)) - (d\*Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(9\*a\*b) - (Sinh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x])/(9\*a^(4/3)\*b^(2/3)) - (d\*Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x])/(9\*a\*b) + ((-1)^(1/3)\*Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x])/(9\*a^(4/3)\*b^(2/3))

**Rubi [A]** time = 1.32363, antiderivative size = 695, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292}

$$\frac{(-1)^{2/3} \cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1} \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} - \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(x\*Cosh[c + d\*x])/(a + b\*x^3)^2,x]

```
[Out] Cosh[c + d*x]/(3*a*b*x) - Cosh[c + d*x]/(3*b*x*(a + b*x^3)) - ((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]/(9*a^(4/3)*b^(2/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(4/3)*b^(2/3)) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a*b) - (d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a*b) - (d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a*b) + (d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a*b) + ((-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a^(4/3)*b^(2/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a*b) - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(4/3)*b^(2/3)) - (d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(9*a*b) + ((-1)^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(4/3)*b^(2/3)))
```

### Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^p_, x_Symbol]
:> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

### Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^p_, x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol]
:> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] := Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

### Rubi steps



$$\begin{aligned}
\int \frac{x \cosh(c + dx)}{(a + bx^3)^2} dx &= -\frac{\cosh(c + dx)}{3bx(a + bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx}{3b} + \frac{d \int \frac{\sinh(c+dx)}{x(a+bx^3)} dx}{3b} \\
&= -\frac{\cosh(c + dx)}{3bx(a + bx^3)} - \frac{\int \left( \frac{\cosh(c+dx)}{ax^2} - \frac{bx \cosh(c+dx)}{a(a+bx^3)} \right) dx}{3b} + \frac{d \int \left( \frac{\sinh(c+dx)}{ax} - \frac{bx^2 \sinh(c+dx)}{a(a+bx^3)} \right) dx}{3b} \\
&= -\frac{\cosh(c + dx)}{3bx(a + bx^3)} + \frac{\int \frac{x \cosh(c+dx)}{a+bx^3} dx}{3a} - \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{3ab} - \frac{d \int \frac{x^2 \sinh(c+dx)}{a+bx^3} dx}{3a} + \frac{d \int \frac{\sinh(c+dx)}{x} dx}{3ab} \\
&= \frac{\cosh(c + dx)}{3abx} - \frac{\cosh(c + dx)}{3bx(a + bx^3)} + \frac{\int \left( -\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a} + \sqrt[3]{bx})} - \frac{(-1)^{2/3} \cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a} - \sqrt[3]{-1}\sqrt[3]{bx})} + \frac{\sqrt[3]{-1} \cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a} + (-1)^{2/3}\sqrt[3]{bx})} \right) dx}{3a} \\
&= \frac{\cosh(c + dx)}{3abx} - \frac{\cosh(c + dx)}{3bx(a + bx^3)} + \frac{d\text{Chi}(dx) \sinh(c)}{3ab} + \frac{d \cosh(c) \text{Shi}(dx)}{3ab} - \frac{\int \frac{\cosh(c+dx)}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{9a^{4/3}\sqrt[3]{b}} + \frac{\int \frac{\sinh(c+dx)}{\sqrt[3]{a} - \sqrt[3]{bx}} dx}{9ab^{2/3}} \\
&= \frac{\cosh(c + dx)}{3abx} - \frac{\cosh(c + dx)}{3bx(a + bx^3)} - \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \int \frac{\cosh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{\sqrt[3]{a} + \sqrt[3]{bx}} dx}{9a^{4/3}\sqrt[3]{b}} - \frac{\left(d \cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\right) \int \frac{\sinh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{\sqrt[3]{a} - \sqrt[3]{bx}} dx}{9ab^{2/3}} \\
&= \frac{\cosh(c + dx)}{3abx} - \frac{\cosh(c + dx)}{3bx(a + bx^3)} - \frac{(-1)^{2/3} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9ab^{2/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.188285, size = 387, normalized size = 0.56

$$(a + bx^3) \text{RootSum}\left[\#1^3 b + a \&, \frac{-\sinh(\#1d+c)\text{Chi}(d(x-\#1))-\#1d \sinh(\#1d+c)\text{Chi}(d(x-\#1))+\cosh(\#1d+c)\text{Chi}(d(x-\#1))+\#1d \cosh(\#1d+c)\text{Chi}(d(x-\#1))}{\#1^3 b + a \&}\right]$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x\*Cosh[c + d\*x])/(a + b\*x^3)^2,x]

[Out] (6\*b\*x^2\*Cosh[c + d\*x] + (a + b\*x^3)\*RootSum[a + b\*#1^3 &, (Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] - CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] - Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + d\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)]\*#1 - d\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1]\*#1 - d\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1 + d\*Sin

```
h[c + d*#1]*SinhIntegral[d*(x - #1)]*#1/#1 & ] - (a + b*x^3)*RootSum[a + b
*#1^3 & , (-(Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]) - CoshIntegral[d*(x -
#1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - Sinh[c + d
*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#
1 + d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhInt
egral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1/#1 & ]
)/(18*a*b*(a + b*x^3))
```

**Maple [C]** time = 0.037, size = 395, normalized size = 0.6

$$\frac{d^3 e^{-dx-c} x^2}{6a(bd^3x^3 + ad^3)} - \frac{d}{18ab} \sum_{_R1=\text{RootOf}(b_Z^3-3_Z^2bc+3_Zbc^2+ad^3-bc^3)} \frac{(_R1^2 - _R1c + _R1 + c) e^{-_R1} \text{Ei}(1, dx - _R1 + c)}{_R1^2 - 2_R1c + c^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(d*x+c)/(b*x^3+a)^2,x)
```

```
[Out] 1/6*d^3*exp(-d*x-c)*x^2/a/(b*d^3*x^3+a*d^3)-1/18*d/a/b*sum(( _R1^2- _R1*c+_R1
+c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*
b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*d*c/a/b*sum(( _R1-c+2)/(_R1^2-2*_R1*c+c^2)
*exp(-_R1)*Ei(1,d*x-_R1+c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*
c^3))+1/6*d^3*exp(d*x+c)*x^2/a/(b*d^3*x^3+a*d^3)+1/18*d/a/b*sum(( _R1^2- _R1*
c-_R1-c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*
_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*d*c/a/b*sum(( _R1-c-2)/(_R1^2-2*_R1*c
+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c), _R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.39469, size = 4891, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x^3+a)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/36*(12*a*b*d^2*x^2*cosh(d*x + c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b - \sqrt{-3}*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b - \sqrt{-3}*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) + c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b - \sqrt{-3}*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b - \sqrt{-3}*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*cosh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) - c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b + \sqrt{-3}*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + \sqrt{-3}*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) - c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b + \sqrt{-3}*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + \sqrt{-3}*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*cosh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) + c) + 2*((a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^{(1/3)})*cosh(c + (-a*d^3/b)^{(1/3)}) - 2*((a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^{(1/3)})*cosh(-c + (a*d^3/b)^{(1/3)}) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b - \sqrt{-3}*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b - \sqrt{-3}*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*sinh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) + c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b - \sqrt{-3}*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b - \sqrt{-3}*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*sinh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) - c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^{(2/3)}*((b^2*x^3 + a*b + \sqrt{-3}*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + \sqrt{-3}*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*s \end{aligned}$$

```
inh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (2*(a*b*d^3*x^3 + a^2*d^3)*co
sh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^(2/3
)*((b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 +
a*b + sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(
1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((a
*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x +
c)^2 - (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*b)*
sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) +
2*((a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d
*x + c)^2 + (a*d^3/b)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a
*b)*sinh(d*x + c)^2))*Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3))
/((a^2*b^2*d^2*x^3 + a^3*b*d^2)*cosh(d*x + c)^2 - (a^2*b^2*d^2*x^3 + a^3*b*
d^2)*sinh(d*x + c)^2)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x\*\*3+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(dx + c)}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x^3+a)^2,x, algorithm="giac")

[Out] integrate(x\*cosh(d\*x + c)/(b\*x^3 + a)^2, x)

$$3.105 \quad \int \frac{\cosh(c+dx)}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=739

result too large to display

```
[Out] Cosh[c + d*x]/(3*a*b*x^2) - Cosh[c + d*x]/(3*b*x^2*(a + b*x^3)) - (2*(-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a^(5/3)*b^(1/3)) + (2*(-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]/(9*a^(5/3)*b^(1/3)) + (2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + ((-1)^(2/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a^(4/3)*b^(2/3)) + (2*(-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a^(5/3)*b^(1/3)) + (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(4/3)*b^(2/3)) + (2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3)) - ((-1)^(1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(4/3)*b^(2/3)) + (2*(-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3))
```

**Rubi [A]** time = 1.32306, antiderivative size = 739, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 8, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5279, 5293, 3297, 3303, 3298, 3301, 5281, 5292}

$$\frac{d \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{4/3}b^{2/3}} + \frac{(-1)^{2/3}d \sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{4/3}b^{2/3}} - \frac{\sqrt[3]{-1}d \sinh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\dots\right)}{9a^{4/3}b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(a + b\*x^3)^2,x]

```
[Out] Cosh[c + d*x]/(3*a*b*x^2) - Cosh[c + d*x]/(3*b*x^2*(a + b*x^3)) - (2*(-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a^(5/3)*b^(1/3)) + (2*(-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]/(9*a^(5/3)*b^(1/3)) + (2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) + ((-1)^(2/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(4/3)*b^(2/3)) - ((-1)^(2/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a^(4/3)*b^(2/3)) + (2*(-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a^(5/3)*b^(1/3)) + (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(4/3)*b^(2/3)) + (2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3)) - ((-1)^(1/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(4/3)*b^(2/3)) + (2*(-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3))
```

$$\begin{aligned} & /3*d)/b^{(1/3)} - d*x]/(9*a^{(5/3)*b^{(1/3)}}) + (2*(-1)^{(2/3)*Cosh[c - ((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)}}] * CoshIntegral[-(((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)}}) - d*x])/(9*a^{(5/3)*b^{(1/3)}}) + (2*Cosh[c - (a^{(1/3)*d}/b^{(1/3)})] * CoshIntegral[(a^{(1/3)*d}/b^{(1/3)} + d*x])/(9*a^{(5/3)*b^{(1/3)}}) + (d*CoshIntegral[(a^{(1/3)*d}/b^{(1/3)} + d*x]*Sinh[c - (a^{(1/3)*d}/b^{(1/3)})]/(9*a^{(4/3)*b^{(2/3)}}) + ((-1)^{(2/3)*d*CoshIntegral[(-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)} - d*x]*Sinh[c + ((-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)}])]/(9*a^{(4/3)*b^{(2/3)}}) - ((-1)^{(1/3)*d*CoshIntegral[-(((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)}}) - d*x]*Sinh[c - ((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)}])]/(9*a^{(4/3)*b^{(2/3)}}) - ((-1)^{(2/3)*d*Cosh[c + ((-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)})] * SinhIntegral[(-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)} - d*x])/(9*a^{(4/3)*b^{(2/3)}}) + (2*(-1)^{(1/3)*Sinh[c + ((-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)})] * SinhIntegral[(-1)^{(1/3)*a^{(1/3)*d}/b^{(1/3)} - d*x])/(9*a^{(5/3)*b^{(1/3)}}) + (d*Cosh[c - (a^{(1/3)*d}/b^{(1/3)})] * SinhIntegral[(a^{(1/3)*d}/b^{(1/3)} + d*x])/(9*a^{(4/3)*b^{(2/3)}}) + (2*Sinh[c - (a^{(1/3)*d}/b^{(1/3)})] * SinhIntegral[(a^{(1/3)*d}/b^{(1/3)} + d*x])/(9*a^{(5/3)*b^{(1/3)}}) - ((-1)^{(1/3)*d*Cosh[c - ((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)})] * SinhIntegral[(-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)} + d*x])/(9*a^{(4/3)*b^{(2/3)}}) + (2*(-1)^{(2/3)*Sinh[c - ((-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)})] * SinhIntegral[(-1)^{(2/3)*a^{(1/3)*d}/b^{(1/3)} + d*x])/(9*a^{(5/3)*b^{(1/3)}}) \end{aligned}$$

### Rule 5279

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x]/(b*n*(p + 1)), x] + (-Dist[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x], x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[n, 2]
```

### Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
```

```
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{(a+bx^3)^2} dx &= -\frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2 \int \frac{\cosh(c+dx)}{x^3(a+bx^3)} dx}{3b} + \frac{d \int \frac{\sinh(c+dx)}{x^2(a+bx^3)} dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2 \int \left( \frac{\cosh(c+dx)}{ax^3} - \frac{b \cosh(c+dx)}{a(a+bx^3)} \right) dx}{3b} + \frac{d \int \left( \frac{\sinh(c+dx)}{ax^2} - \frac{bx \sinh(c+dx)}{a(a+bx^3)} \right) dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx^2(a+bx^3)} + \frac{2 \int \frac{\cosh(c+dx)}{a+bx^3} dx}{3a} - \frac{2 \int \frac{\cosh(c+dx)}{x^3} dx}{3ab} - \frac{d \int \frac{x \sinh(c+dx)}{a+bx^3} dx}{3a} + \frac{d \int \frac{\sinh(c+dx)}{x^2} dx}{3ab} \\
&= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{d \sinh(c+dx)}{3abx} + \frac{2 \int \left( -\frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{bx})} - \frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx})} - \frac{\cosh(c+dx)}{3a^{2/3}} \right) dx}{3a} \\
&= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2 \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx}{9a^{5/3}} - \frac{2 \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx}} dx}{9a^{5/3}} - \frac{2 \int \frac{\cosh(c+dx)}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}} dx}{9a^{5/3}} + \dots \\
&= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{3ab} + \frac{d^2 \sinh(c) \text{Shi}(dx)}{3ab} - \frac{(d^2 \cosh(c)) \int \frac{\cosh(dx)}{x} dx}{3ab} \\
&= \frac{\cosh(c+dx)}{3abx^2} - \frac{\cosh(c+dx)}{3bx^2(a+bx^3)} - \frac{2\sqrt[3]{-1} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{2(-1)^{2/3} \cosh(c)}{3ab}
\end{aligned}$$

**Mathematica [C]** time = 0.193522, size = 387, normalized size = 0.52

$$(a+bx^3) \text{RootSum}\left[\#1^3b+a\&, \frac{-2 \sinh(\#1d+c) \text{Chi}(d(x-\#1)) - \#1d \sinh(\#1d+c) \text{Chi}(d(x-\#1)) + 2 \cosh(\#1d+c) \text{Chi}(d(x-\#1)) + \#1d \cosh(\#1d+c) \text{Chi}(d(x-\#1))}{3ab}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(a + b\*x^3)^2, x]

[Out] (6\*b\*x\*Cosh[c + d\*x] + (a + b\*x^3)\*RootSum[a + b\*#1^3 & , (2\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] - 2\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] - 2\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + 2\*Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + d\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)]\*#1 - d\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1]\*#1 - d\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1 +



$$\frac{d \sinh[c + d \cdot \#1] \operatorname{SinhIntegral}[d \cdot (x - \#1)] \cdot \#1 / \#1^2 \& ] - (a + b \cdot x^3) \operatorname{RootSum}[a + b \cdot \#1^3 \& , (-2 \cdot \operatorname{Cosh}[c + d \cdot \#1] \cdot \operatorname{CoshIntegral}[d \cdot (x - \#1)] - 2 \cdot \operatorname{CoshIntegral}[d \cdot (x - \#1)] \cdot \sinh[c + d \cdot \#1] - 2 \cdot \operatorname{Cosh}[c + d \cdot \#1] \cdot \operatorname{SinhIntegral}[d \cdot (x - \#1)] + d \cdot \operatorname{Cosh}[c + d \cdot \#1] \cdot \operatorname{CoshIntegral}[d \cdot (x - \#1)] \cdot \#1 + d \cdot \operatorname{CoshIntegral}[d \cdot (x - \#1)] \cdot \sinh[c + d \cdot \#1] \cdot \#1 + d \cdot \operatorname{Cosh}[c + d \cdot \#1] \cdot \operatorname{SinhIntegral}[d \cdot (x - \#1)] \cdot \#1 + d \cdot \sinh[c + d \cdot \#1] \cdot \operatorname{SinhIntegral}[d \cdot (x - \#1)] \cdot \#1) / \#1^2 \& ]}{(18 \cdot a \cdot b \cdot (a + b \cdot x^3))}$$

**Maple [C]** time = 0.026, size = 226, normalized size = 0.3

$$\frac{d^3 e^{-dx-c} x}{6a(bd^3x^3 + ad^3)} - \frac{d^2}{18ab} \sum_{_R1=\operatorname{RootOf}(b_Z^3-3_Z^2bc+3_Zbc^2+ad^3-bc^3)} \frac{(_R1 - c + 2) e^{-_R1} \operatorname{Ei}(1, dx - _R1 + c)}{-_R1^2 - 2\_R1c + c^2} + \frac{d^3 e^{dx+c}}{6a(bd^3x^3 + ad^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/(b\*x^3+a)^2,x)

[Out]  $\frac{1}{6} d^3 \exp(-d \cdot x - c) \cdot x / a / (b \cdot d^3 \cdot x^3 + a \cdot d^3) - \frac{1}{18} d^2 / a / b \cdot \sum((\_R1 - c + 2) / (\_R1^2 - 2 \cdot \_R1 \cdot c + c^2) \cdot \exp(-\_R1) \cdot \operatorname{Ei}(1, d \cdot x - \_R1 + c), \_R1 = \operatorname{RootOf}(b \cdot Z^3 - 3 \cdot Z^2 \cdot b \cdot c + 3 \cdot Z \cdot b \cdot c^2 + a \cdot d^3 - b \cdot c^3)) + \frac{1}{6} d^3 \exp(d \cdot x + c) \cdot x / a / (b \cdot d^3 \cdot x^3 + a \cdot d^3) + \frac{1}{18} d^2 / a / b \cdot \sum((\_R1 - c - 2) / (\_R1^2 - 2 \cdot \_R1 \cdot c + c^2) \cdot \exp(\_R1) \cdot \operatorname{Ei}(1, -d \cdot x + \_R1 - c), \_R1 = \operatorname{RootOf}(b \cdot Z^3 - 3 \cdot Z^2 \cdot b \cdot c + 3 \cdot Z \cdot b \cdot c^2 + a \cdot d^3 - b \cdot c^3))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.3497, size = 5042, normalized size = 6.82

result too large to display



$$2*(-a*d^3/b)^{(1/3)}*((b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\cosh(d*x + c)^2 - (b*x^3 - \sqrt{-3}*(b*x^3 + a) + a)*\sinh(d*x + c)^2)*\text{Ei}(-d*x + 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\sinh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) + c) + 2*((-a*d^3/b)^{(2/3)}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2) + 2*(-a*d^3/b)^{(1/3)}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2))*\text{Ei}(-d*x + (-a*d^3/b)^{(1/3)})*\sinh(c + (-a*d^3/b)^{(1/3)}) - 2*((a*d^3/b)^{(2/3)}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2) + 2*(a*d^3/b)^{(1/3)}*((b*x^3 + a)*\cosh(d*x + c)^2 - (b*x^3 + a)*\sinh(d*x + c)^2))*\text{Ei}(d*x + (a*d^3/b)^{(1/3)})*\sinh(-c + (a*d^3/b)^{(1/3)})/((a^2*b*d*x^3 + a^3*d)*\cosh(d*x + c)^2 - (a^2*b*d*x^3 + a^3*d)*\sinh(d*x + c)^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x\*\*3+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(bx^3+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(b\*x^3+a)^2,x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/(b\*x^3 + a)^2, x)

$$3.106 \quad \int \frac{\cosh(c+dx)}{x(a+bx^3)^2} dx$$

**Optimal.** Leaf size=697

result too large to display

```
[Out] Cosh[c + d*x]/(3*a*b*x^3) - Cosh[c + d*x]/(3*b*x^3*(a + b*x^3)) + (Cosh[c]*
CoshIntegral[d*x])/a^2 - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInte
gral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(3*a^2) - (Cosh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])
/(3*a^2) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3)
+ d*x]/(3*a^2) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1
/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) + ((-1)^(1/3)*d*CoshIntegral[((-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(9*a
^(5/3)*b^(1/3)) - ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) +
(Sinh[c]*SinhIntegral[d*x])/a^2 - ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)
*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a^(5/3)
*b^(1/3)) + (Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(3*a^2) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*
SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3)) - (Sinh[c - (a
^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(3*a^2) - ((-1)
^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3)) - (Sinh[c - ((-1)^(2/3)*a^(1
/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(3*a^2)
```

---

**Rubi [A]** time = 1.47159, antiderivative size = 697, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292, 5280}

$$-\frac{d \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}d \sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{(-1)^{2/3}d \sinh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{9a^{5/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(x*(a + b*x^3)^2), x]
```

```
[Out] Cosh[c + d*x]/(3*a*b*x^3) - Cosh[c + d*x]/(3*b*x^3*(a + b*x^3)) + (Cosh[c]*
CoshIntegral[d*x])/a^2 - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInte
```

```

gral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(3*a^2) - (Cosh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])
/(3*a^2) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3)
+ d*x]/(3*a^2) - (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1
/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) + ((-1)^(1/3)*d*CoshIntegral[(-1)^(1/
3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]/(9*a
^(5/3)*b^(1/3)) - ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(9*a^(5/3)*b^(1/3)) +
(Sinh[c]*SinhIntegral[d*x])/a^2 - ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*SinhIntegral[(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(9*a^(5/3
)*b^(1/3)) + (Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(3*a^2) - (d*Cosh[c - (a^(1/3)*d)/b^(1/3)]*
SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3)) - (Sinh[c - (a
^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(3*a^2) - ((-1)
^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(-1)^(2/3)*
a^(1/3)*d)/b^(1/3) + d*x]/(9*a^(5/3)*b^(1/3)) - (Sinh[c - ((-1)^(2/3)*a^(1
/3)*d)/b^(1/3)]*SinhIntegral[(-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(3*a^2)

```

### Rule 5291

```

Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])

```

### Rule 5293

```

Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])

```

### Rule 3297

```

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]

```

### Rule 3303

```

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f

```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x(a+bx^3)^2} dx &= -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^4(a+bx^3)} dx}{b} + \frac{d \int \frac{\sinh(c+dx)}{x^3(a+bx^3)} dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} - \frac{\int \left( \frac{\cosh(c+dx)}{ax^4} - \frac{b \cosh(c+dx)}{a^2x} + \frac{b^2x^2 \cosh(c+dx)}{a^2(a+bx^3)} \right) dx}{b} + \frac{d \int \left( \frac{\sinh(c+dx)}{ax^3} - \frac{b \sinh(c+dx)}{a(a+bx^3)} \right) dx}{3b} \\
&= -\frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\int \frac{\cosh(c+dx)}{x} dx}{a^2} - \frac{\int \frac{\cosh(c+dx)}{x^4} dx}{ab} - \frac{b \int \frac{x^2 \cosh(c+dx)}{a+bx^3} dx}{a^2} - \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{3a} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{3a} \\
&= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} - \frac{d \sinh(c+dx)}{6abx^2} - \frac{b \int \left( \frac{\cosh(c+dx)}{3b^{2/3}(\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{\cosh(c+dx)}{3b^{2/3}(-\sqrt[3]{-1}\sqrt[3]{a} + \sqrt[3]{bx})} + \frac{\cosh(c+dx)}{3b^{2/3}(\sqrt[3]{-1}\sqrt[3]{a} - \sqrt[3]{bx})} \right) dx}{a^2} \\
&= \frac{\cosh(c+dx)}{3abx^3} - \frac{d^2 \cosh(c+dx)}{6abx} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{\sqrt[3]{b} \int \frac{\cosh(c+dx)}{x} dx}{3b^{2/3}} \\
&= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{d^3 \int \frac{\sinh(c+dx)}{x} dx}{6ab} + \frac{(d^3 \cosh(c+dx))}{3b^{2/3}} \\
&= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^2} - \frac{\sinh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^2} \\
&= \frac{\cosh(c+dx)}{3abx^3} - \frac{\cosh(c+dx)}{3bx^3(a+bx^3)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^2} - \frac{\sinh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{3a^2}
\end{aligned}$$

**Mathematica [C]** time = 8.61079, size = 5530, normalized size = 7.93

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d\*x]/(x\*(a + b\*x^3)^2), x]

[Out] Result too large to show

**Maple [C]** time = 0.072, size = 338, normalized size = 0.5

$$\frac{e^{-dx-c}d^3}{6a((dx+c)^3b-3(dx+c)^2bc+3(dx+c)bc^2+ad^3-bc^3)} - \frac{e^{-c}\text{Ei}(1,dx)}{2a^2} + \frac{1}{18a^2b} \sum_{_R1=\text{RootOf}(b_Z^3-3_Z^2bc+3_Zbc^2+ad^3-bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/x/(b\*x^3+a)^2,x)

[Out]  $\frac{1}{6} \exp(-d*x-c) d^3/a / ((d*x+c)^3*b-3*(d*x+c)^2*b*c+3*(d*x+c)*b*c^2+a*d^3-b*c^3) - 1/2/a^2 * \exp(-c) * \text{Ei}(1,d*x) + 1/18/a^2/b * \sum(( -a*d^3+3*_R1^2*b-6*_R1*b*c+3*b*c^2) / (_R1^2-2*_R1*c+c^2) * \exp(-_R1) * \text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)) + 1/6 * \exp(d*x+c) * d^3/a / ((d*x+c)^3*b-3*(d*x+c)^2*b*c+3*(d*x+c)*b*c^2+a*d^3-b*c^3) - 1/2/a^2 * \exp(c) * \text{Ei}(1,-d*x) + 1/18/a^2/b * \sum((a*d^3+3*_R1^2*b-6*_R1*b*c+3*b*c^2) / (_R1^2-2*_R1*c+c^2) * \exp(_R1) * \text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x^3+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.32726, size = 4420, normalized size = 6.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x^3+a)^2,x, algorithm="fricas")

[Out]  $-1/36 * ((6*(b*x^3+a) * \cosh(d*x+c)^2 - 6*(b*x^3+a) * \sinh(d*x+c)^2 - (a*d^3/b)^{1/3} * ((b*x^3 + \sqrt{-3}*(b*x^3+a) + a) * \cosh(d*x+c)^2 - (b*x^3 + \sqrt{-3}*(b*x^3+a) + a) * \sinh(d*x+c)^2)) * \text{Ei}(d*x - 1/2*(a*d^3/b)^{1/3} * ($



$$\begin{aligned}
& \sqrt{-3} + 1) * \cosh(1/2 * (a * d^3 / b)^{1/3} * (\sqrt{-3} + 1) + c) + (6 * (b * x^3 + a) * \cosh(d * x + c)^2 - 6 * (b * x^3 + a) * \sinh(d * x + c)^2 - (-a * d^3 / b)^{1/3} * ((b * x^3 + \sqrt{-3} * (b * x^3 + a) + a) * \cosh(d * x + c)^2 - (b * x^3 + \sqrt{-3} * (b * x^3 + a) + a) * \sinh(d * x + c)^2)) * \operatorname{Ei}(-d * x - 1/2 * (-a * d^3 / b)^{1/3} * (\sqrt{-3} + 1)) * \cosh(1/2 * (-a * d^3 / b)^{1/3} * (\sqrt{-3} + 1) - c) + (6 * (b * x^3 + a) * \cosh(d * x + c)^2 - 6 * (b * x^3 + a) * \sinh(d * x + c)^2 - (a * d^3 / b)^{1/3} * ((b * x^3 - \sqrt{-3} * (b * x^3 + a) + a) * \cosh(d * x + c)^2 - (b * x^3 - \sqrt{-3} * (b * x^3 + a) + a) * \sinh(d * x + c)^2)) * \operatorname{Ei}(d * x + 1/2 * (a * d^3 / b)^{1/3} * (\sqrt{-3} - 1)) * \cosh(1/2 * (a * d^3 / b)^{1/3} * (\sqrt{-3} - 1) - c) + (6 * (b * x^3 + a) * \cosh(d * x + c)^2 - 6 * (b * x^3 + a) * \sinh(d * x + c)^2 - (-a * d^3 / b)^{1/3} * ((b * x^3 - \sqrt{-3} * (b * x^3 + a) + a) * \cosh(d * x + c)^2 - (b * x^3 - \sqrt{-3} * (b * x^3 + a) + a) * \sinh(d * x + c)^2)) * \operatorname{Ei}(-d * x + 1/2 * (-a * d^3 / b)^{1/3} * (\sqrt{-3} - 1)) * \cosh(1/2 * (-a * d^3 / b)^{1/3} * (\sqrt{-3} - 1) + c) + 2 * (3 * (b * x^3 + a) * \cosh(d * x + c)^2 - 3 * (b * x^3 + a) * \sinh(d * x + c)^2 + (-a * d^3 / b)^{1/3} * ((b * x^3 + a) * \cosh(d * x + c)^2 - (b * x^3 + a) * \sinh(d * x + c)^2)) * \operatorname{Ei}(-d * x + (-a * d^3 / b)^{1/3}) * \cosh(c + (-a * d^3 / b)^{1/3}) + 2 * (3 * (b * x^3 + a) * \cosh(d * x + c)^2 - 3 * (b * x^3 + a) * \sinh(d * x + c)^2 + (a * d^3 / b)^{1/3} * ((b * x^3 + a) * \cosh(d * x + c)^2 - (b * x^3 + a) * \sinh(d * x + c)^2)) * \operatorname{Ei}(d * x + (a * d^3 / b)^{1/3}) * \cosh(-c + (a * d^3 / b)^{1/3}) + (6 * (b * x^3 + a) * \cosh(d * x + c)^2 - 6 * (b * x^3 + a) * \sinh(d * x + c)^2 - (a * d^3 / b)^{1/3} * ((b * x^3 + \sqrt{-3} * (b * x^3 + a) + a) * \cosh(d * x + c)^2 - (b * x^3 + \sqrt{-3} * (b * x^3 + a) + a) * \sinh(d * x + c)^2)) * \operatorname{Ei}(d * x - 1/2 * (a * d^3 / b)^{1/3} * (\sqrt{-3} + 1)) * \sinh(1/2 * (a * d^3 / b)^{1/3} * (\sqrt{-3} + 1) + c) + (6 * (b * x^3 + a) * \cosh(d * x + c)^2 - 6 * (b * x^3 + a) * \sinh(d * x + c)^2 - (-a * d^3 / b)^{1/3} * ((b * x^3 + \sqrt{-3} * (b * x^3 + a) + a) * \cosh(d * x + c)^2 - (b * x^3 + \sqrt{-3} * (b * x^3 + a) + a) * \sinh(d * x + c)^2)) * \operatorname{Ei}(-d * x - 1/2 * (-a * d^3 / b)^{1/3} * (\sqrt{-3} + 1)) * \sinh(1/2 * (-a * d^3 / b)^{1/3} * (\sqrt{-3} + 1) - c) - (6 * (b * x^3 + a) * \cosh(d * x + c)^2 - 6 * (b * x^3 + a) * \sinh(d * x + c)^2 - (a * d^3 / b)^{1/3} * ((b * x^3 - \sqrt{-3} * (b * x^3 + a) + a) * \cosh(d * x + c)^2 - (b * x^3 - \sqrt{-3} * (b * x^3 + a) + a) * \sinh(d * x + c)^2)) * \operatorname{Ei}(d * x + 1/2 * (a * d^3 / b)^{1/3} * (\sqrt{-3} - 1)) * \sinh(1/2 * (a * d^3 / b)^{1/3} * (\sqrt{-3} - 1) - c) - (6 * (b * x^3 + a) * \cosh(d * x + c)^2 - 6 * (b * x^3 + a) * \sinh(d * x + c)^2 - (-a * d^3 / b)^{1/3} * ((b * x^3 - \sqrt{-3} * (b * x^3 + a) + a) * \cosh(d * x + c)^2 - (b * x^3 - \sqrt{-3} * (b * x^3 + a) + a) * \sinh(d * x + c)^2)) * \operatorname{Ei}(-d * x + 1/2 * (-a * d^3 / b)^{1/3} * (\sqrt{-3} - 1)) * \sinh(1/2 * (-a * d^3 / b)^{1/3} * (\sqrt{-3} - 1) + c) - 2 * (3 * (b * x^3 + a) * \cosh(d * x + c)^2 - 3 * (b * x^3 + a) * \sinh(d * x + c)^2 + (-a * d^3 / b)^{1/3} * ((b * x^3 + a) * \cosh(d * x + c)^2 - (b * x^3 + a) * \sinh(d * x + c)^2)) * \operatorname{Ei}(-d * x + (-a * d^3 / b)^{1/3}) * \sinh(c + (-a * d^3 / b)^{1/3}) - 2 * (3 * (b * x^3 + a) * \cosh(d * x + c)^2 - 3 * (b * x^3 + a) * \sinh(d * x + c)^2 + (a * d^3 / b)^{1/3} * ((b * x^3 + a) * \cosh(d * x + c)^2 - (b * x^3 + a) * \sinh(d * x + c)^2)) * \operatorname{Ei}(d * x + (a * d^3 / b)^{1/3}) * \sinh(-c + (a * d^3 / b)^{1/3}) - 12 * a * \cosh(d * x + c) - 18 * ((b * x^3 + a) * \operatorname{Ei}(d * x) + (b * x^3 + a) * \operatorname{Ei}(-d * x)) * \cosh(c) - 18 * ((b * x^3 + a) * \operatorname{Ei}(d * x) - (b * x^3 + a) * \operatorname{Ei}(-d * x)) * \sinh(c) / ((a^2 * b * x^3 + a^3) * \cosh(d * x + c)^2 - (a^2 * b * x^3 + a^3) * \sinh(d * x + c)^2)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x\*\*3+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(bx^3 + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/x/(b\*x^3+a)^2,x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((b\*x^3 + a)^2\*x), x)

$$3.107 \quad \int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} dx$$

**Optimal.** Leaf size=784

result too large to display

```
[Out] -(x^3*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh[c + d*x]/(6*b^2*(a + b*x^3)
) - ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(1/3)*b^(8/3)) + ((-1)^(1/3)*d^
2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/
3)*d)/b^(1/3)) - d*x])/(54*a^(1/3)*b^(8/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(
1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*d*
CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(27*
a^(2/3)*b^(7/3)) - (2*(-1)^(1/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) +
(2*(-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) - (d*x*Sinh[c +
d*x])/(18*b^2*(a + b*x^3)) + (2*(-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(2/3)*
b^(7/3)) + ((-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhInt
egral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a^(1/3)*b^(8/3)) + (2*d*Co
sh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^
(2/3)*b^(7/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d
)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*(-1)^(2/3)*d*Cosh[c - ((-1)^(2/
3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(
27*a^(2/3)*b^(7/3)) + ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1
/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3
))
```

**Rubi [A]** time = 1.6262, antiderivative size = 784, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {5291, 5289, 5280, 3303, 3298, 3301, 5290, 5293}

$$\frac{2d \sinh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(xd + \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{27a^{2/3}b^{7/3}} - \frac{2\sqrt[3]{-1}d \sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{27a^{2/3}b^{7/3}} + \frac{2(-1)^{2/3}d \sinh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{27a^{2/3}b^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*Cosh[c + d\*x])/(a + b\*x^3)^3,x]

```
[Out] -(x^3*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh[c + d*x]/(6*b^2*(a + b*x^3)
) - ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(1/3)*b^(8/3)) + ((-1)^(1/3)*d^
2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/
3)*d)/b^(1/3)) - d*x])/(54*a^(1/3)*b^(8/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(
1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*d*
CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(27*
a^(2/3)*b^(7/3)) - (2*(-1)^(1/3)*d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1
/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) +
(2*(-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh
[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(2/3)*b^(7/3)) - (d*x*Sinh[c +
d*x])/(18*b^2*(a + b*x^3)) + (2*(-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(27*a^(2/3)*
b^(7/3)) + ((-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(54*a^(1/3)*b^(8/3)) + (2*d*Co
sh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(27*a^
(2/3)*b^(7/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d
)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3)) + (2*(-1)^(2/3)*d*Cosh[c - ((-1)^(2/
3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/
(27*a^(2/3)*b^(7/3)) + ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1
/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(54*a^(1/3)*b^(8/3
))
```

### Rule 5291

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

### Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] := Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

### Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5290

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Sinh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sy
mbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} dx &= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} + \frac{\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} dx}{2b} + \frac{d \int \frac{x^3 \sinh(c+dx)}{(a+bx^3)^2} dx}{6b} \\
&= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{dx \sinh(c+dx)}{18b^2(a+bx^3)} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{18b^2} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{6b^2} + \frac{d^2 \int \frac{\sinh(c+dx)}{a+bx^3} dx}{6b^2} \\
&= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{dx \sinh(c+dx)}{18b^2(a+bx^3)} + \frac{d \int \left( -\frac{\sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{bx})} - \frac{\sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx})} \right) dx}{18b^2} \\
&= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{dx \sinh(c+dx)}{18b^2(a+bx^3)} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx}{54a^{2/3}b^2} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx}} dx}{54a^{2/3}b^2} \\
&= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{dx \sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\left( d \cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \right) \int \frac{\sinh\left(\frac{\sqrt[3]{ad}}{\sqrt[3]{b}} + dx\right)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx}{54a^{2/3}b^2} \\
&= -\frac{x^3 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{6b^2(a+bx^3)} - \frac{(-1)^{2/3}d^2 \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{54\sqrt[3]{ab^8/3}} + \frac{\sqrt[3]{-1}d^2}{\dots}
\end{aligned}$$

**Mathematica [C]** time = 0.595173, size = 397, normalized size = 0.51

$$d\text{RootSum}\left[\#1^3 b + a \&, \frac{4 \sinh(\#1 d + c) \text{Chi}(d(x - \#1)) - \#1 d \sinh(\#1 d + c) \text{Chi}(d(x - \#1)) - 4 \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \#1 d \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) - 4 \sinh(\#1 d + c) \text{Chi}(d(x - \#1))}{\#1^2}\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*Cosh[c + d\*x])/(a + b\*x^3)^3,x]

[Out] (d\*RootSum[a + b\*#1^3 & , (-4\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] + 4\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] + 4\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] - 4\*Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + d\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)]\*#1 - d\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1]\*#1 - d\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1 + d\*Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1)/#1^2 & ] + d\*RootSum[a + b\*#1^3 & , (4\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] + 4\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] + 4\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + 4\*Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]

] + d\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)]\*#1 + d\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1]\*#1 + d\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1 + d\*Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1/#1^2 & ] - (6\*b\*(3\*(a + 2\*b\*x^3)\*Cosh[c + d\*x] + d\*x\*(a + b\*x^3)\*Sinh[c + d\*x]))/(a + b\*x^3)^2/(108\*b^3)

**Maple [C]** time = 0.52, size = 2448, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*cosh(d\*x+c)/(b\*x^3+a)^3,x)

[Out] 1/108/d^3/a^2/b^3\*sum((10\*\_R1^2\*a\*b\*c^2\*d^3-\_R1^2\*b^2\*c^5-\_R1\*a^2\*d^6-10\*\_R1\*a\*b\*c^3\*d^3+2\*\_R1\*b^2\*c^6-4\*a^2\*c\*d^6+5\*a\*b\*c^4\*d^3-b^2\*c^7-10\*\_R1^2\*a\*b\*c\*d^3-20\*\_R1^2\*b^2\*c^4+20\*\_R1\*a\*b\*c^2\*d^3+34\*\_R1\*b^2\*c^5+4\*a^2\*d^6+10\*a\*b\*c^3\*d^3-14\*b^2\*c^6-10\*\_R1\*a\*b\*c\*d^3-20\*\_R1\*b^2\*c^4-10\*a\*b\*c^2\*d^3+10\*b^2\*c^5))/(\_R1^2-2\*\_R1\*c+c^2)\*exp(-\_R1)\*Ei(1,d\*x-\_R1+c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))+1/36\*d^7\*exp(-d\*x-c)/b/(b^2\*d^6\*x^6+2\*a\*b\*d^6\*x^3+a^2\*d^6)\*x^4+1/36\*d^7\*exp(-d\*x-c)\*a/b^2/(b^2\*d^6\*x^6+2\*a\*b\*d^6\*x^3+a^2\*d^6)\*x+1/108/d^3\*c^5/a^2/b\*sum((\_R1^2-2\*\_R1\*c+c^2+6\*\_R1-6\*c+10)/(\_R1^2-2\*\_R1\*c+c^2)\*exp(-\_R1)\*Ei(1,d\*x-\_R1+c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))-5/108/d^3\*c^4/a^2/b^2\*sum((\_R1^2\*b\*c-2\*\_R1\*b\*c^2-a\*d^3+b\*c^3+4\*\_R1^2\*b-2\*\_R1\*b\*c-2\*b\*c^2+4\*\_R1\*b+6\*b\*c)/(\_R1^2-2\*\_R1\*c+c^2)\*exp(-\_R1)\*Ei(1,d\*x-\_R1+c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))+5/54/d^3\*c^3/a^2/b^2\*sum((\_R1^2\*b\*c^2-\_R1\*a\*d^3-2\*\_R1\*b\*c^3-a\*c\*d^3+b\*c^4+8\*\_R1^2\*b\*c-10\*\_R1\*b\*c^2-2\*a\*d^3+2\*b\*c^3+8\*\_R1\*b\*c+2\*b\*c^2)/(\_R1^2-2\*\_R1\*c+c^2)\*exp(-\_R1)\*Ei(1,d\*x-\_R1+c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))-1/6\*d^6\*exp(-d\*x-c)/b/(b^2\*d^6\*x^6+2\*a\*b\*d^6\*x^3+a^2\*d^6)\*x^3+5/54/d^3\*c^2/a^2/b^2\*sum((\_R1^2\*a\*d^3-\_R1^2\*b\*c^3+\_R1\*a\*c\*d^3+2\*\_R1\*b\*c^4+a\*c^2\*d^3-b\*c^5-12\*\_R1^2\*b\*c^2+18\*\_R1\*b\*c^3+6\*a\*c\*d^3-6\*b\*c^4-12\*\_R1\*b\*c^2-2\*a\*d^3+2\*b\*c^3)/(\_R1^2-2\*\_R1\*c+c^2)\*exp(-\_R1)\*Ei(1,d\*x-\_R1+c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))-1/12\*d^6\*exp(-d\*x-c)\*a/b^2/(b^2\*d^6\*x^6+2\*a\*b\*d^6\*x^3+a^2\*d^6)-5/108/d^3\*c/a^2/b^3\*sum((4\*\_R1^2\*a\*b\*c\*d^3-\_R1^2\*b^2\*c^4-2\*\_R1\*a\*b\*c^2\*d^3+2\*\_R1\*b^2\*c^5-a^2\*d^6+2\*a\*b\*c^3\*d^3-b^2\*c^6-2\*\_R1^2\*a\*b\*d^3-16\*\_R1^2\*b^2\*c^3+4\*\_R1\*a\*b\*c\*d^3+26\*\_R1\*b^2\*c^4+10\*a\*b\*c^2\*d^3-10\*b^2\*c^5-2\*\_R1\*a\*b\*d^3-16\*\_R1\*b^2\*c^3-6\*a\*b\*c\*d^3+6\*b^2\*c^4)/(\_R1^2-2\*\_R1\*c+c^2)\*exp(-\_R1)\*Ei(1,d\*x-\_R1+c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))+1/108/d^3\*c^5/a^2/b\*sum((\_R1^2-2\*\_R1\*c+c^2-6\*\_R1+6\*c+10)/(\_R1^2-2\*\_R1\*c+c^2)\*exp(\_R1)\*Ei(1,-d\*x+\_R1-c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))-5/108/d^3\*c^4/a^2/b^2\*sum((\_R1^2\*b\*c-2\*\_R1\*b\*c^2-a\*d^3+b\*c^3-4\*\_R1^2\*b+2\*\_R1\*b\*c+2\*b\*c^2+4\*\_R1\*b+6\*b\*c)/(\_R1^2-2\*\_R1\*c+c^2)\*exp(\_R1)\*Ei(1,-d\*x+\_R1-c),\_R1=RootOf(\_Z^3\*b-3\*\_Z^2\*b\*c+3\*\_Z\*b\*c^2+a\*d^3-b\*c^3))

$$\begin{aligned}
& x+_R1-c), \_R1=\text{RootOf}(\_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+5/54/d^3*c^3 \\
& /a^2/b^2*\text{sum}((\_R1^2*b*c^2-\_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+1 \\
& 0*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(\_R1^2-2*_R1*c+c^2)*\text{exp}(\_R1) \\
& *Ei(1,-d*x+_R1-c), \_R1=\text{RootOf}(\_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+5/5 \\
& 4/d^3*c^2/a^2/b^2*\text{sum}((\_R1^2*a*d^3-\_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^ \\
& 2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*a* \\
& d^3+2*b*c^3)/(\_R1^2-2*_R1*c+c^2)*\text{exp}(\_R1)*Ei(1,-d*x+_R1-c), \_R1=\text{RootOf}(\_Z^3* \\
& b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/12*d^6*\text{exp}(d*x+c)*a/b^2/(b^2*d^6*x^ \\
& 6+2*a*b*d^6*x^3+a^2*d^6)-1/6*d^6*\text{exp}(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^ \\
& 2*d^6)*x^3+1/108/d^3/a^2/b^3*\text{sum}((10*_R1^2*a*b*c^2*d^3-\_R1^2*b^2*c^5-\_R1*a^ \\
& 2*d^6-10*_R1*a*b*c^3*d^3+2*_R1*b^2*c^6-4*a^2*c*d^6+5*a*b*c^4*d^3-b^2*c^7+10 \\
& *_R1^2*a*b*c*d^3+20*_R1^2*b^2*c^4-20*_R1*a*b*c^2*d^3-34*_R1*b^2*c^5-4*a^2*d \\
& ^6-10*a*b*c^3*d^3+14*b^2*c^6-10*_R1*a*b*c*d^3-20*_R1*b^2*c^4-10*a*b*c^2*d^3 \\
& +10*b^2*c^5)/(\_R1^2-2*_R1*c+c^2)*\text{exp}(\_R1)*Ei(1,-d*x+_R1-c), \_R1=\text{RootOf}(\_Z^3* \\
& b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-5/108/d^3*c/a^2/b^3*\text{sum}((4*_R1^2*a*b* \\
& c*d^3-\_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b \\
& ^2*c^6+2*_R1^2*a*b*d^3+16*_R1^2*b^2*c^3-4*_R1*a*b*c*d^3-26*_R1*b^2*c^4-10*a \\
& *b*c^2*d^3+10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/ \\
& (\_R1^2-2*_R1*c+c^2)*\text{exp}(\_R1)*Ei(1,-d*x+_R1-c), \_R1=\text{RootOf}(\_Z^3*b-3*_Z^2*b*c+3 \\
& *_Z*b*c^2+a*d^3-b*c^3))-1/36*d^7*\text{exp}(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^ \\
& 2*d^6)*x^4-1/36*d^7*\text{exp}(d*x+c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x
\end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*cosh(d\*x+c)/(b\*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 3.09944, size = 6886, normalized size = 8.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*cosh(d\*x+c)/(b\*x^3+a)^3,x, algorithm="fricas")



```
[Out] 1/216*(((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2
*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b
^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 +
2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 -
(b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x
+ c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(
1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 -
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^
3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^
3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*
b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) +
1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)^(2/3)*((b^2*
x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)
^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sin
h(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b
^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - s
qrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + ((-
a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3
+ a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 +
2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x
^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6
+ 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2)
)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*
(sqrt(-3) - 1) + c) - 2*((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh
(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(
1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
a^2)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3
)) - 2*((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2
*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*
a*b*x^3 + a^2)*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2)*sinh(d*x + c)^
2))*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + ((a*d^3/b)^(2/3)
*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d
*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt
(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 +
a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x - 1/2*
(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c
) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a
*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2
*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(-a*d^3/b)^(1/3)*((b^2*x^6 +
2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (
b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x
+ c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)
```

$$\begin{aligned} &^{(1/3)} * (\sqrt{-3} + 1) - c) - ((a*d^3/b)^{(2/3)} * ((b^2*x^6 + 2*a*b*x^3 + a^2 + \\ &\sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + \\ &\sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \sinh(d*x + c)^2 - 4*(a*d^3/b)^{(1/3)} * ((b^2*x^6 + 2*a*b*x^3 + a^2 - \\ &\sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - \\ &\sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \sinh(d*x + c)^2)) * \text{Ei}(d*x + 1/2*(a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1) \\ &)) * \sinh(1/2*(a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1) - c) - ((-a*d^3/b)^{(2/3)} * ((b^2*x^6 + 2*a*b*x^3 + a^2 + \\ &\sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + \\ &\sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \sinh(d*x + c)^2) - 4*(-a*d^3/b)^{(1/3)} * ((b^2*x^6 + 2*a*b*x^3 + a^2 - \\ &\sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - \\ &\sqrt{-3}) * (b^2*x^6 + 2*a*b*x^3 + a^2)) * \sinh(d*x + c)^2)) * \text{Ei}(-d*x + 1/2*(-a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1) \\ &)) * \sinh(1/2*(-a*d^3/b)^{(1/3)} * (\sqrt{-3} - 1) + c) + 2 * ((-a*d^3/b)^{(2/3)} * ((b^2*x^6 + 2*a*b*x^3 + a^2) * \\ &\cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2) * \sinh(d*x + c)^2) - 4*(-a*d^3/b)^{(1/3)} * ((b^2*x^6 + 2*a*b*x^3 + a^2) * \\ &\cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2) * \sinh(d*x + c)^2)) * \text{Ei}(-d*x + (-a*d^3/b)^{(1/3)}) * \sinh(c + (-a*d^3/b)^{(1/3)}) + 2 * ((a*d^3/b)^{(2/3)} * ((b^2*x^6 + 2*a*b*x^3 + a^2) * \\ &\cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2) * \sinh(d*x + c)^2) - 4*(a*d^3/b)^{(1/3)} * ((b^2*x^6 + 2*a*b*x^3 + a^2) * \\ &\cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2) * \sinh(d*x + c)^2)) * \text{Ei}(d*x + (a*d^3/b)^{(1/3)}) * \sinh(-c + (a*d^3/b)^{(1/3)}) - 36*(2*a*b*x^3 + a^2) * \cosh(d*x + c) - 12 * (a*b*d*x^4 + a^2*d*x) * \sinh(d*x + c) / ((a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2) * \cosh(d*x + c)^2 - (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2) * \sinh(d*x + c)^2) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*cosh(d\*x+c)/(b\*x\*\*3+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \cosh(dx + c)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^5*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

$$3.108 \quad \int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} dx$$

**Optimal.** Leaf size=1105

result too large to display

```
[Out] Cosh[c + d*x]/(9*a*b^2*x) - (x^2*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh[
c + d*x]/(9*b^2*x*(a + b*x^3)) - ((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d
)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(4/3)*
b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*a^(2/3)*b^(7/3)) + ((-1)^(
1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)) - d*x]/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Cosh[c - (
(-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)) - d*x]/(54*a^(2/3)*b^(7/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegr
al[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(4/3)*b^(5/3)) + (d^2*Cosh[c - (a^(1/3
)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(54*a^(2/3)*b^(7/3))
- (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)]
)/(27*a*b^2) - (d*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c
+ ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]/(27*a*b^2) - (d*CoshIntegral[-(((-1)^(2
/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]/(2
7*a*b^2) - (d*Sinh[c + d*x]/(18*b^2*(a + b*x^3)) + (d*Cosh[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(2
7*a*b^2) + ((-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(4/3)*b^(5/3)) + ((-1)^(1/3)
*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x]/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - (a^(1/3)*d)/b^(1/
3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a*b^2) - (Sinh[c - (a^(1/3
)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(4/3)*b^(5/3))
+ (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*
x]/(54*a^(2/3)*b^(7/3)) - (d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Sinh
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(27*a*b^2) + ((-1)^(1/3)*Si
nh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/
b^(1/3) + d*x]/(27*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d^2*Sinh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(5
4*a^(2/3)*b^(7/3))
```

---

**Rubi [A]** time = 1.85304, antiderivative size = 1105, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} =$

0.474, Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292, 5288, 5281}

result too large to display

Antiderivative was successfully verified.

[In] Int[(x^4\*Cosh[c + d\*x])/(a + b\*x^3)^3,x]

[Out] Cosh[c + d\*x]/(9\*a\*b^2\*x) - (x^2\*Cosh[c + d\*x])/(6\*b\*(a + b\*x^3)^2) - Cosh[c + d\*x]/(9\*b^2\*x\*(a + b\*x^3)) - ((-1)^(2/3)\*Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(27\*a^(4/3)\*b^(5/3)) - ((-1)^(1/3)\*d^2\*Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(54\*a^(2/3)\*b^(7/3)) + ((-1)^(1/3)\*Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(27\*a^(4/3)\*b^(5/3)) + ((-1)^(2/3)\*d^2\*Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(54\*a^(2/3)\*b^(7/3)) - (Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x]/(27\*a^(4/3)\*b^(5/3)) + (d^2\*Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x]/(54\*a^(2/3)\*b^(7/3)) - (d\*CoshIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x]\*Sinh[c - (a^(1/3)\*d)/b^(1/3)])/(27\*a\*b^2) - (d\*CoshIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]\*Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)])/(27\*a\*b^2) - (d\*CoshIntegral[-(((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]\*Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)])/(27\*a\*b^2) - (d\*Sinh[c + d\*x])/(18\*b^2\*(a + b\*x^3)) + (d\*Cosh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(27\*a\*b^2) + ((-1)^(2/3)\*Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(27\*a^(4/3)\*b^(5/3)) + ((-1)^(1/3)\*d^2\*Sinh[c + ((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(1/3)\*a^(1/3)\*d)/b^(1/3) - d\*x]/(54\*a^(2/3)\*b^(7/3)) - (d\*Cosh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x]/(27\*a\*b^2) - (Sinh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x]/(27\*a^(4/3)\*b^(5/3)) + (d^2\*Sinh[c - (a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[(a^(1/3)\*d)/b^(1/3) + d\*x]/(54\*a^(2/3)\*b^(7/3)) - (d\*Cosh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x]/(27\*a\*b^2) + ((-1)^(1/3)\*Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x]/(27\*a^(4/3)\*b^(5/3)) + ((-1)^(2/3)\*d^2\*Sinh[c - ((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3)]\*SinhIntegral[((-1)^(2/3)\*a^(1/3)\*d)/b^(1/3) + d\*x]/(54\*a^(2/3)\*b^(7/3))

### Rule 5291

Int[Cosh[(c\_.) + (d\_.)\*(x\_.)]\*(x\_.)^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(x^(m - n + 1)\*(a + b\*x^n)^(p + 1)\*Cosh[c + d\*x])/(b\*n\*(p + 1)), x] + (-Dist[(m - n + 1)/(b\*n\*(p + 1)), Int[x^(m - n)\*(a + b\*x^n)^(p + 1)\*Cosh[c + d\*x], x], x] - Dist[d/(b\*n\*(p + 1)), Int[x^(m - n + 1)\*(a + b\*x^n)^(p + 1)\*Cosh[c + d\*x], x], x]

```
n)^(p + 1)*Sinh[c + d*x], x], x) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

### Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5292

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] := Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

### Rule 5288

```

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] := Simp[(e^m*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])

```

### Rule 5281

```

Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

```

### Rubi steps

$$\begin{aligned}
\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} dx &= -\frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} + \frac{\int \frac{x \cosh(c+dx)}{(a+bx^3)^2} dx}{3b} + \frac{d \int \frac{x^2 \sinh(c+dx)}{(a+bx^3)^2} dx}{6b} \\
&= -\frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx}{9b^2} + \frac{d \int \frac{\sinh(c+dx)}{x(a+bx^3)} dx}{9b^2} + \frac{d^2 \int \frac{\cosh(c+dx)}{a+bx^3} dx}{18b^2} \\
&= -\frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\int \left( \frac{\cosh(c+dx)}{ax^2} - \frac{bx \cosh(c+dx)}{a(a+bx^3)} \right) dx}{9b^2} + \frac{d \int \left( \frac{\sinh(c+dx)}{ax} - \frac{bx \sinh(c+dx)}{a(a+bx^3)} \right) dx}{9b^2} \\
&= -\frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2(a+bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^2} dx}{9ab^2} + \frac{\int \frac{x \cosh(c+dx)}{a+bx^3} dx}{9ab} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{18ab} \\
&= \frac{\cosh(c+dx)}{9ab^2 x} - \frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2(a+bx^3)} + \frac{\int \left( -\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[3]{a}+\sqrt[3]{bx})} - \frac{(-1)^{2/3} \cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}} \right) dx}{54a^{2/3}b^{7/3}} \\
&= \frac{\cosh(c+dx)}{9ab^2 x} - \frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{\sqrt[3]{-1}d^2 \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - d\right)}{54a^{2/3}b^{7/3}} \\
&= \frac{\cosh(c+dx)}{9ab^2 x} - \frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{\sqrt[3]{-1}d^2 \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - d\right)}{54a^{2/3}b^{7/3}} \\
&= \frac{\cosh(c+dx)}{9ab^2 x} - \frac{x^2 \cosh(c+dx)}{6b(a+bx^3)^2} - \frac{\cosh(c+dx)}{9b^2 x(a+bx^3)} - \frac{(-1)^{2/3} \cosh\left(c + \frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - d\right)}{27a^{4/3}b^{5/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.524039, size = 675, normalized size = 0.61

$$\text{RootSum}\left[\#1^3 b + a \&, \frac{-2\#1^2 b d \sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + 2\#1^2 b d \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + 2\#1^2 b d \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - 2\#1^2 b d \cosh(\#1 d + c)}{54 a^{2/3} b^{7/3}}\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*Cosh[c + d\*x])/(a + b\*x^3)^3,x]



```
[Out] (RootSum[a + b*x^3 & , (a*d^2*Cosh[c + d*x]*CoshIntegral[d*(x - #1)] - a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*x] - a*d^2*Cosh[c + d*x]*SinhIntegral[d*(x - #1)] + a*d^2*Sinh[c + d*x]*SinhIntegral[d*(x - #1)] + 2*b*Cosh[c + d*x]*CoshIntegral[d*(x - #1)]*#1 - 2*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*x]*#1 - 2*b*Cosh[c + d*x]*SinhIntegral[d*(x - #1)]*#1 + 2*b*Sinh[c + d*x]*SinhIntegral[d*(x - #1)]*#1 + 2*b*d*Cosh[c + d*x]*CoshIntegral[d*(x - #1)]*#1^2 - 2*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*x]*#1^2 - 2*b*d*Cosh[c + d*x]*SinhIntegral[d*(x - #1)]*#1^2 + 2*b*d*Sinh[c + d*x]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] - RootSum[a + b*x^3 & , (-a*d^2*Cosh[c + d*x]*CoshIntegral[d*(x - #1)]) - a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*x] - a*d^2*Cosh[c + d*x]*SinhIntegral[d*(x - #1)] - a*d^2*Sinh[c + d*x]*SinhIntegral[d*(x - #1)] - 2*b*Cosh[c + d*x]*CoshIntegral[d*(x - #1)]*#1 - 2*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*x]*#1 - 2*b*Cosh[c + d*x]*SinhIntegral[d*(x - #1)]*#1 - 2*b*Sinh[c + d*x]*SinhIntegral[d*(x - #1)]*#1 + 2*b*d*Cosh[c + d*x]*CoshIntegral[d*(x - #1)]*#1^2 + 2*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*x]*#1^2 + 2*b*d*Cosh[c + d*x]*SinhIntegral[d*(x - #1)]*#1^2 + 2*b*d*Sinh[c + d*x]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + (6*b*Cosh[d*x]*(b*x^2*(-a + 2*b*x^3)*Cosh[c] - a*d*(a + b*x^3)*Sinh[c]))/(a + b*x^3)^2 + (6*b*(-a*d*(a + b*x^3)*Cosh[c]) + b*x^2*(-a + 2*b*x^3)*Sinh[c])*Sinh[d*x])/(a + b*x^3)^2)/(108*a*b^3)
```

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**Maple [C]** time = 0.143, size = 1927, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*cosh(d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] 1/27/d^2*c^3/a^2/b^2*sum((R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(R1^2-2*_R1*c+c^2)*exp(-R1)*Ei(1,d*x-R1+c),_R1=RootOf(Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108/d^2*c^4/a^2/b*sum((R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(R1^2-2*_R1*c+c^2)*exp(-R1)*Ei(1,d*x-R1+c),_R1=RootOf(Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/27/d^2*c/a^2/b^2*sum((R1^2*a*d^3-R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5-12*_R1^2*b*c^2+18*_R1*b*c^3+6*a*c*d^3-6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(R1^2-2*_R1*c+c^2)*exp(-R1)*Ei(1,d*x-R1+c),_R1=RootOf(Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18/d^2*c^2/a^2/b^2*sum((R1^2*b*c^2-R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(R1^2-2*_R1*c+c^2)*exp(-R1)*Ei(1,d*x-R1+c),_R1=RootOf(Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/108/d^2/a^2/b^3*sum((4*_R1^2*a*b*c*d^3-R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6-
```

$$\begin{aligned}
& 2*_R1^2*a*b*d^3-16*_R1^2*b^2*c^3+4*_R1*a*b*c*d^3+26*_R1*b^2*c^4+10*a*b*c^2*d^3-10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2)*\exp(-_R1)*\text{Ei}(1,d*x-_R1+c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*d^6*\exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^5-1/36*d^6*\exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2+1/36*d^7*\exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+1/36*d^7*\exp(-d*x-c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)+1/108/d^2/a^2/b^3*\text{sum}((4*_R1^2*a*b*c*d^3-_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6+2*_R1^2*a*b*d^3+16*_R1^2*b^2*c^3-4*_R1*a*b*c*d^3-26*_R1*b^2*c^4-10*a*b*c^2*d^3+10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*d^6*\exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^5-1/36*d^6*\exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2-1/36*d^7*\exp(d*x+c)*a/b^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/27/d^2*c/a^2/b^2*\text{sum}((*_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/27/d^2*c^3/a^2/b^2*\text{sum}((*_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18/d^2*c^2/a^2/b^2*\text{sum}((*_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108/d^2*c^4/a^2/b*\text{sum}((*_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*\exp(_R1)*\text{Ei}(1,-d*x+_R1-c), _R1=\text{RootOf}(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/36*d^7*\exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3
\end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*cosh(d\*x+c)/(b\*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 3.4755, size = 10106, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & -1/216*((4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x + c)^2 - 4* \\ & (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c)^2 - 2*(a*d^3/b)^( \\ & 2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3})*(b^3*x^6 + 2*a*b^2*x^3 + a^ \\ & 2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3})*(b^3*x^6 \\ & + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) + (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 \\ & + 2*a^2*b*d^3*x^3 + a^3*d^3 + \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a \\ & ^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + \sqrt{-3}) \\ & *(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2)*\text{Ei}(d*x \\ & - 1/2*(a*d^3/b)^(1/3)*(\sqrt{-3} + 1))*\cosh(1/2*(a*d^3/b)^(1/3)*(\sqrt{-3} + \\ & 1) + c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x + c)^2 - \\ & 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c)^2 + 2*(-a*d^3/b) \\ & ^{(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3})*(b^3*x^6 + 2*a*b^2*x^3 \\ & + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3})*(b^3*x^6 \\ & + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) + (-a*d^3/b)^(1/3)*((a*b^2*d^3 \\ & *x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^ \\ & 3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 \\ & + \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2))*\text{E} \\ & \text{i}(-d*x - 1/2*(-a*d^3/b)^(1/3)*(\sqrt{-3} + 1))*\cosh(1/2*(-a*d^3/b)^(1/3)*(\sqrt{-3} \\ & + 1) - c) + (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x \\ & + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c)^2 - 2* \\ & (a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3})*(b^3*x^6 + 2*a*b \\ & ^2*x^3 + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3} \\ & )*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) + (a*d^3/b)^(1/3)*((a*b \\ & ^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b* \\ & d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^ \\ & 3*d^3 - \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c) \\ & ^2))*\text{Ei}(d*x + 1/2*(a*d^3/b)^(1/3)*(\sqrt{-3} - 1))*\cosh(1/2*(a*d^3/b)^(1/3)* \\ & (\sqrt{-3} - 1) - c) - (4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\cosh(d \\ & *x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c)^2 + \\ & 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3})*(b^3*x^6 + 2 \\ & *a*b^2*x^3 + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{-3}) \\ & *(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) + (-a*d^3/b)^(1/3)* \\ & ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a \\ & ^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 \\ & + a^3*d^3 - \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x \\ & + c)^2))*\text{Ei}(-d*x + 1/2*(-a*d^3/b)^(1/3)*(\sqrt{-3} - 1))*\cosh(1/2*(-a*d^3/b) \\ & ^{(1/3)*(\sqrt{-3} - 1) + c) - 2*(2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d \\ & ^3)*\cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d* \\ & x + c)^2 - 2*(-a*d^3/b)^(2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*\cosh(d*x + c \\ & )^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*(\end{aligned}$$

$$\begin{aligned}
& (a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - (a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2) \operatorname{Ei}(-dx + (-a^3 d^3/b)^{1/3}) \\
& ) \cosh(c + (-a^3 d^3/b)^{1/3}) + 2(2(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - 2(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx \\
& * x + c)^2 + 2(a^3 d^3/b)^{2/3}((b^3 x^6 + 2 a^2 b x^3 + a^2 b) \cosh(dx + c)^2 - (b^3 x^6 + 2 a^2 b x^3 + a^2 b) \sinh(dx + c)^2) - (a^3 d^3/b)^{1/3}(( \\
& a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - (a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2)) \operatorname{Ei}(dx + (a^3 d^3/b)^{1/3}) \c \\
& osh(-c + (a^3 d^3/b)^{1/3}) + (4(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - 4(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + \\
& c)^2 - 2(a^3 d^3/b)^{2/3}((b^3 x^6 + 2 a^2 b x^3 + a^2 b - \sqrt{-3})(b^3 x^6 + 2 a^2 b x^3 + a^2 b)) \cosh(dx + c)^2 - (b^3 x^6 + 2 a^2 b x^3 + a^2 b \\
& - \sqrt{-3})(b^3 x^6 + 2 a^2 b x^3 + a^2 b)) \sinh(dx + c)^2) + (a^3 d^3/b)^{1/3}((a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3 + \sqrt{-3})(a^2 b^2 d^3 x^6 + \\
& 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - (a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3 + \sqrt{-3})(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3)) \sinh \\
& (dx + c)^2) \operatorname{Ei}(dx - 1/2(a^3 d^3/b)^{1/3}(\sqrt{-3} + 1)) \sinh(1/2(a^3 d^3/b)^{1/3}(\sqrt{-3} + 1) + c) - (4(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - 4(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + \\
& c)^2 + 2(-a^3 d^3/b)^{2/3}((b^3 x^6 + 2 a^2 b x^3 + a^2 b - \sqrt{-3})(b^3 x^6 + 2 a^2 b x^3 + a^2 b)) \cosh(dx + c)^2 - (b^3 x^6 + 2 a^2 b x^3 + a^2 b \\
& - \sqrt{-3})(b^3 x^6 + 2 a^2 b x^3 + a^2 b)) \sinh(dx + c)^2) + (-a^3 d^3/b)^{1/3}((a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3 + \sqrt{-3})(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3)) \cosh(dx + c)^2 - (a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3 + \sqrt{-3})(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3)) \sinh(dx + c)^2) \operatorname{Ei}(-dx - 1/2(-a^3 d^3/b)^{1/3}(\sqrt{-3} + 1)) \sinh(1/2(-a^3 d^3/b)^{1/3}(\sqrt{-3} + 1) - c) - (4(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - 4(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2 - 2(a^3 d^3/b)^{2/3}((b^3 x^6 + 2 a^2 b x^3 + a^2 b + \sqrt{-3})(b^3 x^6 + 2 a^2 b x^3 + a^2 b)) \cosh(dx + c)^2 - (b^3 x^6 + 2 a^2 b x^3 + a^2 b + \sqrt{-3})(b^3 x^6 + 2 a^2 b x^3 + a^2 b)) \sinh(dx + c)^2) + (a^3 d^3/b)^{1/3}((a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3 - \sqrt{-3})(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3)) \cosh(dx + c)^2 - (a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3 - \sqrt{-3})(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3)) \sinh(dx + c)^2) \operatorname{Ei}(dx + 1/2(a^3 d^3/b)^{1/3}(\sqrt{-3} - 1)) \sinh(1/2(a^3 d^3/b)^{1/3}(\sqrt{-3} - 1) - c) + (4(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - 4(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2 + 2(-a^3 d^3/b)^{2/3}((b^3 x^6 + 2 a^2 b x^3 + a^2 b + \sqrt{-3})(b^3 x^6 + 2 a^2 b x^3 + a^2 b)) \cosh(dx + c)^2 - (b^3 x^6 + 2 a^2 b x^3 + a^2 b + \sqrt{-3})(b^3 x^6 + 2 a^2 b x^3 + a^2 b)) \sinh(dx + c)^2) + (-a^3 d^3/b)^{1/3}((a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3 - \sqrt{-3})(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3)) \cosh(dx + c)^2 - (a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3 - \sqrt{-3})(a^2 b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3)) \sinh(dx + c)^2) \operatorname{Ei}(-dx + 1/2(-a^3 d^3/b)^{1/3}(\sqrt{-3} - 1)) \sinh(1/2(-a^3 d^3/b)^{1/3}(\sqrt{-3} - 1) + c) + 2(2(a^2 b^2 d^3 x^6 + 2 a
\end{aligned}$$

$$\begin{aligned}
& 2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c)^2 - 2*(-a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*\sinh(d*x + c)^2) \\
& - (-a*d^3/b)^{(1/3)}*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c)^2))*\text{Ei}(-d*x + (-a*d^3/b)^{(1/3)})*\sinh(c + (-a*d^3/b)^{(1/3)}) - 2*(2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x + c)^2 - 2*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c)^2 + 2*(a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*\sinh(d*x + c)^2) - (a*d^3/b)^{(1/3)}*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c)^2))*\text{Ei}(d*x + (a*d^3/b)^{(1/3)})*\sinh(-c + (a*d^3/b)^{(1/3)}) - 12*(2*a*b^2*d^2*x^5 - a^2*b*d^2*x^2)*\cosh(d*x + c) + 12*(a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c))/((a^2*b^4*d^2*x^6 + 2*a^3*b^3*d^2*x^3 + a^4*b^2*d^2)*\cosh(d*x + c)^2 - (a^2*b^4*d^2*x^6 + 2*a^3*b^3*d^2*x^3 + a^4*b^2*d^2)*\sinh(d*x + c)^2)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*cosh(d\*x+c)/(b\*x\*\*3+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \cosh(dx + c)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*cosh(d\*x+c)/(b\*x^3+a)^3,x, algorithm="giac")

[Out] integrate(x^4\*cosh(d\*x + c)/(b\*x^3 + a)^3, x)

$$3.109 \quad \int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} dx$$

**Optimal.** Leaf size=776

result too large to display

```
[Out] Cosh[c + d*x]/(18*a*b^2*x^2) - (x*Cosh[c + d*x])/((6*b*(a + b*x^3)^2) - Cosh
[c + d*x]/(18*b^2*x^2*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)
/3)*d]/b^(1/3))*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(5
/3)*b^(4/3)) - (d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*a*b^2) + ((-1)^(2/3)*Cosh[c - ((-
1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3))
- d*x]/(27*a^(5/3)*b^(4/3)) - (d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]/(54*a*b^2) + (Cos
h[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(
5/3)*b^(4/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d
)/b^(1/3) + d*x]/(54*a*b^2) + (d*Sinh[c + d*x])/(18*a*b^2*x) - (d*Sinh[c +
d*x])/(18*b^2*x*(a + b*x^3)) + ((-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d]/
b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(5/3)*b^
(4/3)) + (d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*a*b^2) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]
*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[
c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(54*a*b^2
) + ((-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[c - ((-1)
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x]/(54*a*b^2)
```

**Rubi [A]** time = 2.65879, antiderivative size = 776, normalized size of antiderivative = 1., number of steps used = 71, number of rules used = 10, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$ , Rules used = {5291, 5279, 5293, 3297, 3303, 3298, 3301, 5281, 5292, 5290}

$$-\frac{\sqrt[3]{-1} \cosh\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{27a^{5/3}b^{4/3}} + \frac{(-1)^{2/3} \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}} + \frac{\cosh\left(c - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{27a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] Cosh[c + d*x]/(18*a*b^2*x^2) - (x*Cosh[c + d*x])/(6*b*(a + b*x^3)^2) - Cosh
[c + d*x]/(18*b^2*x^2*(a + b*x^3)) - ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/
3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(5
/3)*b^(4/3)) - (d^2*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*a*b^2) + ((-1)^(2/3)*Cosh[c - ((-
1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3))
- d*x]/(27*a^(5/3)*b^(4/3)) - (d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]/(54*a*b^2) + (Cos
h[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(
5/3)*b^(4/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d
)/b^(1/3) + d*x]/(54*a*b^2) + (d*Sinh[c + d*x])/(18*a*b^2*x) - (d*Sinh[c +
d*x])/(18*b^2*x*(a + b*x^3)) + ((-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(5/3)*b^
(4/3)) + (d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*a*b^2) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]
*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[
c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(54*a*b^2
) + ((-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)
^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(5/3)*b^(4/3)) - (d^2*Sinh[c - ((-1)
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d
*x]/(54*a*b^2)
```

### Rule 5291

```
Int[Cosh[(c_) + (d_)*(x_)]*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1
)), x] + (-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1
)*Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^
n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1]
&& IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

### Rule 5279

```
Int[Cosh[(c_) + (d_)*(x_)]*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Si
mp[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x])/(b*n*(p + 1)), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[n, 2]
```

### Rule 5293

```
Int[Cosh[(c_) + (d_)*(x_)]*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
```

2] || EqQ[p, -1])

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 5292

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 5290

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1
```



)), x] + (-Dist[(m - n + 1)/(b\*n\*(p + 1)), Int[x^(m - n)\*(a + b\*x^n)^(p + 1) \* Sinh[c + d\*x], x], x] - Dist[d/(b\*n\*(p + 1)), Int[x^(m - n + 1)\*(a + b\*x^n)^(p + 1) \* Cosh[c + d\*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3 \cosh(c + dx)}{(a + bx^3)^3} dx &= -\frac{x \cosh(c + dx)}{6b(a + bx^3)^2} + \frac{\int \frac{\cosh(c+dx)}{(a+bx^3)^2} dx}{6b} + \frac{d \int \frac{x \sinh(c+dx)}{(a+bx^3)^2} dx}{6b} \\
 &= -\frac{x \cosh(c + dx)}{6b(a + bx^3)^2} - \frac{\cosh(c + dx)}{18b^2x^2(a + bx^3)} - \frac{d \sinh(c + dx)}{18b^2x(a + bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^3(a+bx^3)} dx}{9b^2} + \frac{d^2 \int \frac{\cosh(c+dx)}{x(a+bx^3)} dx}{18b^2} \\
 &= -\frac{x \cosh(c + dx)}{6b(a + bx^3)^2} - \frac{\cosh(c + dx)}{18b^2x^2(a + bx^3)} - \frac{d \sinh(c + dx)}{18b^2x(a + bx^3)} - \frac{\int \left( \frac{\cosh(c+dx)}{ax^3} - \frac{b \cosh(c+dx)}{a(a+bx^3)} \right) dx}{9b^2} + \frac{d^2 \int \frac{\cosh(c+dx)}{x(a+bx^3)} dx}{18b^2} \\
 &= -\frac{x \cosh(c + dx)}{6b(a + bx^3)^2} - \frac{\cosh(c + dx)}{18b^2x^2(a + bx^3)} - \frac{d \sinh(c + dx)}{18b^2x(a + bx^3)} - \frac{\int \frac{\cosh(c+dx)}{x^3} dx}{9ab^2} + \frac{\int \frac{\cosh(c+dx)}{a+bx^3} dx}{9ab} + \frac{d^2 \int \frac{\cosh(c+dx)}{x(a+bx^3)} dx}{18b^2} \\
 &= \frac{\cosh(c + dx)}{18ab^2x^2} - \frac{x \cosh(c + dx)}{6b(a + bx^3)^2} - \frac{\cosh(c + dx)}{18b^2x^2(a + bx^3)} - \frac{d \sinh(c + dx)}{18b^2x(a + bx^3)} + \frac{\int \left( -\frac{\cosh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{bx})} - \frac{d \sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{bx})} \right) dx}{18b^2} \\
 &= \frac{\cosh(c + dx)}{18ab^2x^2} - \frac{x \cosh(c + dx)}{6b(a + bx^3)^2} - \frac{\cosh(c + dx)}{18b^2x^2(a + bx^3)} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{18ab^2} + \frac{d \sinh(c + dx)}{18ab^2x} - \frac{d \sinh(c + dx)}{18ab^2x} \\
 &= \frac{\cosh(c + dx)}{18ab^2x^2} - \frac{x \cosh(c + dx)}{6b(a + bx^3)^2} - \frac{\cosh(c + dx)}{18b^2x^2(a + bx^3)} + \frac{d^2 \cosh(c) \text{Chi}(dx)}{18ab^2} + \frac{d \sinh(c + dx)}{18ab^2x} - \frac{d \sinh(c + dx)}{18ab^2x} \\
 &= \frac{\cosh(c + dx)}{18ab^2x^2} - \frac{x \cosh(c + dx)}{6b(a + bx^3)^2} - \frac{\cosh(c + dx)}{18b^2x^2(a + bx^3)} - \frac{\sqrt[3]{-1} \cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - a\right)}{27a^{5/3}b^{4/3}}
 \end{aligned}$$

**Mathematica [C]** time = 0.636095, size = 429, normalized size = 0.55

$$\text{RootSum} \left[ \#1^3 b + a \&, \frac{-\#1^2 d^2 \sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + \#1^2 d^2 \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \#1^2 d^2 \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - \#1^2 d^2 \cosh(\#1 d + c) \text{Shi}(d(x - \#1))}{27a^{5/3}b^{4/3}} \right]$$



$$R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36/d*c/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/108/d/a^2/b^2*sum((_R1^2*a*d^3-_R1^2*b*c^3+_R1*a*c*d^3+2*_R1*b*c^4+a*c^2*d^3-b*c^5+12*_R1^2*b*c^2-18*_R1*b*c^3-6*a*c*d^3+6*b*c^4-12*_R1*b*c^2-2*a*d^3+2*b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36*d^7*exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^5+1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2+1/108/d*c^3/a^2/b*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36*d^6*exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4-1/18*d^6*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.63863, size = 6494, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x^3+a)^3,x, algorithm="fricas")

[Out] 
$$-1/108*(((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3))*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3))*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 +$$

$$\begin{aligned}
& a^3 d^3 \sinh(dx + c)^2 - (-a d^3/b)^{1/3} ((b^3 x^6 + 2 a b^2 x^3 + a^2 b + \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b)) \cosh(dx + c)^2 - (b^3 x^6 + 2 a b^2 x^3 + a^2 b + \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b) \sinh(dx + c)^2) \operatorname{Ei}(-dx - 1/2(-a d^3/b)^{1/3}(\sqrt{-3} + 1)) \cosh(1/2(-a d^3/b)^{1/3}(\sqrt{-3} + 1) - c) + ((a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - (a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2 + (a d^3/b)^{1/3} ((b^3 x^6 + 2 a b^2 x^3 + a^2 b - \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b)) \cosh(dx + c)^2 - (b^3 x^6 + 2 a b^2 x^3 + a^2 b - \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b) \sinh(dx + c)^2) \operatorname{Ei}(dx + 1/2(a d^3/b)^{1/3}(\sqrt{-3} - 1)) \cosh(1/2(a d^3/b)^{1/3}(\sqrt{-3} - 1) - c) + ((a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - (a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2 - (-a d^3/b)^{1/3} ((b^3 x^6 + 2 a b^2 x^3 + a^2 b - \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b)) \cosh(dx + c)^2 - (b^3 x^6 + 2 a b^2 x^3 + a^2 b - \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b) \sinh(dx + c)^2) \operatorname{Ei}(-dx + 1/2(-a d^3/b)^{1/3}(\sqrt{-3} - 1)) \cosh(1/2(-a d^3/b)^{1/3}(\sqrt{-3} - 1) + c) + ((a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - (a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2 + 2(-a d^3/b)^{1/3} ((b^3 x^6 + 2 a b^2 x^3 + a^2 b) \cosh(dx + c)^2 - (b^3 x^6 + 2 a b^2 x^3 + a^2 b) \sinh(dx + c)^2) \operatorname{Ei}(-dx + (-a d^3/b)^{1/3}) \cosh(c + (-a d^3/b)^{1/3}) + ((a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - (a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2 - 2(a d^3/b)^{1/3} ((b^3 x^6 + 2 a b^2 x^3 + a^2 b) \cosh(dx + c)^2 - (b^3 x^6 + 2 a b^2 x^3 + a^2 b) \sinh(dx + c)^2) \operatorname{Ei}(dx + (a d^3/b)^{1/3}) \cosh(-c + (a d^3/b)^{1/3}) + ((a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - (a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2 + (a d^3/b)^{1/3} ((b^3 x^6 + 2 a b^2 x^3 + a^2 b + \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b)) \cosh(dx + c)^2 - (b^3 x^6 + 2 a b^2 x^3 + a^2 b + \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b) \sinh(dx + c)^2) \operatorname{Ei}(dx - 1/2(a d^3/b)^{1/3}(\sqrt{-3} + 1)) \sinh(1/2(a d^3/b)^{1/3}(\sqrt{-3} + 1) + c) + ((a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - (a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2 - (-a d^3/b)^{1/3} ((b^3 x^6 + 2 a b^2 x^3 + a^2 b + \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b)) \cosh(dx + c)^2 - (b^3 x^6 + 2 a b^2 x^3 + a^2 b + \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b) \sinh(dx + c)^2) \operatorname{Ei}(-dx - 1/2(-a d^3/b)^{1/3}(\sqrt{-3} + 1)) \sinh(1/2(-a d^3/b)^{1/3}(\sqrt{-3} + 1) - c) - ((a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - (a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2 + (a d^3/b)^{1/3} ((b^3 x^6 + 2 a b^2 x^3 + a^2 b - \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b)) \cosh(dx + c)^2 - (b^3 x^6 + 2 a b^2 x^3 + a^2 b - \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b) \sinh(dx + c)^2) \operatorname{Ei}(dx + 1/2(a d^3/b)^{1/3}(\sqrt{-3} - 1)) \sinh(1/2(a d^3/b)^{1/3}(\sqrt{-3} - 1) - c) - ((a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \cosh(dx + c)^2 - (a b^2 d^3 x^6 + 2 a^2 b d^3 x^3 + a^3 d^3) \sinh(dx + c)^2 - (-a d^3/b)^{1/3} ((b^3 x^6 + 2 a b^2 x^3 + a^2 b - \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b)) \cosh(dx + c)^2 - (b^3 x^6 + 2 a b^2 x^3 + a^2 b - \sqrt{-3})(b^3 x^6 + 2 a b^2 x^3 + a^2 b) \sinh(dx + c)^2) \operatorname{Ei}(-dx +
\end{aligned}$$

$$\begin{aligned} & \frac{1}{2}(-a^3d^3/b)^{1/3}(\sqrt{-3} - 1) \sinh\left(\frac{1}{2}(-a^3d^3/b)^{1/3}(\sqrt{-3} - 1) + c\right) - \left((a^2b^2d^3x^6 + 2a^2b^2d^3x^3 + a^3d^3) \cosh(dx + c)^2 - (a^2b^2d^3x^6 + 2a^2b^2d^3x^3 + a^3d^3) \sinh(dx + c)^2 + 2(-a^3d^3/b)^{1/3}((b^3x^6 + 2a^2b^2x^3 + a^2b) \cosh(dx + c)^2 - (b^3x^6 + 2a^2b^2x^3 + a^2b) \sinh(dx + c)^2)\right) \operatorname{Ei}(-dx + (-a^3d^3/b)^{1/3}) \sinh\left(c + (-a^3d^3/b)^{1/3}\right) - \left((a^2b^2d^3x^6 + 2a^2b^2d^3x^3 + a^3d^3) \cosh(dx + c)^2 - (a^2b^2d^3x^6 + 2a^2b^2d^3x^3 + a^3d^3) \sinh(dx + c)^2 - 2(a^3d^3/b)^{1/3}((b^3x^6 + 2a^2b^2x^3 + a^2b) \cosh(dx + c)^2 - (b^3x^6 + 2a^2b^2x^3 + a^2b) \sinh(dx + c)^2)\right) \operatorname{Ei}(dx + (a^3d^3/b)^{1/3}) \sinh\left(-c + (a^3d^3/b)^{1/3}\right) - 6(a^2b^2d^2x^4 - 2a^2b^2d^2x) \cosh(dx + c) - 6(a^2b^2d^2x^5 + a^2b^2d^2x^2) \sinh(dx + c) / \left((a^2b^4d^2x^6 + 2a^3b^3d^2x^3 + a^4b^2d) \cosh(dx + c)^2 - (a^2b^4d^2x^6 + 2a^3b^3d^2x^3 + a^4b^2d) \sinh(dx + c)^2\right) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*cosh(d\*x+c)/(b\*x\*\*3+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \cosh(dx + c)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*cosh(d\*x+c)/(b\*x^3+a)^3,x, algorithm="giac")

[Out] integrate(x^3\*cosh(d\*x + c)/(b\*x^3 + a)^3, x)

$$3.110 \quad \int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3} dx$$

**Optimal.** Leaf size=781

result too large to display

```
[Out] -Cosh[c + d*x]/(6*b*(a + b*x^3)^2) + ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*
a^(4/3)*b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]
*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]/(54*a^(4/3)*b^(5/3)
) + (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d
*x])/ (54*a^(4/3)*b^(5/3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh
[c - (a^(1/3)*d)/b^(1/3)])/(27*a^(5/3)*b^(4/3)) - ((-1)^(1/3)*d*CoshIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)])/(27*a^(5/3)*b^(4/3)) + ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1
/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(27*a^(5/3
)*b^(4/3)) + (d*Sinh[c + d*x])/(18*a*b^2*x^2) - (d*Sinh[c + d*x])/(18*b^2*x
^2*(a + b*x^3)) + ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Si
nhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(5/3)*b^(4/3)) - ((
-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*a^(4/3)*b^(5/3)) + (d*Cosh[c - (a^(1/3)
*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(5/3)*b^(4/3))
+ (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x
])/ (54*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(
1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(5/3)*b^(4/
3)) - ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral
[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(54*a^(4/3)*b^(5/3))
```

**Rubi [A]** time = 1.4493, antiderivative size = 781, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {5289, 5278, 5292, 3297, 3303, 3298, 3301, 5280, 5293}

$$\frac{(-1)^{2/3}d^2 \cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} + c\right) \text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{54a^{4/3}b^{5/3}} - \frac{\sqrt[3]{-1}d^2 \cosh\left(c - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(-xd - \frac{(-1)^{2/3}\sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}} + \frac{d^2 \cosh\left(c - \frac{\sqrt[3]{a}}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

```
[Out] -Cosh[c + d*x]/(6*b*(a + b*x^3)^2) + ((-1)^(2/3)*d^2*Cosh[c + ((-1)^(1/3)*a
^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*
a^(4/3)*b^(5/3)) - ((-1)^(1/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]
*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]/(54*a^(4/3)*b^(5/3)
) + (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d
*x]/(54*a^(4/3)*b^(5/3)) + (d*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh
[c - (a^(1/3)*d)/b^(1/3)]/(27*a^(5/3)*b^(4/3)) - ((-1)^(1/3)*d*CoshIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(
1/3)]/(27*a^(5/3)*b^(4/3)) + ((-1)^(2/3)*d*CoshIntegral[-(((-1)^(2/3)*a^(1
/3)*d)/b^(1/3) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]/(27*a^(5/3
)*b^(4/3)) + (d*Sinh[c + d*x]/(18*a*b^2*x^2) - (d*Sinh[c + d*x]/(18*b^2*x
^2*(a + b*x^3)) + ((-1)^(1/3)*d*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Si
nhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(5/3)*b^(4/3)) - ((
-1)^(2/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(
1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*a^(4/3)*b^(5/3)) + (d*Cosh[c - (a^(1/3)
*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(5/3)*b^(4/3))
+ (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x
]/(54*a^(4/3)*b^(5/3)) + ((-1)^(2/3)*d*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(
1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(5/3)*b^(4/
3)) - ((-1)^(1/3)*d^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral
[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(54*a^(4/3)*b^(5/3))
```

### Rule 5289

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p
_), x_Symbol] :> Simp[(e^m*(a + b*x^n)^(p + 1)*Cosh[c + d*x]/(b*n*(p + 1))
, x] - Dist[(d*e^m)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x
], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0
] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

### Rule 5278

```
Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*Sinh[(c_.) + (d_.)*(x_.)], x_Symbol] :> Si
mp[(x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x]/(b*n*(p + 1)), x] + (-Dis
t[(-n + 1)/(b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x]/x^n, x],
x] - Dist[d/(b*n*(p + 1)), Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x]
, x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p,
-1] && GtQ[n, 2]
```

### Rule 5292

```
Int[(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*Sinh[(c_.) + (d_.)*(x_.)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5280

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := Int[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps



$$\begin{aligned}
\int \frac{x^2 \cosh(c + dx)}{(a + bx^3)^3} dx &= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} + \frac{d \int \frac{\sinh(c+dx)}{(a+bx^3)^2} dx}{6b} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} - \frac{d \int \frac{\sinh(c+dx)}{x^3(a+bx^3)} dx}{9b^2} + \frac{d^2 \int \frac{\cosh(c+dx)}{x^2(a+bx^3)} dx}{18b^2} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} - \frac{d \int \left( \frac{\sinh(c+dx)}{ax^3} - \frac{b \sinh(c+dx)}{a(a+bx^3)} \right) dx}{9b^2} + \frac{d^2 \int \left( \frac{\cosh(c+dx)}{ax^2} - \frac{bx \cosh(c+dx)}{a(a+bx^3)} \right) dx}{18b^2} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} - \frac{d \int \frac{\sinh(c+dx)}{x^3} dx}{9ab^2} + \frac{d \int \frac{\sinh(c+dx)}{a+bx^3} dx}{9ab} + \frac{d^2 \int \frac{\cosh(c+dx)}{x^2} dx}{18ab^2} \\
&= -\frac{d^2 \cosh(c + dx)}{18ab^2 x} - \frac{\cosh(c + dx)}{6b(a + bx^3)^2} + \frac{d \sinh(c + dx)}{18ab^2 x^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} + \frac{d \int \left( -\frac{\sinh(c+dx)}{3a^{2/3}(-\sqrt[3]{a}-\sqrt[3]{bx})} \right) dx}{18ab^2} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} + \frac{d \sinh(c + dx)}{18ab^2 x^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}-\sqrt[3]{bx}} dx}{27a^{5/3}b} - \frac{d \int \frac{\sinh(c+dx)}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx}} dx}{27a^{5/3}b} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} + \frac{d^3 \text{Chi}(dx) \sinh(c)}{18ab^2} + \frac{d \sinh(c + dx)}{18ab^2 x^2} - \frac{d \sinh(c + dx)}{18b^2 x^2 (a + bx^3)} + \frac{d^3 \cosh(c) \text{Shi}(dx)}{18ab^2} \\
&= -\frac{\cosh(c + dx)}{6b(a + bx^3)^2} + \frac{(-1)^{2/3} d^2 \cosh\left(c + \frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}}\right) \text{Chi}\left(\frac{\sqrt[3]{-1} \sqrt[3]{ad}}{\sqrt[3]{b}} - dx\right)}{54a^{4/3}b^{5/3}} - \frac{\sqrt[3]{-1} d^2 \cosh\left(c - \frac{(-1)^{2/3} \sqrt[3]{ad}}{\sqrt[3]{b}}\right)}{54a^{4/3}b^{5/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.421524, size = 423, normalized size = 0.54

$$d\text{RootSum}\left[\#1^3 b + a \&, \frac{-2 \sinh(\#1 d + c) \text{Chi}(d(x - \#1)) - \#1 d \sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + 2 \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + \#1 d \cosh(\#1 d + c) \text{Chi}(d(x - \#1))}{\#1^3 b + a}\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Cosh[c + d\*x])/(a + b\*x^3)^3,x]

```
[Out] -(d*RootSum[a + b*#1^3 & , (2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] + d*RootSum[a + b*#1^3 & , (-2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (6*b*Cosh[d*x]*(-3*a*Cosh[c] + d*x*(a + b*x^3)*Sinh[c]))/(a + b*x^3)^2 - (6*b*(d*x*(a + b*x^3)*Cosh[c] - 3*a*Sinh[c])*Sinh[d*x])/(a + b*x^3)^2)/(108*a*b^2)
```

**Maple [C]** time = 0.086, size = 994, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(d*x+c)/(b*x^3+a)^3,x)
```

```
[Out] -1/36*d^7*exp(-d*x-c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4-1/36*d^7*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/12*d^6*exp(-d*x-c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/108/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4+8*_R1^2*b*c-10*_R1*b*c^2-2*a*d^3+2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108*c^2/a^2/b*sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/54*c/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/36*d^7*exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^4+1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x-1/12*d^6*exp(d*x+c)/b/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)-1/108/a^2/b^2*sum((_R1^2*b*c^2-_R1*a*d^3-2*_R1*b*c^3-a*c*d^3+b*c^4-8*_R1^2*b*c+10*_R1*b*c^2+2*a*d^3-2*b*c^3+8*_R1*b*c+2*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/108*c^2/a^2/b*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/54*c/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b*c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

$c^2+a*d^3-b*c^3$ )

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate( $x^2*\cosh(d*x+c)/(b*x^3+a)^3,x$ , algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.53715, size = 6869, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate( $x^2*\cosh(d*x+c)/(b*x^3+a)^3,x$ , algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/216*(36*a^2*\cosh(d*x + c) + ((a*d^3/b)^{(2/3)}*((b^2*x^6 + 2*a*b*x^3 + a^2 \\ & - \sqrt{-3}*(b^2*x^6 + 2*a*b*x^3 + a^2))*\cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b \\ & *x^3 + a^2 - \sqrt{-3}*(b^2*x^6 + 2*a*b*x^3 + a^2))*\sinh(d*x + c)^2) + 2*(a* \\ & d^3/b)^{(1/3)}*((b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}*(b^2*x^6 + 2*a*b*x^3 + \\ & a^2))*\cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}*(b^2*x^6 + 2* \\ & a*b*x^3 + a^2))*\sinh(d*x + c)^2))*\text{Ei}(d*x - 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + \\ & 1))*\cosh(1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) + c) + ((-a*d^3/b)^{(2/3)}*((b^2* \\ & x^6 + 2*a*b*x^3 + a^2 - \sqrt{-3}*(b^2*x^6 + 2*a*b*x^3 + a^2))*\cosh(d*x + c) \\ & ^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - \sqrt{-3}*(b^2*x^6 + 2*a*b*x^3 + a^2))*\sin \\ & h(d*x + c)^2) + 2*(-a*d^3/b)^{(1/3)}*((b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}*( \\ & b^2*x^6 + 2*a*b*x^3 + a^2))*\cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + \\ & \sqrt{-3}*(b^2*x^6 + 2*a*b*x^3 + a^2))*\sinh(d*x + c)^2))*\text{Ei}(-d*x - 1/2*(-a*d \\ & ^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\cosh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) - c) + \\ & ((a*d^3/b)^{(2/3)}*((b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}*(b^2*x^6 + 2*a*b*x \\ & ^3 + a^2))*\cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}*(b^2*x^6 \\ & + 2*a*b*x^3 + a^2))*\sinh(d*x + c)^2) + 2*(a*d^3/b)^{(1/3)}*((b^2*x^6 + 2*a*b \\ & *x^3 + a^2 - \sqrt{-3}*(b^2*x^6 + 2*a*b*x^3 + a^2))*\cosh(d*x + c)^2 - (b^2*x \\ & ^6 + 2*a*b*x^3 + a^2 - \sqrt{-3}*(b^2*x^6 + 2*a*b*x^3 + a^2))*\sinh(d*x + c)^ \\ & 2))*\text{Ei}(d*x + 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\cosh(1/2*(a*d^3/b)^{(1/3)}*( \\ & \sqrt{-3} - 1) - c) + ((-a*d^3/b)^{(2/3)}*((b^2*x^6 + 2*a*b*x^3 + a^2 + \sqrt{-3}(- \end{aligned}$$



$$\begin{aligned} &^2) * \text{Ei}(d*x + (a*d^3/b)^{1/3}) * \sinh(-c + (a*d^3/b)^{1/3}) - 12*(a*b*d*x^4 + \\ &a^2*d*x) * \sinh(d*x + c) / ((a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) * \cosh(d*x + \\ &c)^2 - (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) * \sinh(d*x + c)^2) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*cosh(d\*x+c)/(b\*x\*\*3+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \cosh(dx + c)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*cosh(d\*x+c)/(b\*x^3+a)^3,x, algorithm="giac")

[Out] integrate(x^2\*cosh(d\*x + c)/(b\*x^3 + a)^3, x)

$$3.111 \quad \int \frac{x \cosh(c+dx)}{(a+bx^3)^3} dx$$

**Optimal.** Leaf size=1147

result too large to display

```
[Out] -Cosh[c + d*x]/(18*a*b^2*x^4) + (2*Cosh[c + d*x])/(9*a^2*b*x) - Cosh[c + d*
x]/(6*b*x*(a + b*x^3)^2) + Cosh[c + d*x]/(18*b^2*x^4*(a + b*x^3)) - (2*(-1)
^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^
(1/3)*d)/b^(1/3) - d*x]/(27*a^(7/3)*b^(2/3)) + ((-1)^(1/3)*d^2*Cosh[c + ((
-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
d*x])/(54*a^(5/3)*b^(4/3)) + (2*(-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)
/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]/(27*a^(7/3)
*b^(2/3)) - ((-1)^(2/3)*d^2*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshI
ntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(54*a^(5/3)*b^(4/3)) - (2
*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27
*a^(7/3)*b^(2/3)) - (d^2*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)
*d)/b^(1/3) + d*x])/(54*a^(5/3)*b^(4/3)) - (2*d*CoshIntegral[(a^(1/3)*d)/b
^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)]/(27*a^2*b) - (2*d*CoshIntegral
[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)]/(27*a^2*b) - (2*d*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*
x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]/(27*a^2*b) + (d*Sinh[c + d*x])
/(18*a*b^2*x^3) - (d*Sinh[c + d*x])/(18*b^2*x^3*(a + b*x^3)) + (2*d*Cosh[c
+ ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/
3) - d*x]/(27*a^2*b) + (2*(-1)^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/
3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(27*a^(7/3)*b^(2/3)
) - ((-1)^(1/3)*d^2*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c -
(a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(27*a^2*b) -
(2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x]/(
27*a^(7/3)*b^(2/3)) - (d^2*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1
/3)*d)/b^(1/3) + d*x]/(54*a^(5/3)*b^(4/3)) - (2*d*Cosh[c - ((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(27*a^
2*b) + (2*(-1)^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[
((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]/(27*a^(7/3)*b^(2/3)) - ((-1)^(2/3)*d
^2*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)
*d)/b^(1/3) + d*x]/(54*a^(5/3)*b^(4/3))
```

---

**Rubi [A]** time = 3.19773, antiderivative size = 1147, normalized size of antiderivative = 1., number of steps used = 89, number of rules used = 9, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} =$

0.529, Rules used = {5291, 5293, 3297, 3303, 3298, 3301, 5292, 5290, 5281}

result too large to display

Antiderivative was successfully verified.

[In] Int[(x\*Cosh[c + d\*x])/(a + b\*x^3)^3,x]

[Out] 
$$\begin{aligned} & -\text{Cosh}[c + d*x]/(18*a*b^2*x^4) + (2*\text{Cosh}[c + d*x])/(9*a^2*b*x) - \text{Cosh}[c + d*x]/(6*b*x*(a + b*x^3)^2) + \text{Cosh}[c + d*x]/(18*b^2*x^4*(a + b*x^3)) - (2*(-1)^{2/3}*\text{Cosh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[((-1)^{1/3}*a^{1/3}*d)/b^{1/3} - d*x])/(27*a^{7/3}*b^{2/3}) + ((-1)^{1/3}*d^2*\text{Cosh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[((-1)^{1/3}*a^{1/3}*d)/b^{1/3} - d*x])/(54*a^{5/3}*b^{4/3}) + (2*(-1)^{1/3}*\text{Cosh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[-(((-1)^{2/3}*a^{1/3}*d)/b^{1/3}) - d*x])/(27*a^{7/3}*b^{2/3}) - ((-1)^{2/3}*d^2*\text{Cosh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[-(((-1)^{2/3}*a^{1/3}*d)/b^{1/3}) - d*x])/(54*a^{5/3}*b^{4/3}) - (2*\text{Cosh}[c - (a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(27*a^{7/3}*b^{2/3}) - (d^2*\text{Cosh}[c - (a^{1/3}*d)/b^{1/3}]*\text{CoshIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(54*a^{5/3}*b^{4/3}) - (2*d*\text{CoshIntegral}[(a^{1/3}*d)/b^{1/3} + d*x]*\text{Sinh}[c - (a^{1/3}*d)/b^{1/3}])/(27*a^2*b) - (2*d*\text{CoshIntegral}[((-1)^{1/3}*a^{1/3}*d)/b^{1/3} - d*x]*\text{Sinh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}])/(27*a^2*b) - (2*d*\text{CoshIntegral}[-(((-1)^{2/3}*a^{1/3}*d)/b^{1/3}) - d*x]*\text{Sinh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}])/(27*a^2*b) + (d*\text{Sinh}[c + d*x])/(18*a*b^2*x^3) - (d*\text{Sinh}[c + d*x])/(18*b^2*x^3*(a + b*x^3)) + (2*d*\text{Cosh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[((-1)^{1/3}*a^{1/3}*d)/b^{1/3} - d*x])/(27*a^2*b) + (2*(-1)^{2/3}*\text{Sinh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[((-1)^{1/3}*a^{1/3}*d)/b^{1/3} - d*x])/(27*a^{7/3}*b^{2/3}) - ((-1)^{1/3}*d^2*\text{Sinh}[c + ((-1)^{1/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[(a^{1/3}*d)/b^{1/3} - d*x])/(54*a^{5/3}*b^{4/3}) - (2*d*\text{Cosh}[c - (a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(27*a^2*b) - (2*\text{Sinh}[c - (a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(27*a^{7/3}*b^{2/3}) - (d^2*\text{Sinh}[c - (a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(54*a^{5/3}*b^{4/3}) - (2*d*\text{Cosh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(27*a^2*b) + (2*(-1)^{1/3}*\text{Sinh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(27*a^{7/3}*b^{2/3}) - ((-1)^{2/3}*d^2*\text{Sinh}[c - ((-1)^{2/3}*a^{1/3}*d)/b^{1/3}]*\text{SinhIntegral}[(a^{1/3}*d)/b^{1/3} + d*x])/(54*a^{5/3}*b^{4/3}) \end{aligned}$$

### Rule 5291

Int[Cosh[(c\_.) + (d\_.)\*(x\_.)]\*(x\_.)^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(x^(m - n + 1)\*(a + b\*x^n)^(p + 1)\*Cosh[c + d\*x])/(b\*n\*(p + 1)), x] + (-Dist[(m - n + 1)/(b\*n\*(p + 1)), Int[x^(m - n)\*(a + b\*x^n)^(p + 1)

```
) * Cosh[c + d*x], x], x] - Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

### Rule 5293

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5292

```
Int[(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*Sinh[(c_.) + (d_.)*(x_.)], x_Symbol] := Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```



Rule 5290

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Simp[(x^(m - n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*x])/(b*n*(p + 1)), x] +
(-Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] -
Dist[d/(b*n*(p + 1)), Int[x^(m - n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*x], x], x]) /;
FreeQ[{a, b, c, d}, x] && ILtQ[p, -1] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Rule 5281

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
:> Int[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /;
FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\int \frac{x \cosh(c+dx)}{(a+bx^3)^3} dx &= -\frac{\cosh(c+dx)}{6bx(a+bx^3)^2} - \frac{\int \frac{\cosh(c+dx)}{x^2(a+bx^3)^2} dx}{6b} + \frac{d \int \frac{\sinh(c+dx)}{x(a+bx^3)^2} dx}{6b} \\
&= -\frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2x^3(a+bx^3)} + \frac{2 \int \frac{\cosh(c+dx)}{x^5(a+bx^3)} dx}{9b^2} - \frac{d \int \frac{\sinh(c+dx)}{x^4(a+bx^3)} dx}{18b^2} \\
&= -\frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2x^3(a+bx^3)} + \frac{2 \int \left( \frac{\cosh(c+dx)}{ax^5} - \frac{b \cosh(c+dx)}{a^2x^2} + \frac{b^2x \cosh(c+dx)}{a^2(a+bx^3)} \right) dx}{9b^2} \\
&= -\frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} - \frac{d \sinh(c+dx)}{18b^2x^3(a+bx^3)} + \frac{2 \int \frac{x \cosh(c+dx)}{a+bx^3} dx}{9a^2} + \frac{2 \int \frac{\cosh(c+dx)}{x^5} dx}{9ab^2} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} - \frac{d^2 \cosh(c+dx)}{36ab^2x^2} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} + \frac{2d \sinh(c+dx)}{18ab^2x^3} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{d^2 \cosh(c+dx)}{108ab^2x^2} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} + \frac{2d \cosh(c+dx)}{18ab^2x^3} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} + \frac{d \sinh(c+dx)}{18ab^2x^3} + \frac{d^3 \sinh(c+dx)}{108ab^2x} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} + \frac{d^4 \cosh(c) \text{Chi}(dx)}{36ab^2} - \frac{2d^3 \sinh(c) \text{Chi}(dx)}{108ab^2} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} - \frac{d^4 \cosh(c) \text{Chi}(dx)}{108ab^2} - \frac{2d^3 \sinh(c) \text{Chi}(dx)}{108ab^2} \\
&= -\frac{\cosh(c+dx)}{18ab^2x^4} + \frac{2 \cosh(c+dx)}{9a^2bx} - \frac{\cosh(c+dx)}{6bx(a+bx^3)^2} + \frac{\cosh(c+dx)}{18b^2x^4(a+bx^3)} - \frac{2(-1)^{2/3} \cosh\left(c + \frac{\sqrt[3]{-1}}{\sqrt[3]{b}}\right)}{27a^{7/3}b^{2/3}}
\end{aligned}$$

**Mathematica [C]** time = 0.457546, size = 669, normalized size = 0.58

$$\text{RootSum}\left[\#1^3 b + a \&, \frac{-4\#1^2 b d \sinh(\#1 d + c) \text{Chi}(d(x - \#1)) + 4\#1^2 b d \cosh(\#1 d + c) \text{Chi}(d(x - \#1)) + 4\#1^2 b d \sinh(\#1 d + c) \text{Shi}(d(x - \#1)) - 4\#1^2 b d \cosh(\#1 d + c) \text{Shi}(d(x - \#1))}{(a + b x^3)^3}, x\right]$$

Antiderivative was successfully verified.

[In] Integrate[(x\*Cosh[c + d\*x])/(a + b\*x^3)^3,x]

[Out] (RootSum[a + b\*#1^3 & , (-a\*d^2\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)]) + a\*d^2\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] + a\*d^2\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] - a\*d^2\*Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + 4\*b\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)]\*#1 - 4\*b\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1]\*#1 - 4\*b\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1 + 4\*b\*Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1 + 4\*b\*d\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)]\*#1^2 - 4\*b\*d\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1]\*#1^2 - 4\*b\*d\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1^2 + 4\*b\*d\*Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1^2)/#1^2 & ] - RootSum[a + b\*#1^3 & , (a\*d^2\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)] + a\*d^2\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1] + a\*d^2\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] + a\*d^2\*Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)] - 4\*b\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)]\*#1 - 4\*b\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1]\*#1 - 4\*b\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1 - 4\*b\*Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1 + 4\*b\*d\*Cosh[c + d\*#1]\*CoshIntegral[d\*(x - #1)]\*#1^2 + 4\*b\*d\*CoshIntegral[d\*(x - #1)]\*Sinh[c + d\*#1]\*#1^2 + 4\*b\*d\*Cosh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1^2 + 4\*b\*d\*Sinh[c + d\*#1]\*SinhIntegral[d\*(x - #1)]\*#1^2)/#1^2 & ] + (6\*b\*Cosh[d\*x]\*(b\*x^2\*(7\*a + 4\*b\*x^3)\*Cosh[c] + a\*d\*(a + b\*x^3)\*Sinh[c]))/(a + b\*x^3)^2 + (6\*b\*(a\*d\*(a + b\*x^3)\*Cosh[c] + b\*x^2\*(7\*a + 4\*b\*x^3)\*Sinh[c])\*Sinh[d\*x])/(a + b\*x^3)^2)/(108\*a^2\*b^2)

**Maple [C]** time = 0.063, size = 810, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*cosh(d\*x+c)/(b\*x^3+a)^3,x)

[Out] 1/9\*d^6\*exp(-d\*x-c)/a^2/(b^2\*d^6\*x^6+2\*a\*b\*d^6\*x^3+a^2\*d^6)\*b\*x^5-1/36\*d^7\*exp(-d\*x-c)/a/(b^2\*d^6\*x^6+2\*a\*b\*d^6\*x^3+a^2\*d^6)\*x^3+7/36\*d^6\*exp(-d\*x-c)/a/(b^2\*d^6\*x^6+2\*a\*b\*d^6\*x^3+a^2\*d^6)\*x^2-1/36\*d^7\*exp(-d\*x-c)/b/(b^2\*d^6\*x

$$\begin{aligned} & ^6+2*a*b*d^6*x^3+a^2*d^6)-1/108*d/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+ \\ & b*c^3+4*_R1^2*b-2*_R1*b*c-2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(- \\ & _R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1 \\ & /108*d*c/a^2/b*sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp \\ & (-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3) \\ & )+1/9*d^6*exp(d*x+c)/a^2/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*b*x^5+1/36*d^7 \\ & *exp(d*x+c)/a/(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^3+7/36*d^6*exp(d*x+c)/a \\ & /(b^2*d^6*x^6+2*a*b*d^6*x^3+a^2*d^6)*x^2+1/36*d^7*exp(d*x+c)/b/(b^2*d^6*x^6 \\ & +2*a*b*d^6*x^3+a^2*d^6)-1/108*d/a^2/b^2*sum((_R1^2*b*c-2*_R1*b*c^2-a*d^3+b* \\ & c^3-4*_R1^2*b+2*_R1*b*c+2*b*c^2+4*_R1*b+6*b*c)/(_R1^2-2*_R1*c+c^2)*exp(_R1) \\ & *Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/1 \\ & 08*d*c/a^2/b*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp( \\ & _R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x^3+a)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 3.0848, size = 10103, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x^3+a)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/216*((8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x + c)^2 - 8* \\ & (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c)^2 - 4*(a*d^3/b)^( \\ & 2/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3})*(b^3*x^6 + 2*a*b^2*x^3 + a^ \\ & 2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3})*(b^3*x^6 \\ & + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((a*b^2*d^3*x^6 \\ & + 2*a^2*b*d^3*x^3 + a^3*d^3 + \sqrt{-3})*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a \\ & ^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + \sqrt{-3})* \\ & (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh(d*x + c)^2))*Ei(d*x \end{aligned}$$



$$\begin{aligned}
& /3)*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + \sqrt{-3}*(a*b^2*d^3*x^6 + \\
& 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3 \\
& *x^3 + a^3*d^3 + \sqrt{-3}*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\sinh \\
& (d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\sinh(1/2*(a*d^3/ \\
& b)^{(1/3)}*(\sqrt{-3} + 1) + c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^ \\
& 3)*\cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x \\
& + c)^2 + 4*(-a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - \sqrt{-3}*(b^ \\
& 3*x^6 + 2*a*b^2*x^3 + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^ \\
& 2*b - \sqrt{-3}*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) - (-a*d^3/ \\
& b)^{(1/3)}*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 + \sqrt{-3}*(a*b^2*d^3*x \\
& x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2* \\
& b*d^3*x^3 + a^3*d^3 + \sqrt{-3}*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)) \\
& *\sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1))*\sinh(1/2* \\
& (-a*d^3/b)^{(1/3)}*(\sqrt{-3} + 1) - c) - (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 \\
& + a^3*d^3)*\cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)* \\
& \sinh(d*x + c)^2 - 4*(a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + \sqrt{- \\
& 3}*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x \\
& ^3 + a^2*b + \sqrt{-3}*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) - ( \\
& a*d^3/b)^{(1/3)}*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - \sqrt{-3}*(a*b^ \\
& 2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + \\
& 2*a^2*b*d^3*x^3 + a^3*d^3 - \sqrt{-3}*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3 \\
& *d^3))*\sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1))*\sinh( \\
& 1/2*(a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) - c) + (8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x \\
& ^3 + a^3*d^3)*\cosh(d*x + c)^2 - 8*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^ \\
& 3)*\sinh(d*x + c)^2 + 4*(-a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + s \\
& qrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b \\
& ^2*x^3 + a^2*b + \sqrt{-3}*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*\sinh(d*x + c)^2) \\
& - (-a*d^3/b)^{(1/3)}*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - \sqrt{-3})* \\
& (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3))*\cosh(d*x + c)^2 - (a*b^2*d^3*x \\
& ^6 + 2*a^2*b*d^3*x^3 + a^3*d^3 - \sqrt{-3}*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 \\
& + a^3*d^3))*\sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) \\
& )*\sinh(1/2*(-a*d^3/b)^{(1/3)}*(\sqrt{-3} - 1) + c) + 2*(4*(a*b^2*d^3*x^6 + 2*a \\
& ^2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^ \\
& 3 + a^3*d^3)*\sinh(d*x + c)^2 - 4*(-a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + \\
& a^2*b)*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*\sinh(d*x + c)^2) \\
& + (-a*d^3/b)^{(1/3)}*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x + \\
& c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c)^2))*Ei(-d* \\
& x + (-a*d^3/b)^{(1/3)}*\sinh(c + (-a*d^3/b)^{(1/3)}) - 2*(4*(a*b^2*d^3*x^6 + 2* \\
& a^2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x + c)^2 - 4*(a*b^2*d^3*x^6 + 2*a^2*b*d^3*x \\
& ^3 + a^3*d^3)*\sinh(d*x + c)^2 + 4*(a*d^3/b)^{(2/3)}*((b^3*x^6 + 2*a*b^2*x^3 + \\
& a^2*b)*\cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*\sinh(d*x + c)^2) \\
& + (a*d^3/b)^{(1/3)}*((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\cosh(d*x + c \\
& )^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c)^2))*Ei(d*x \\
& + (a*d^3/b)^{(1/3)}*\sinh(-c + (a*d^3/b)^{(1/3)}) - 12*(4*a*b^2*d^2*x^5 + 7*a^2 \\
& *b*d^2*x^2)*\cosh(d*x + c) - 12*(a^2*b*d^3*x^3 + a^3*d^3)*\sinh(d*x + c))/((a
\end{aligned}$$

$$^3*b^3*d^2*x^6 + 2*a^4*b^2*d^2*x^3 + a^5*b*d^2)*\cosh(d*x + c)^2 - (a^3*b^3*d^2*x^6 + 2*a^4*b^2*d^2*x^3 + a^5*b*d^2)*\sinh(d*x + c)^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x\*\*3+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \cosh(dx + c)}{(bx^3 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*cosh(d\*x+c)/(b\*x^3+a)^3,x, algorithm="giac")

[Out] integrate(x\*cosh(d\*x + c)/(b\*x^3 + a)^3, x)





# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71     else # result do not contain complex
72         # this assumes optimal do not as well
73         if debug then
74             print("result do not contain complex, this assumes optimal do not
as well");
75         fi;
76         if leaf_count_result<=2*leaf_count_optimal then
77             if debug then
78                 print("leaf_count_result<=2*leaf_count_optimal");
79             fi;
80             return "A";
81         else
82             if debug then
83                 print("leaf_count_result>2*leaf_count_optimal");
84             fi;
85             return "B";
86         end if
87     end if
88     else #ExpnType(result) > ExpnType(optimal)
89         if debug then
90             print("ExpnType(result) > ExpnType(optimal)");
91         fi;
92         return "C";
93     end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+^') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```

```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```



```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

## 4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```

```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183                 else:
184                     return "B"
185             else: #result contains complex but optimal is not
186                 return "C"
187         else: # result do not contain complex, this assumes optimal do not as
188             well
189                 if leaf_count_result <= 2*leaf_count_optimal:
190                     return "A"
191                 else:
192                     return "B"
193         else:
194             return "C"
```